Influence of Lamb Wave and Structural Parameters on Damage Localization: A Comprehensive Analysis Using Numerical Simulations

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ABSTRACT

The structural health monitoring (SHM) field extensively uses Lamb waves for defect detection due to their extended propagation capability and sensitivity to various damage types. However, the Lamb wave exhibits a few limitations caused by its multimode characteristics and intrinsic dispersion behaviour. It is challenging to localize structural defects due to the presence of many dispersive modes at a particular frequency band. Varying the central frequency of a Lamb wave across its dispersion curve range for a specific structural thickness is investigatedhere to study its behaviour in damage localization. Numerical simulations using the finite element method in three dimensions show fifteen aluminium plate analyses in the time domain. A through-thickness hole is created as a defect in the plate. The least dispersed symmetric Lamb mode (S0) with an increasing central frequency in the specified range is actuated in each scenario. The output of out-of-plane signals is utilized in damage location imaging. Lamb wave frequency accuracy varies significantly, as seen in the images of the damage areas. The damage area grows as the central frequency of the actuation signal increases. The precise location is estimated in each of the fifteen cases; however, the projected damage zone differs. According to the imaging study, damage localization is improved using low-frequency Lamb modes.

INTRODUCTION

Guided waves are quickly becoming a popular tool for structural health monitoring (SHM) and non-destructive testing & evaluation (NDT&E) by the significant areas they examine, requiring minimal contact with the structure. The guided waves that propagate within thin-walled structures are referred to as Lamb waves (LWs) [1], [2]. A diverse group of researchers made significant contributions to the field of SHM and NDT&E

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through the application of Lamb wave–based methods [3], [4]. While the LW–based methods show progressive results in identifying and locating damages in structural components, there are still several complex issues due to the intricacy of their multimodal behaviour. A minimum of two fundamental modes, namely the symmetric mode (S0) and the antisymmetric mode (A0), coexist simultaneously at any frequency of LWs. As the frequency and plate thickness increase, other Lamb wave modes, such as S1 and A1, become apparent. Different wave modes propagate at different velocities, making the interpretation of Lamb wave signals highly challenging. Typically, LW-based SHM applications utilize narrowband excitation. The frequency is carefully selected to ensure that the frequency and plate thickness product is below the cutoff value decided from the dispersion curve. This restriction allows for the existence of only two fundamental modes within the selected frequency band.

Lamb waves are a type of guided waves that travel between two parallel surfaces, such as those seen in plate structures. Numerous textbooks and literature provide comprehensive information on the basics of Lamb waves [5], [6], [7]. The Lamb waves are classified into symmetric modes (S0, S1, S2, …) and antisymmetric modes (A0, A1, A2, …) based on the surface particle motion relative to the mid-plane. These modes are mathematically defined by the Rayleigh-Lamb equations. The group velocity dispersion curves of various modes are also computed using the equations. These dispersion curves demonstrate the existence of several modes of Lamb waves in the structures and their velocity's reliance on frequency. It is evident that there are specific frequencies at which all modes, except for the two fundamental A0 and S0 modes, are no longer present. In our work, we analyzed both the velocity–frequency curves and the velocity–frequency·thickness curve.

Numerous studies widely explored and utilized guided modes in the 100-300 kHz frequency band. For instance, Köhler et al. examined the interaction between Lamb waves and drilled holes in an aluminium plate, exploring the potential for accurately determining their size [8], [9]. Chandrasekaran et al. developed a technique called Higher Order Mode Cluster (HOMC) that utilizes Lamb modes in the frequency·thickness range of around 20 MHz·mm to identify pitting and corrosion in plates [10]. Their approach offers the advantage of producing a wave packet that is nearly non-dispersive, resulting in excellent spatial resolution. In this work, the authors have varied the Lamb wave central frequency and the plate thickness in individual cases to study their effect on the efficacy of the damage localization imaging algorithm. The investigation uses numerical simulations of plate structure with a single defect at a fixed location for each analysis.

