Interaction between Mass and Stiffness Parameters of Connections for Structural Parameter Estimation of a Steel Grid

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ABSTRACT

In any structural system, the mechanical properties of its connections impact the structure's global performance. Particularly, the rigidity of joints plays a substantial role in the system's vibration response. Since connections typically include several components with complex geometry, they are usually oversimplified in the modeling process. Therefore, the analytical model is typically a low-fidelity representation of the connections' performance. Although simplifying joints for modeling purposes is a good practice for engineering applications, it may reduce the accuracy of the model predictions. Thus, it is essential to reconcile accuracy with efficiency, particularly for complex joint modeling. In addition to the rigidity, the mass of connections may be another influential structural parameter for the system's global response. The joint mass may be underestimated in the modeling procedure because the elements of joints in the steel connection regions, including plates, stiffeners, and bolts, are not often modeled in simplified analyses. Hence, as the analytical model is updated, connections' mass and stiffness need to be estimated and incorporated into the updated model. This paper studies the interaction of mass and stiffness estimation for bolted connections of an experimental steel grid. The structure used in this research is a well-known structure built, instrumented, and tested at the University of Central Florida. The laboratory setup simulates the behavior of deck-on-beam bridges. In its initial analytical model, all grid joints are assumed to be completely fixed, and no extra mass is considered in the joint regions. The model updating procedure is limited to estimating joints' mass and stiffness values. The modal information of the structure is extracted from the measured frequency response functions of the grid excited in a vertical hammer impact test. Stiffness- and flexibility-based error functions are employed for the model updating. The results demonstrate how the selection of joints' uncertain parameters can affect the procedure of finding suitable structural layouts for the semi-rigid connections of the grid. The model updating procedure may be generalized to condition assessment and structural health monitoring of bridges, especially when their connections are subject to damage and failure.

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INTRODUCTION

The transmission of forces throughout a system depends on the connections between its mechanical or structural members. Oversimplified joints are often modeled to represent semi-rigid connections, which could lead to uncertainties in developing reliable analytical simulations. Introducing and calibrating multiple stiffness parameters for sophisticated connections may alleviate the problems of finding a stiffness scheme that accurately represents the realistic mechanical behavior of semi-rigid joints. Moreover, the constituent elements utilized in steel connection areas, such as plates, stiffeners, and bolts, entail additional localized mass, increasing the weight of the joints. Therefore, it is necessary to appropriately estimate both mass and stiffness parameters of connections to prevent significant inaccuracies in the system's dynamic response.

Wu and Li adopted weighted least squares and Bayesian estimation methods to identify the connection stiffness of beam-column joints [1]. Sanayei et al. utilized their proposed combined multiple parameter estimation algorithms to update the finite element model of a laboratory grid structure whose members were connected by bolted joints [2]. Subsequently, Santini-Bell et al. developed similar research using the test data of the same laboratory grid structure [3]. Altunisik et al. presented a finite element model updating procedure for an arch-type steel laboratory bridge model with semi-rigid connections [4]. Basaga et al. updated the stiffness of connections in the analytical models of two laboratory structures using their proposed model updating algorithm [5]. Zapico-Valle et al. proposed two models comprising beams to reproduce the dynamic behavior of a beam-column bolted moment connection [6]. Using the method proposed in Ref. [2] for parametrization of joint models, Sanayei et al. estimated stiffness and mass values for the connection zones of the University of Central Florida (UCF) benchmark laboratory structural grid [7]. Dai proposed a finite element model updating technique based on uniform design [8]. Mehrkash and Santini-Bell updated the finite element model of the laboratory steel grid mentioned in Ref. [7] using experimental modal data [9]. Their model updating was limited to the stiffness estimation for the grid joints, simulated by simplified rotational partial fixities.

