Improving Hybrid Model Accuracy for Structural Analysis: The Effects of Parameterized Joint Stiffness on System Equivalent Model Mixing

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ABSTRACT

System equivalent model mixing (SEMM) is a hybrid model that integrates experimental data and numerical analysis to obtain an unmeasured degree-of-freedom (DoF). SEMM extends the frequency response functions obtained from experimental data to a finite element method (FEM) model, resulting in a highly accurate and precise hybrid model that enables the measurement of joint behavior and characteristics. By leveraging the precision of experimental data and the expansion of numerical models, SEMM can acquire frequency response functions from inaccessible joints. This approach provides researchers with more complete information on joint behavior and characteristics than traditional modeling approaches that rely solely on either experimental or numerical data, allowing for more accurate and comprehensive analysis. A common method for implementing SEMM involves using a virtual point transformation (VPT) that accounts for the rotational degrees of freedom of the joint by employing three tri-axial sensors near the joint, or by attaching a transmission simulator (TS) to the joint to mitigate the influence of rotational degrees of freedom. However, the accuracy of the expansion DoF using SEMM is determined not only by the initial boundary condition of the analytical model but also by the boundary condition error between the experiment and the analytical model. Therefore, the accurate establishment of system boundary conditions in the numerical model is crucial for SEMM modeling to improve the reliability and correctness of the resulting hybrid model. This paper describes the development of hybrid models for structural analysis by incorporating parameterized joint stiffness to optimize boundary conditions in numerical models and enhance the accuracy of the resulting hybrid models. The accuracy of the proposed approach based on SEMM is evaluated using the frequency response assurance criterion (FRAC). The results show that the FRAC is sensitive to changes in boundary stiffness and that the accuracy of FRFs is directly proportional to the similarity between the numerical and experimental boundaries. To further investigate the impact of the measured DoF, we applied classical sequential sensor placement optimization techniques to select the optimal DoF in the experiment and evaluated their performance accordingly.

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INTRODUCTION

During system analysis and identification, both experimental systems and analytical models may exhibit various errors arising from multiple factors, such as measurement inaccuracies and model biases. One notable type of error is systematic errors, including non-uniform geometry, which possess characteristics that are challenging to capture entirely in the analysis model. Nonetheless, unlike experiments, analytical models offer an abundance of degree-of-freedom (DoF), allowing for the estimation of dynamic behaviors at unmeasurable points. In essence, experiments and analysis are mutually complementary, requiring the use of hybrid measurement and estimation techniques that effectively combine both experimental and analytical results.

System Equivalent Model Mixing (SEMM) is a hybrid dynamic measurement technique classified under the category of direct updating methods. It aims to minimize the discrepancy between the actual structure and the analytical model, thereby expanding the full analytical DoF beyond the limitations of experimental DoF [1-7]. SEMM utilizes the Lagrange multiplier frequency-based substructuring (LM-FBS) method to effectively capture the dynamic behavior of assembled substructures using the compatibility and the force equilibrium equation at the interface.

Various studies have been conducted utilizing SEMM, mainly for the purpose of joint identification [3], expansion to rotational DoF [4], and enhancement of SEMM model accuracy [5, 6]. The identification of joints in assembled structures, such as bolted or welded joints, poses a challenge for analytical modeling. Consequently, joint identification is typically performed using iterative methods to minimize discrepancies between experimental and analytical results [3]. Furthermore, SEMM enables the estimation of rotational DoF, which is difficult to directly measure in experiments. In order to accommodate for additional rotational DoF, SEMM can be utilized with the virtual point transformation (VPT) technique [4]. In addition, studies have been conducted to mitigate the expansion error in SEMM by selecting a sensor excluded from the SEMM process. This selection is based on comparing the correlation with the SEMM model, taking into account the sensor's location [5, 6]. In another application of SEMM, researchers have employed vision sensors that provide a low resolution with the full DoF data. These vision sensors were combined with vibration sensor data, for the purpose of the hybrid measurement [7].

SEMM has the advantage of enabling non-parametric direct virtual sensing through the expansion of the experiment DoF. However, a common challenge encountered in the SEMM process is the variation in accuracy, which is dependent on the choice of analysis model. Specifically, the accuracy of the SEMM model can vary significantly based on the specific boundary conditions employed in the analysis model. Therefore, this paper suggests improving SEMM accuracy through analytical model updating, particularly for parameterizable variables, in order to prevent overfitting of the updated model. In this context, the joint stiffness seves as a parameterizable variable, and it can be calculated based on the interface admittance and the connectivity of the assembled system.