METHODOLOGY

Thirty models (fifteen pristine state and fifteen defective state) are simulated in Abaqus and studied to comprehensively analyze the effect of the structural parameter (thickness) and Lamb wave parameter (central frequency) variations [11]. The models are square plate-like structures for the isotropic aluminium alloy, whose properties are listed in Table I. The list of various models with their specifications (thickness and LW frequency) is summarized in Table II. An array of thirty-six transducer points is marked on the surface of the plate around its centre for LW actuation and data acquisition purposes. At one of these points, the LW is actuated, while the

TABLE I. PHYSICAL PROPERTIES OF ALUMINIUM

Physical properties	**Value**
Elastic modulus	68.9 GPa
Density	2710 kg/m^3
Poisson's ratio	0.33

TABLE II. PARAMETERS OF LAMB WAVE AND STRUCTURE

Model	**Frequency (kHz)**		**Model**	**Thickness (mm)**		**Model**	**Frequency · Thickness = 0.2 MHz·mm**
1.	100		**6.**	1		**11.**	25 kHz, 8 mm
2.	200		**7.**	2		**12.**	50 kHz, 4 mm
3.	300		**8.**	3		**13.**	100 kHz, 2mm
4.	400		**9.**	4		**14.**	200 kHz, 1mm
5.	500		**10.**	5		**15.**	400 kHz, 0.5 mm

propagated waves are sensed and recorded at all the other points. The process is repeated until all the points are considered actuation points. This procedure is repeated for all the models for data acquisition.

FE Simulation

The focus structure is a square plate with an edge dimension of 1000 mm. A through-hole of 20 mm radius is created at (25 mm, 25 mm) from the centre of the plate as a defect (Figure 1). An actuation signal is excited by a one-cycle Hanning windowed tone-burst (Figure 2). As per the dispersion curve, only A0 and S0 modes are present below 1 MHz. As the frequency increases, other wave modes, such as A1, S1, A2, and S2, become apparent (Figure 3). The parameter variations used for each analysis are mentioned below:

- In simulations for the first analysis, the thickness of the plate is set at 2 mm. The actuation signal is a tone burst with central frequencies 100 kHz, 200 kHz, …, and 500 kHz.
- Conversely, for the second analysis, the thickness of the plate is varied from 1 mm, 2 mm, …, and 5 mm, keeping the actuation signal central frequency set at 100 kHz.
- The third analysis is needed after the results from the previous two studies, where the LW signal with 100 kHz on a 2 mm plate showed the best result in both cases. Hence, the third analysis consists of cases where both the central frequency of LW and the thickness of the plate vary in proportion, such that their product is 0.2 MHz·mm.

Almost 2 million elements are formed in each case, and the history output is requested for all thirty-six sensing points.

Signal Processing for Damage Localization

After all the data acquisition is done, Matlab is used for data processing and imaging algorithms [12]. The following steps are followed for the damage localization using the imaging algorithm:

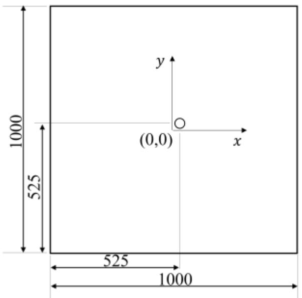

Figure 1. Plate with a hole defect at (25, 25)

