Amplification and Filtering of Shear Horizontal Waves Using Flexural Resonance-Based Metamaterials

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ABSTRACT

We investigate the use of a flexural resonance based metamaterial to focus shear-horizontal modes in a thin-walled structure. Shear horizontal waves have many advantages for structural heath monitoring, but their low amplitude and in-plane displacements can make them difficult to detect with sufficient signal-to-noise ratios. Through simulations, we demonstrate that resonance based elastic metamaterials can be used to amplify these modes prior to capture.

INTRODUCTION

Guided waves are highly effective tools to detect damage in thin-walled structures [1]. Large areas of structures can be covered with a few sensor/actuator pairs. However, attenuation of the propagating waves can reduce the signal-to-noise ratio of the collected signal. For example, composite structures have inherent material damping properties higher than metallic structures. Therefore, it is often necessary to amplify the guided waves prior to detection. Recently, researchers have explored the use of phononic metamaterial based lens for this purpose [2, 3]. These metamaterials function as a graded-index (GRIN) lens and manipulate the local velocity of the guided waves.

To augment the change in local velocity of the guided wave, and thus increase the focusing factor of the metamaterial, researchers have also designed resonance based metamaterials. By exploiting the resonance modes of the unit cell, the local change in velocity is drastically increased [4]. The resonance mode interacts with the dispersion behavior of the unit cell to create a hybridized mode. With the goal of manipulating the antisymmetric modes (A) in the structure, the resonance condition is based on the longitudinal resonance mode of the unit cell, which couples strongly to the A modes.

Alternatively, the A and S modes also weakly couple with the flexural resonance modes of a unit cell [5]. However, this coupling has a high quality factor (Q factor) in the dispersion behavior. Additionally, the flexural resonance often occurs a lower frequencies, which can be used to avoid the first $A_{\tilde{n}}$ bandgap. Rahman et al [6] demonstrated that the flexural resonance condition can be applied to focus A modes in a plate

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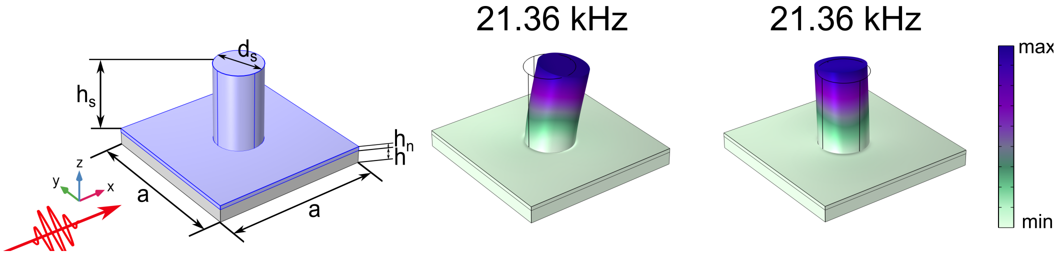

Figure 1. (left) Unit cell for elastic metamaterial to focus SH waves in an aluminum plate. a = 10 mm, h = 0.84 mm, h_n = 0.254 mm, h_s = 5 mm, d_s = 3 mm.
(right) First flexural resonance mode for the unit cell.

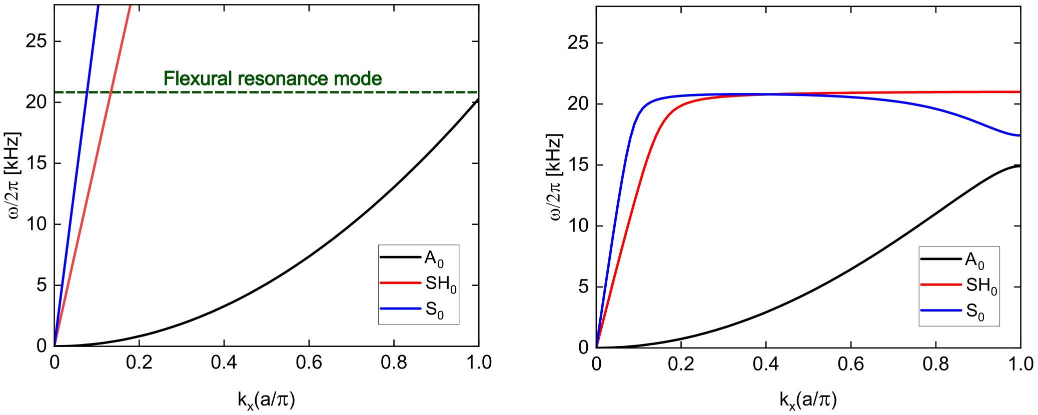

Figure 2. (left) Calculated dispersion curves for A_0, S_0, and SH_0 modes in aluminum plate. Frequency of flexural resonance mode from figure 1 is also indicated. (right) Calculated dispersion curves for same modes for plate with metamaterial, i.e. unit cell from figure 1.

with a relatively high amplification of the signal. In this paper, we investigate the use of the flexural resonance based metamaterial to focus shear-horizontal (SH) modes in the structure. SH modes are non-dispersive and have no out-of-plane particle displacement, therefore they are less susceptible to the surrounding environment [7]. Therefore, the SH modes can be excellent inspection tools for structures.