THEORY

The equation describing the coupling of the substructure system, accounting for joint stiffness, is as follows [1]:

$$Y_{\mathrm{ab}} = Y - YB^T (Z_j^{-1} + BYB^T)^{-1}BY \quad (1)$$

Where the assembled admittance Y_{ab} can be expressed in terms of block-wised structure admittance, joint stiffness, and Boolean matrix, which denoted as Y, Z_j, and B, respectively. Multiplying both sides by B yields the following expression:

$$BY_{\mathrm{ab}} = BY - BYB^T (Z_j^{-1} + BYB^T)^{-1}BY \quad (2)$$

It is the operation of extracting the column of the interface DOF, and it is summarized as follows:

$$(Z_j^{-1} + BYB^T)(BYB^T)^{-1} B(Y - Y_{\mathrm{ab}}) = BY \quad (3)$$

By multiplying the latter part by $\times B^T$, equation (3) can be expressed as:

$$Z_j = (BYB^T)^{-1}[B(Y - Y_{\mathrm{ab}}) B^T](BYB^T)^{-1} \quad (4)$$

Therefore, the dynamic stiffness of the joint can be represented as the interface admittance of the assembly structure obtained from the experiment and the interface admittance of the substructure in the interpretation.

SEMM process is also similar to the previous joint identification process [2].

$$Y_{\mathrm{SEMM}} = Y - YB^T (BYB^T)^{-1}BY \quad (5)$$

Where Y is block-wise matrix combined with 3 types of admittance matrices: Y_{PAR}, Y_{OV}, Y_{REM}. Y_{PAR} corresponds to the full DoF (N_g) admittance matrix of the parent model, obtained through a simulation. Y_{OV} represents the overlay model's admittance matrix, considering only measured DoF (N_m) and derived from experimental data. Y_{REM} is the condensed admittance matrix of the removed model, which is a subset of the parent model containing only the measured DoF. The SEMM process involves coupling substructures with an overlay model and decoupling with a removed model. The decoupling process can be represented as a coupling process with a minus sign applied to admittance. In the MATLAB function, Y is constructed as blkdiag (Ypar, Yov, -Yrem). By combining these matrices, as illustrated in Figure 1, the resulting matrix Y_{SEMM} creates a hybrid model that incorporates a full DoF.

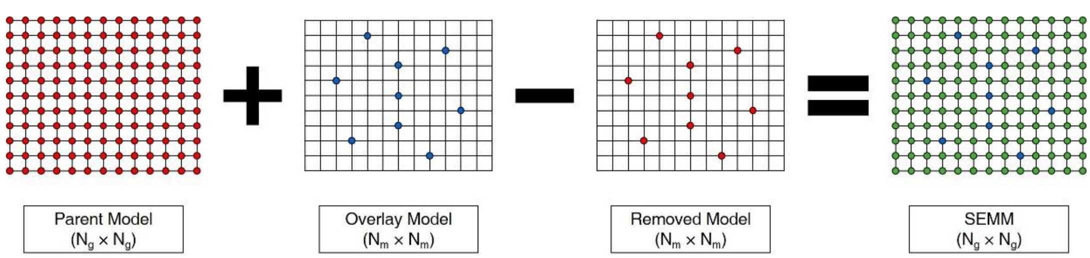

Figure 1. Schematic Diagram of the SEMM Process.

NUMERICAL SIMULATIONS

A numerical simulation is performed using a 6-DoF model with two substructures and two joints. As shown in Figure 2, the two substructures are connected by two joints (k_{25}, k_{46}), and the initial joint stiffness of the parent model was set differently from the value of the overlay model. The non-parametric points are the two stiffness (k_{11}, k_{33}) on the left side of the structure 1. These points also have different values in stiffness between parent model and overlay model. TABLE I provides the mass and stiffness values for each DoF.

The non-parametric joint stiffness in the parent model ranged from 20% to 180% of the overlay model value, divided into a total of 17 cases, with each case representing a 10% increment. In both cases, the parametric joint stiffness was set to 2000N/m, which corresponds to 200% of the overlay model value. Equation (4) was employed to calculate and update the joint stiffness for each case, and the accuracy was assessed by comparing the frequency response using the frequency response assessment (FRAC) method. The FRAC can be expressed as follows:

$$FRAC_{pq} = \frac{\left|\sum Y_{\mathrm{pq}}(w) H_{pq}^{*}(w)\right|^{2}}{\left(\sum Y_{\mathrm{pq}}(w) Y_{\mathrm{pq}}^{*}(w)\right)\left(\sum H_{\mathrm{pq}}(w) H_{pq}^{*}(w)\right)} \quad (6)$$

Where *Y* and *H* are the admittances of the two comparison objects, p and q represent the p^{th} row and q^{th} column of the admittance, respectively, and symbol (*) denotes complex conjugate.