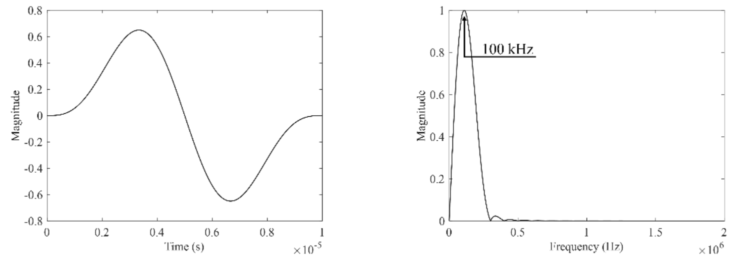

Figure 2. LW actuation signal with one cycle and 100 kHz central frequency

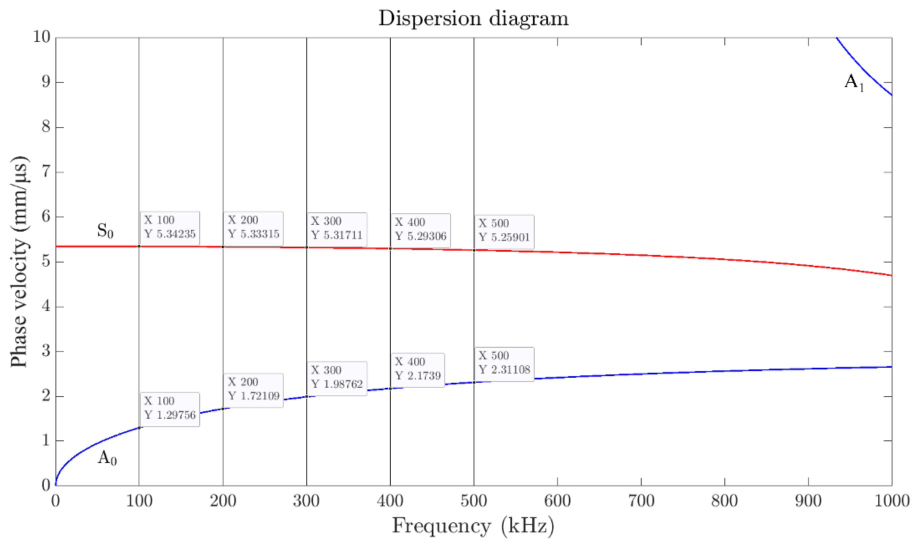

Figure 3. Dispersion curve for a 2 mm plate showing wave velocity at varying central frequencies of LW from 100 kHz to 500 kHz

- The first step for imaging is determining the residual signal, which is obtained by subtracting the pristine condition signal from the current defective condition signal. The residual signal is calculated as, $Residual\ signal\ (RS(t)) = Defective\ state\ signal\ (t) - Pristine\ state\ signal(t)$. Any fluctuation in the obtained residual signal indicates the presence of a defect in the structure. Once a defect is identified, the next step is to localize the defect.
- The time of arrival and time of flight for each actuator-sensor pair is calculated using the following formulae:

$$t_{AP} = \frac{D_{AP}}{v} = \frac{\sqrt{(x_P - x_A)^2 + (y_P - y_A)^2}}{v} \quad (1)$$

$$t_{PS} = \frac{D_{PS}}{v} = \frac{\sqrt{(x_S - x_P)^2 + (y_S - y_P)^2}}{v} \quad (2)$$

$$t_{AS} = \frac{\sqrt{(x_P - x_A)^2 + (y_P - y_A)^2} + \sqrt{(x_S - x_P)^2 + (y_S - y_P)^2}}{v} \quad (3)$$

where t_{AP} is the arrival time of the wave from an actuator A to a random point P on the plate, t_{PS} is the arrival time of the wave from the point P to the sensor S, t_{AS} is the time of flight from the actuator A to the sensor S, v is the wave velocity, and D_{MN} denotes the geometric distance between M and N (M, N: A, P, S).

- The plate is divided into small grids such that each grid P(x,y) denotes the probable location of the defect. The intensity of the signal is obtained at each grid point for each actuator-sensor pair using the corresponding time of flight value. The point at which the highest intensity value is obtained denotes the defect location.

The algorithm is repeated for each of the fifteen models to locate the damage. The results obtained are discussed in the following section.

RESULTS AND DISCUSSION

The damage imaging is done using the above-explained algorithm for localizing the defect. The algorithm's effectiveness for a particular case depends on the location of the highest intensity in the plate and the area covered by the whole plate.

Influence of the Lamb Wave Frequency

The imaging results for the first five defective models are presented in Figure 4. From the obtained images for localized defect, it is observed that as the central frequency of the LW increases, the localized defect area increases up to a specific value (Figure 4(a), 4(b)) and then remains constant (Figure 4(d), 4(e)). It is also observed that in all cases, the algorithm showed the accurate location of the defect, varying only in the localized area. Out of all the cases in the first analysis, the case having a 2 mm plate with 100 kHz central frequency gives the best results for damage localization where the highest intensity values are well within the defect area, as represented by a black circle of radius 20 mm.

Influence of Structural Thickness

The imaging results for the following five defective models are presented in Figure 5. From the obtained images for localized defect, it is observed that when the plate is very thin (1 mm), the intensity is also higher at the boundaries, depicting the fact that the LW is being reflected from the structure boundary (Figure 5(a)). As the plate thickness increases, the localized defect area remains constant (Figure 5(b)-5(e)). It is also observed that in all cases, the algorithm showed the accurate location of the defect. Out of all the cases in the second analysis, the case having an LW central frequency of 100 kHz with a plate thickness of 2 mm to 5 mm gives the best results for damage localization where the highest intensity values are well within the defect area.