Although various techniques are available for identifying connections, the literature reviewed only the related *model updating-based approaches*. Most studies focused on the stiffness of joints, while the connection mass has gained little attention. The current study proposes a procedure for simultaneous stiffness and mass estimation of joints in a laboratory steel grid. The studied structure is the steel grid used in Refs. [7, 9], known as the University of Central Florida (UCF) Grid. The authors estimated the stiffness of the grid connections by updating its simplified analytical model using stiffness-based error functions [9]. In this research, the proposed estimation protocol is developed by incorporating the mass of joints, showing how the interaction of mass and stiffness of the connections can alter the model updating results. The experimental modal data are employed to update the joints' stiffness and mass values. In addition to the stiffness-based error function, the flexibility-based error function is adopted for the structural parameter estimation. Different parametrizations and groupings are considered for the stiffness and mass properties of the grid connections. Finally, the most representative analytical model of the grid is selected based on the connection characteristics.

FORMULATION OF THE STRUCTURAL PARAMETER ESTIMATION

The structural parameter estimation is an inverse problem, i.e., given model outputs, the goal is to determine certain structural parameters of the model. Modal information is promising for various model updating tasks, while the structural parameters of interest can be elemental stiffness and mass values. A couple of the most efficient modal-based error functions are adopted in this research: stiffness- and flexibility-based error functions. Then, the errors of the measured modes are stacked, and the objective functions are computed as the Euclidean norm of the error functions. Finally, the objective functions are minimized through the *fmincon* of the MATLAB Optimization Toolbox™ [10], linked with SAP2000® API [11] for the required structural analyses.

Modal Stiffness-Based Error Function

The modal stiffness-based error function $\mathbf{E}_{\mathrm{ms}}(\boldsymbol{\theta})$ is based on the residual modal elastic and inertia forces predicted at a subset of degrees of freedom, as given by Eq. (1) for each mode [12]:

$$\mathbf{E}_{\mathrm{ms}}(\boldsymbol{\theta}) = \left[\mathbf{K}(\boldsymbol{\theta}) - \omega^2 \mathbf{M}(\boldsymbol{\theta})\right]\boldsymbol{\phi} \tag{1}$$

where $\boldsymbol{\theta}$ is the structural parameter being updated, ω is natural frequency, $\boldsymbol{\phi}$ is mode shape vector, and $\mathbf{K}(\boldsymbol{\theta})$ and $\mathbf{M}(\boldsymbol{\theta})$ are stiffness and mass matrices, respectively. This characteristic equation is partitioned in terms of mode shapes at measured and unmeasured degrees of freedom for each measured mode:

$$\left(\begin{bmatrix}\mathbf{K}_{\mathrm{aa}} & \vdots & \mathbf{K}_{\mathrm{ab}}\\ \cdots & & \cdots \\ \mathbf{K}_{\mathrm{ba}} & \vdots & \mathbf{K}_{\mathrm{bb}}\end{bmatrix} - \lambda \begin{bmatrix}\mathbf{M}_{\mathrm{aa}} & \vdots & \mathbf{M}_{\mathrm{ab}}\\ \cdots & & \cdots \\ \mathbf{M}_{\mathrm{ba}} & \vdots & \mathbf{M}_{\mathrm{bb}}\end{bmatrix}\right)\begin{Bmatrix}\boldsymbol{\phi}_{\mathrm{a}}\\ \cdots \\ \boldsymbol{\phi}_{\mathrm{b}}\end{Bmatrix} = \mathbf{0} \tag{2}$$

where subscripts a and b denote the measured and unmeasured degrees of freedom, respectively, and ω^2 has been shown by λ. This equation is expanded, and the unmeasured mode shapes are condensed out. Hence, Eq. (1) is written in the following form:

$$\mathbf{E}_{\mathrm{ms}} = \left[\left(\mathbf{K}_{\mathrm{aa}} - \lambda\mathbf{M}_{\mathrm{aa}}\right) - \left(\mathbf{K}_{\mathrm{ab}} - \lambda\mathbf{M}_{\mathrm{ab}}\right)\left(\mathbf{K}_{\mathrm{bb}} - \lambda\mathbf{M}_{\mathrm{bb}}\right)^{-1}\left(\mathbf{K}_{\mathrm{ba}} - \lambda\mathbf{M}_{\mathrm{ba}}\right)\right]\boldsymbol{\phi}_{\mathrm{a}} \tag{3}$$

