Simulation for Behavior of Global and Local Coupling Instability in Sandwich Plate

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Abstract. In recent decades, sandwich plates have been widely used in aerospace, ship and other fields. Buckling instability is one of the most common failure modes for such structures under in-plane loading. This paper mainly includes employing higher order hierarchical theory as the deformation assumption of sandwich plate, building macroscopic displacement field envelope with Fourier series, and establishing macroscopic model of sandwich plate. The nonlinear system is solved by asymptotic numerical method. Then the model is used to simulate coupling phenomena of global buckling and local wrinkling, and study the influence of different geometric parameters and material parameters on buckling phenomena.

Introduction

Buckling failure is one of the most common failure modes for such structures under compressive loads. Under the load of in-plane compression, instability phenomenon (global buckling and local wrinkling) of sandwich structures has attracted much attention. For instability modes, many scholars have been studying the global buckling and local wrinkling of sandwich plates in the past few years [1-3]. Based on the elastic bifurcation theory in three-dimensional (3D) model, Grognec et al. proposed an analytical expression [4, 5] for the critical load value of sandwich plate and the corresponding instability mode recently.

At present, researchers can accurately and quickly describe the coupling effect of global buckling and local wrinkling in the instability problem of the sandwich beam [6] and the local wrinkling of the sandwich plate [7]. This paper will study the modeling and simulation of the high order instability mode of the sandwich plate, utilize the multi-scale modeling and simulation method which takes account of calculation efficiency and precision, and consider the coupling effect of two kinds of instability modes. The model of board/entity/plate built by ABAQUS is used as a reference to verify the validity of two-dimensional (2D) Fourier macroscopic model.

Basic Method

Slowly Variable Fourier Coefficient Method

Based on the 2D microcosmic model of sandwich plate proposed by Yu et al. [8], a 2D macroscopic model of sandwich panels is constructed by using the slowly variable Fourier coefficient method, which is to convert the solving unknowns of displacement field (approximate periodic and fast changes) into solving slowly variable unknowns of a 2D macroscopic model by Fourier transform. In this paper, we restrict the generation of fast oscillations in the X direction only, so the assumed displacement function $U(x, y)$ can be expanded into following forms:

$$U(x, y) = \sum_{j=-\infty}^{\infty} U_j(x, y)e^{jsx}$$

(1)
It is shown that the periodic and fast variation is displacement field of the microcosmic model, and the coefficients of Fourier series are the envelopes of the $j$-th harmonic, which are the unknowns of displacement field of 2D macroscopic model.

As shown in Figure 1, this section discusses three macroscopic displacement fields in 2D macroscopic model: $U_0(x, y)$, $U_1(x, y)$ and $U_{-1}(x, y)$. In order to solve the black solid line $U(x, y)$ of the microcosmic displacement field, which is approximately periodic and fast change, the problem can be converted to the following two macroscopic displacement fields by the slowly variable Fourier coefficient method: the red dotted line $U_0(x, y)$ represents macroscopic mean field, the blue dotted line $U_1(x, y)$ and $U_{-1}(x, y)$ are the first order envelope field.

![Figure 1. Schematics of the Fourier expansion.](image)

**Asymptotic Numerical Method**

Combining perturbation method and finite element method, asymptotic numerical method is applied to solve the nonlinear system of sandwich plate model. This method transforms the nonlinear equations into a series of linear equations by means of power series, and the tangent matrix of these linear equations is consistent in each step, in other word, the inverse of a matrix is needed in every step only.

It is assumed that the nonlinear equation can be expressed as $R(V; \lambda) = 0$, where $V$ is an unknown vector, which varies with the change of path parameters $\lambda$. Assuming that solution $(V, \lambda)^j$ of $j$-th is known, the solution of $(j+1)$-th can be obtained by Taylor series expansion:

$$V^{j+1} = V^j + a_1 V_1 + a_2^2 V_2 + \ldots + a^n V_n = V^j + \sum_{p=1}^{n+1} a^n V_p$$

(2)

**Numerical Results**

This model will be used to simulate the global buckling and local wrinkling of instability of sandwich plates. In this chapter, a 2D macroscopic model of rectangular sandwich plate is used for the compression test at both ends. The failure modes and critical loads of this test will be compared with those of the 3D finite element method. The result in Figure 2 demonstrates that the buckling load of 3D finite element model is basically consistent with the one predicted by 2D macroscopic model.

Figure 3 shows the global deformation graph coupled with the global buckling and the local wrinkling in 2D macroscopic model. It can be seen that when coupling instability is produced in the sandwich plate, it leads to a large disturbance on the whole and wavy wrinkles near the middle. The results indicate that although the degree of freedom of macroscopic model is only 0.4% of the one used in the ABAQUS 3D finite element model, it can still simulate the coupling between global buckling and local wrinkling more accurately.
The influence of some physical parameters on coupling phenomena is also analyzed. The larger ratio of length to width and ratio of elastic modulus in the inner to outer layer, the sooner sandwich plate will cause the global buckling. When the wave number reflecting the instability mode of local wrinkling of sandwich plate reaches 35, the local wrinkle occurs first. Corresponding charts are omitted for brevity.

**Conclusion**

This paper uses slowly variable Fourier coefficient method to construct a new 2D Fourier model for studying the instability of sandwich plate. Asymptotic numerical method is introduced to simulate the coupling instability of global buckling and local wrinkling in the plate under uniaxial compression. Calculation results of this model and the 3D numerical value in the ABAQUS are compared.

The results illustrate that the instability mode, critical load and post buckling behavior of 2D macroscopic model can be in good agreement with the data simulated by the finite element model. And calculation efficiency of the macroscopic model is greatly improved compared with the numerical model. Moreover, the simulation results of 2D macroscopic models are insensitive to disturbances and strong in numerical stability. Therefore, the application of this model in simulating the instability can provide theoretical support and technical tools for the development of reasonable preventive measures. It will also provide reference for problems of other instability with characteristics of periodic change.

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**References**