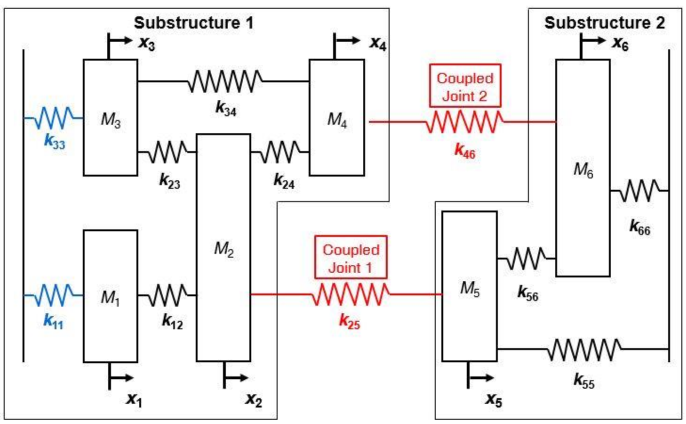

Figure 2. 6 DOF Lumped System.

TABLE I. MASS AND STIFFNESS VALUES OF EACH DEGREES-OF-FREEDOM.

System	Mass(kg)		Stiffness(N/m)		System	Stiffness(N/m)	
Substructure 1	M_1	1	k_{12}	1000	Parent Model	k_{11}	200 ~ 1800
	M_2	1	k_{23}	200		k_{33}	200 ~ 1800
	M_3	1	k_{24}	2000		k_{25}	2000(initial)
	M_4	1	k_{34}	1000		k_{46}	2000(initial)
Substructure 2	M_5	1	k_{55}	500	Overlay Model	k_{11}	1000
	M_6	1	k_{56}	2000		k_{33}	1000
			k_{66}	1000		k_{25}	1000
						k_{46}	1000

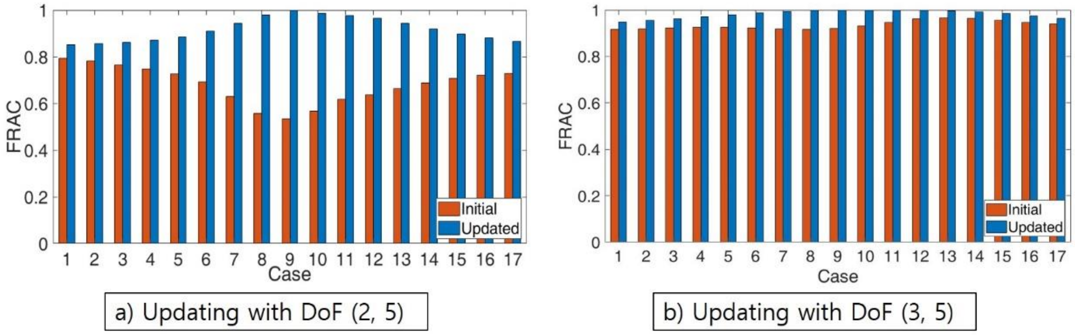

Figure 3. Comparison of FRAC Results for Different Cases of DoF with Initial and Updated SEMM.

RESULTS AND DISCUSSION

The updated stiffness values of parametric joints (k_{25}, k_{46}) are varied from the non-parametric stiffness of the parent model. In Case 1, the updated stiffness values are 1097N/m and 1136.4N/m, representing an increase of 9.7% and 13.6%, respectively, when compared to the overlay model values. In Case 9, where no discrepancy exists between the parent model and the overlay model, the updated stiffness values align with those of the overlay model. However, in Case 17, the updated stiffness values show a change of -3.0% and -4.6%. From case 1 to case 17, while non-parametric stiffness values are changed from −80% to +80%, the updated stiffness values exhibit a relatively smaller range of variation, ranging from 13.6% to -4.6%. This indicates that the updating process for the stiffness values exhibits a low sensitivity.

Figure 3 presents the FRAC of the SEMM model using the initial joint stiffness and updated joint stiffness and compared with the overlay model. Since the joint stiffness of the initial model in each case is larger than the joint stiffness of the overlay model, FRAC of the SEMM with the initial stiffness model is relatively smaller than SEMM model with the updated stiffness. However, despite the discrepancy in between non-parametric joints in substructure 1(k_{11}, k_{33}), it is noteworthy that all the updated joint stiffness values closely align with those of the overlay model. Furthermore, as depicted in the Figure 3a and Figure 3b, the FRAC value is influenced by the DoF utilized in the SEMM updating process. This implies that SEMM updating demonstrates a high level of sensitivity to variations in the location of the sensor.

CONCLUSION AND FUTURE WORK

This paper confirms that updating of the parameterized model in the SEMM process results in enhanced accuracy. Specifically, in the case of updating joint stiffness, the flexibility is calculated based on the difference between the analytical model and the experimental model with respect to joint interfaces. The updating process heavily relies on biased values that are influenced by experimental or systematic errors, making it challenging to completely eliminate the error. Nevertheless, it is crucial to note that despite this challenge, the updating process significantly contributes to reducing the error. In future work, the parameterizable variable, initially referred to as joint stiffness in this paper, will be generalized to encompass various structural properties. These properties will be updated using an objective function that incorporates modal or frequency domain quantities.

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