Modal Flexibility-Based Error Function

The modal flexibility-based error function $\mathbf{E}_{\mathrm{mf}}(\boldsymbol{\theta})$ is based on residual modal displacements predicted at a subset of degrees of freedom. If $\mathbf{K}^{-1}\mathbf{M}$ is denoted by $\mathbf{D}$, the error function can be stated by Eq. (4) for each mode [13]:

$$\mathbf{E}_{\mathrm{mf}}(\boldsymbol{\theta}) = \left[\lambda\mathbf{D}(\boldsymbol{\theta}) - \mathbf{I}\right]\boldsymbol{\phi} \tag{4}$$

where $\mathbf{I}$ is the identity matrix. The dynamic matrix and the mode shapes are partitioned based on the measured and unmeasured degrees of freedom in each mode:

$$\left[\lambda\begin{bmatrix}\mathbf{D}_{aa} & \vdots & \mathbf{D}_{ab}\\ \cdots & & \cdots\\ \mathbf{D}_{ba} & \vdots & \mathbf{D}_{bb}\end{bmatrix}-\mathbf{I}\right]\begin{Bmatrix}\boldsymbol{\phi}_{a}\\ \cdots\\ \boldsymbol{\phi}_{b}\end{Bmatrix}=\mathbf{0} \tag{5}$$

By condensing out the unmeasured degrees of freedom for each mode, Eq. (6) is derived as follows.

$$\mathbf{E}_{mf}=\left[\lambda^{2}\mathbf{D}_{ab}\left(\mathbf{I}-\lambda\mathbf{D}_{bb}\right)^{-1}\mathbf{D}_{ba}+\lambda\mathbf{D}_{aa}-\mathbf{I}\right]\boldsymbol{\phi}_{a} \tag{6}$$

THE UCF GRID AND ITS ANALYTICAL MODEL

The structure used in this research is a laboratory steel grid designed, constructed, instrumented, and tested at the University of Central Florida (UCF). The structure conforms to the anticipated standards for bridges that cover short to medium-range distances. The grid was instrumented with eight vertical accelerometers for an impact hammer test. The layout of the accelerometers, shown by orange circles, and the location of one of the impacts are depicted in Figure 2. The geometrical and mechanical properties of the grid can be found in Ref. [14].

In this paper, the analytical model of the UCF Grid is developed by beam elements in the SAP2000® environment. Figure 1 depicts the grid model, in which the longitudinal girders and the transverse beams are divided into smaller elements to obtain more accurate predictions. In the initial model, all connections between the grid members are modeled as fixed joints. The numerous holes drilled in the connection zones for the connecting bolts may raise uncertainties about the rigidity of the joints. Also, no additional mass is considered in the joint regions. This assumption must be modified later in the model updating procedure because bolts, plates, angles, and sensors make the connection zones heavier. Partial fixities are incorporated at the ends of beams and girders as the rotational springs, while the additional mass is assigned to the nodes representing the joints for updating the finite element model of the grid.


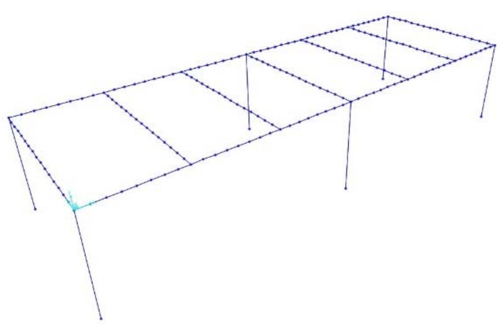

Figure 1. The UCF Grid (left) [14] and its analytical model (right)