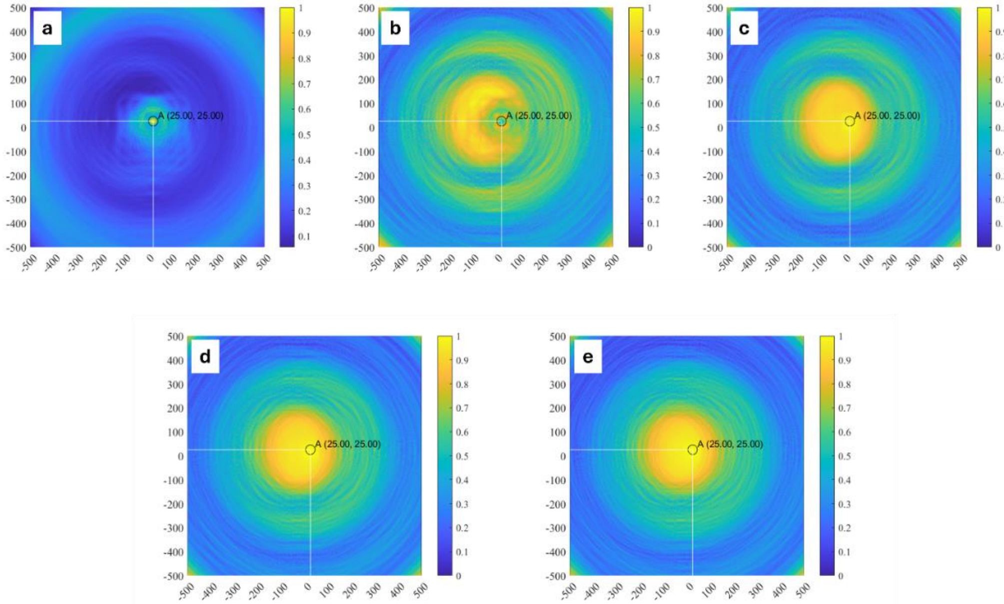

Figure 4. Images showing localized positions of damage exhibited by higher values of intensity and the actual position of damage represented by a black circle, with varying central frequencies of LW: (a) 100 kHz, (b) 200 kHz, (c) 300 kHz, (d) 400 kHz, and (e) 500 kHz.

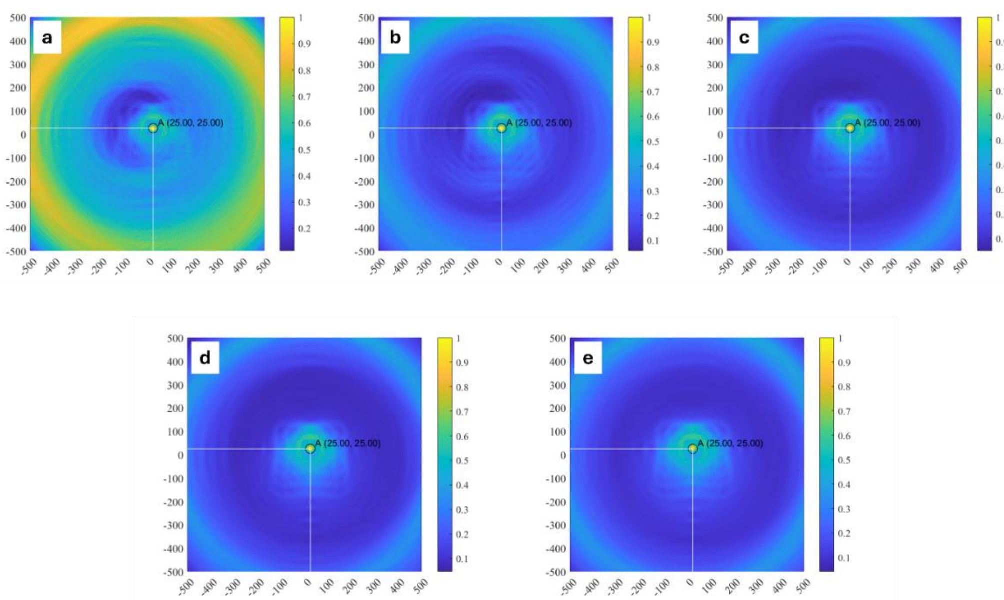

Figure 5. Images showing localized positions of damage exhibited by higher values of intensity and the actual position of damage represented by a black circle, with varying thickness of plate structure: (a) 1 mm, (b) 2 mm, (c) 3 mm, (d) 4 mm, and (e) 5 mm.

Effect of both thickness and frequency variation such that frequency· thickness is constant

It is well observed from the previous results that the common scenario in both analyses, which gives the best localization result, is the one with a 2 mm plate thickness and an actuation signal with 100 kHz central frequency. The product of frequency· thickness results in 0.2 MHz·mm. This result compelled the authors to conduct a third analysis where the product of frequency· thickness is kept constant. This is done by varying both the frequency and plate thickness in proportion such that their product is 0.2 MHz·mm. The thickness is increased from 0.5 mm to 8 mm while the central frequency is decreased from 400 kHz to 25 kHz.