HYBRIDIZATION OF RESONANCE MODES WITH S_0, SH_0, AND A_0 MODES

The metamaterial will be used to focus SH_0 modes in a thin aluminum plate in this example. Figure 1 shows the unit cell of the metamaterial to be used in this paper. It consists of a polymer (Hyper PLA) base layer and pillar, superimposed on the aluminum plate. The unit cell was simulated using the acoustics module in COMSOL, with periodic boundary conditions. For a pillar height of 5mm, the first flexural resonance occurs at 21.36 kHz. Figure 2 plots the dispersion curves for the aluminum plate and the metamaterial unit cell.

There is a clear difference between the two cases. The presence of the metamaterial creates hybridized modes with each of the different modes (A, S, SH), however at

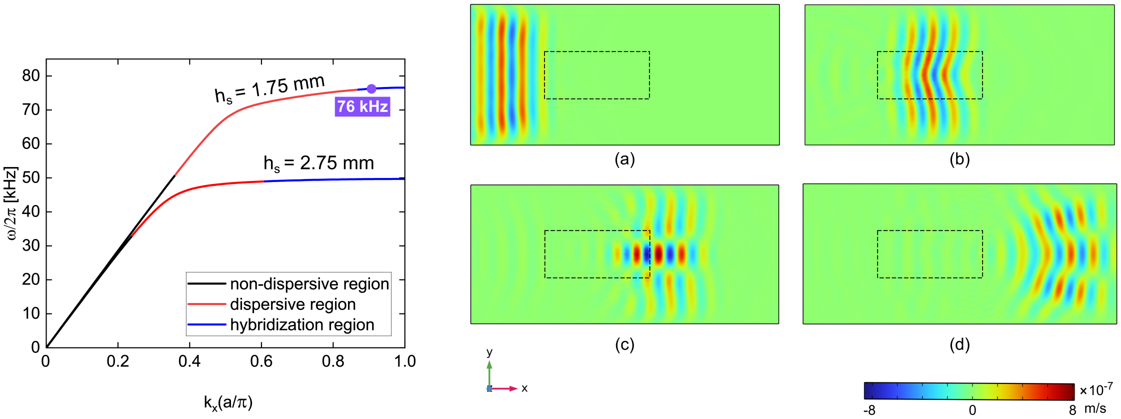

Figure 3. (left) Calculated dispersion curves for two pillar heights for SH_0 mode in aluminum plate. (right) SH_0 mode propagation through plate at (a) 59, (b) 117, (c) 160, and (d) 204 μs. Black rectangle indicates location of metamaterial.

different frequencies. The different forms of the curves are primarily due to the fact that the SH and S modes are non-dispersive in this frequency range, while the A is strongly dispersive. However, all three cases are strongly affected by the metamaterial, and therefore the local velocity of the modes will be altered by the unit cell.

FOCUSING OF SH_0 MODE

An elastic metamaterial was simulated by a distribution of unit cells, varying the height of the pillar, h_s, to create a lens like behavior. The pillar height was varied from 0.15 mm to 1.75 mm and back to 0.15 mm across a single row and 20 columns were simulated in the metamaterial. The simulations were performed using the acoustic module in COMSOL. Figure 3 plots the predicted dispersion curve for the central pillar and the SH_0 mode propagation simulation results for the input SH_0 mode at 76 kHz. The SH_0 can clearly be seen to focus to a location near the right edge of the metamaterial. The mode then diverges after passing through the metamaterial. Although, not presented here, the degree of focusing in this example was stronger than that in other simulations away from the resonance condition.

CONCLUDING REMARKS

Resonance based metamaterials can focus propagating guided modes in thin-walled structures for amplification of the modes prior to detection. As processing of structural health information from these guided modes is often dependent on the accurate measurements of the waveforms of these modes, the use of these metamaterials can potentially improve the quality of the structural health assessments. Prior works have demonstrated such focusing for A and S modes; this paper adds the capability for SH modes, extending their applications in the domain of structural health monitoring. Future studies should address the optimization of these devices.

ACKNOWLEDGMENT

This work was supported by the Office of Naval Research under Grant No N00014-22-1-2365.

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