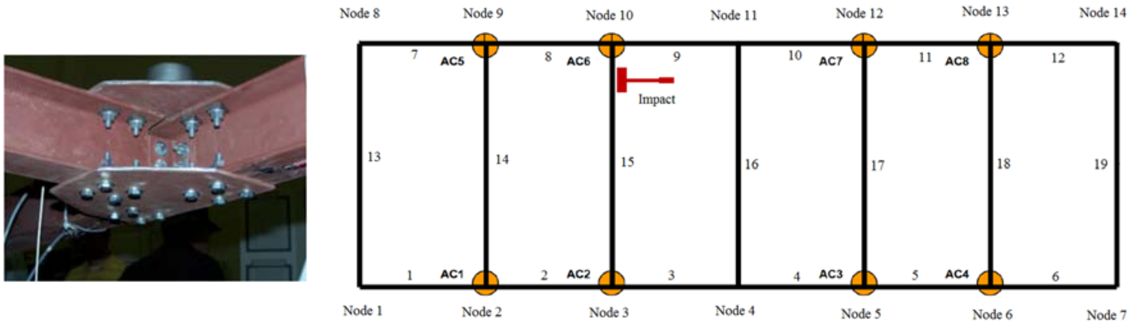

Figure 2. The UCF Grid connection (left) [15] and the plan of the grid instrumentation layout (right) [7]

PARAMETRIZATION

In this study, it is assumed that the only uncertainties that cause errors between the modal output of the actual structure and its analytical model pertain to the simplified modeling of its complex connections. In the initial model, all connections between the grid members are modeled as fixed joints. Nevertheless, the numerous holes drilled in the connection zones for the connecting bolts may raise uncertainties about the rigidity of the joints. Also, initially, no additional mass is considered in the joint regions. In this research, the capabilities of SAP2000® for simulating the rigidity and mass of structural nodes are utilized for modifying the mechanical characteristics of the steel grid connections. To do so, partial fixities are incorporated at the ends of beams and girders as the rotational springs, while the additional mass is assigned to the nodes representing the joints. These two stiffness and mass properties are considered the structural parameters being estimated during the finite element model updating of the grid.

The extracted experimental natural frequencies of the structure for the first 12 modes are given in Table I, in addition to the predicted ones from the analytical model. No springs are considered at the ends of Members 13, 16, and 19 because they do not contribute to the global modes of the structure. Modes 7, 8, and 9 correspond to the local vibrations of the three mentioned members and are not captured by this instrumentation. In addition to Modes 7, 8, and 9, Modes 10 and 11 are not contributed to the estimation procedure, as their MAC values are found to be high.

TABLE I. GRID’S EXPERIMENTAL AND ANALYTICAL NATURAL FREQUENCIES

Mode Number	Experimental Frequency (Hz)	Analytical Frequency (Hz)
1	22.3	22.6
2	26.8	28.4
3	33.3	33.9
4	40.6	43.8
5	64.6	62.4
6	67.6	65.6
7	-	73.2
8	-	73.2
9	-	73.4
10	94.1	96.3
11	96.4	99.5
12	102.4	109.6

TABLE II. DIFFERENT SCENARIOS FOR JOINTS' PARAMETERS BEING UPDATED

Case Number	Longitudinal Springs	Transverse Springs	Mass
1	✓	✓	×
2	✓	×	×
3	×	×	✓
4	✓	×	✓

RESULTS AND DISCUSSION

Four different analytical cases are assessed based on the parametrization and combination of the joints' mechanical properties. These models are considered as shown in Table II. Also, Tables III and IV give the estimated values for Cases 2 and 3 and the lower and upper bounds for the updated parameters.