The imaging results for the five cases for the third analysis are presented in Figure 6. From the obtained images for localized defect, it is observed that when the plate is very thin (0.5 mm), and the LW central frequency is high (400 kHz), the defect area enclosed by the intensity is also larger (Figure 6(a)). As the plate thickness increases and the central frequency decreases, the localized defect area goes beyond the actual defect location. However, its centre remains at the same position (Figure 6(b)-6(e)). Again, out of all the cases in the third analysis as well, the case having an LW central frequency of 100 kHz with a plate thickness of 2 mm gives the best results for damage localization where the highest intensity values are well within the defect area.

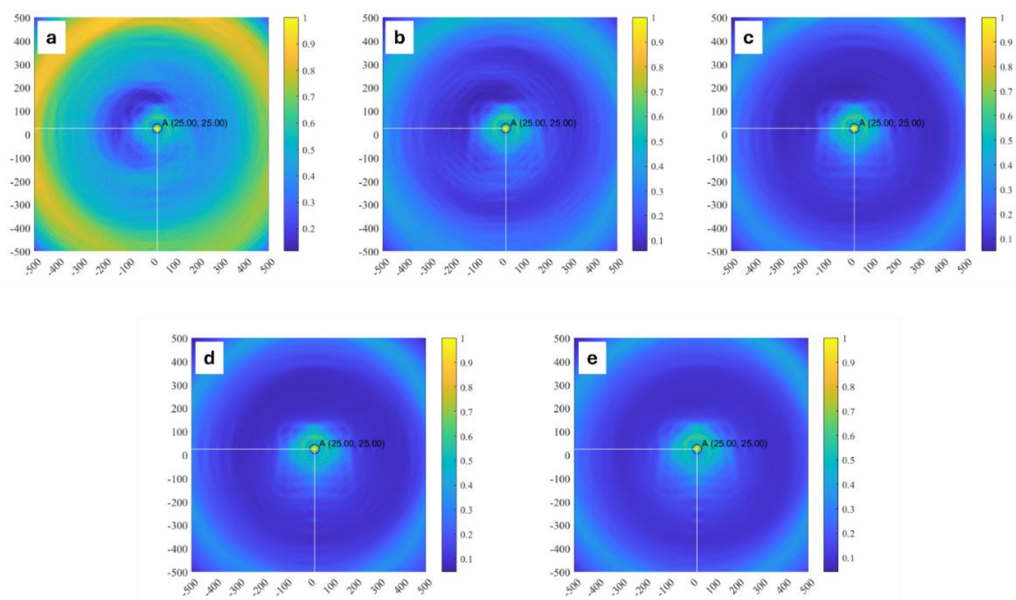

Figure 6. Images showing localized positions of damage exhibited by higher values of intensity and the actual position of damage represented by a black circle, keeping frequency·thickness = 0.2 MHz·mm for the plate structure: (a) 0.5 mm with 400 kHz, (b) 1 mm with 200 kHz, (c) 2 mm with 100 kHz, (d) 4 mm with 50 kHz, and (e) 8 mm with 25 kHz.

CONCLUSION

The study of structural and wave parameter variations showed significant results in determining the central frequency of Lamb wave for SHM and NDT&E purposes. The following points are concluded from the study:

1) *Frequency variation*: Though all the cases showed accurate localization of the defect for the 2 mm plate thickness, the case with 100 kHz central frequency presented the best localization result regarding damage quantification.

2) *Thickness variation*: All the cases considered showed accurate defect localization for the 100 kHz frequency signal. The algorithm is suitable for use in damage imaging for the thicker plates at the specified frequency of 100 kHz as they give almost the same localization area.

3) *Constant frequency·thickness*: Apart from the lower frequency signal (25 and 50 kHz), all other cases showed accurate defect localization for 0.2 MHz·mm. The case with a 100 kHz central frequency and a 2 mm plate thickness presented the best localization result in terms of quantification.

It is concluded that the used algorithm is best suitable for the intermediate central frequency of the Lamb wave where the phase velocity of the fundamental symmetric mode (S0) is almost constant while that for the fundamental antisymmetric mode (A0) is in the moderate range. It is hence determined that SHM using Lamb waves at low frequencies can identify relatively significant defects.

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