For Case 1, two groups of structural parameters are considered. All longitudinal rotational springs are placed in the first group, and every rotational transverse spring is put together in the second group. Based on this grouping, the stiffness variations of the springs create the objective function of interest. It could be shown that this objective function is a one-way surface for both the stiffness- and flexibility-based error functions. The one-way surface of the objective function plot implies that the objective function's sensitivity to the transverse beams' stiffness is negligible. Therefore, only the stiffness of the longitudinal girders should be considered as the updating parameter.

Case 2 is similar to Case 1, but the stiffness of the transverse beams is discarded from the parameter estimation procedure. The estimation results for this case are given in Table III. It is observed that both stiffness- and flexibility-based error functions estimated the stiffness of the longitudinal beams in a straightforward manner.

In Case 3, no stiffness is updated, but the mass of the eight instrumented joints is estimated. All eight joints are assumed to have the same weight and are considered a group of structural parameters. The estimated mass (weight) values are shown in Table IV.

Finally, Case 4 combines Cases 2 and 3, where joints' mass and the longitudinal beams' stiffness are considered as the updating parameters. By examining the objective function, it is observed that the longitudinal stiffness asymptotically converges to large values, which correspond to the complete fixity of the joints. Hence, one can assume the connections are fixed, and only the mass of the joints is updated, like in Case 3.

TABLE III. ESTIMATED VALUES FOR THE LONGITUDINAL SPRINGS (kN.m/rad)

Error Function	Lower Bound	Upper Bound	Initial Value	Estimated Value
Stiffness-based	2000	18000	8000	11306
Flexibility-based	2000	18000	8000	9813

TABLE IV. ESTIMATED VALUES FOR THE WEIGHT OF THE CONNECTIONS (kN)

Error Function	Lower Bound	Upper Bound	Initial Value	Estimated Value
Stiffness-based	0.0001	0.0040	0.0010	0.0013
Flexibility-based	0.0001	0.0040	0.0010	0.0011

TABLE V. NATURAL FREQUENCIES OF THE UPDATED UCF GRID MODEL

Mode No.	Natural Frequencies (Hz)			
	Original Model	Case 2	Case 3	Experimental
1	22.6	22.1	21.9	22.3
2	28.4	27.8	26.9	26.8
3	33.9	33.0	32.9	33.3
4	43.8	42.6	41.4	40.6
5	62.4	61.6	62.0	64.6
6	65.6	64.9	65.4	67.6
7	73.2	73.2	73.2	-
8	73.2	73.2	73.2	-
9	73.4	73.4	73.4	-
10	96.3	96.3	93.9	94.1
11	99.5	99.3	96.6	96.4
12	109.6	106.9	103.9	102.4

MODEL SELECTION

Any of the four previously introduced representations of the steel grid based on the structural parametrization of their connections may be used as an updated analytical model of the system. However, a comparison can be made to examine which one more accurately describes the modal properties of the structure. It was observed that the two-parameter Cases 1 and 4 involved parametrization redundancy so that one of the parameters being updated could be removed from the estimation procedure. Therefore, only Cases 2 and 3 are compared here, and the more representative model based on joint parametrization is proposed. Thus, the structure's natural frequencies predicted by the updated models of Cases 2 and 3 are given in Table V. These are the values corresponding to the average of the estimated parameters obtained by the stiffness- and flexibility-based error functions. The natural frequencies of the original model and the experimental ones are listed again for comparison. For most modes, the errors between the experimental frequencies and the ones of Case 3 are observed to be smaller than the corresponding errors when Case 2 is compared. Both updated models predict natural frequencies closer to the experimentally extracted ones. Particularly, the frequency errors of the updated model in Case 3 show a significant decrease for Modes 2, 4, 11 and 12. The decreases in errors by updating the model using Case 2 are not substantial. Hence, Case 3 can be considered a more representative model of the structure based on the modal characteristics of the system and focusing on the uncertainties of the joints.

CONCLUSIONS

The finite element model of the UCF Grid was updated solely based on the structural parameters of its complex connections. Four different mass and stiffness parametrizations were considered for the connections of the structure. It was demonstrated that the simultaneous consideration of the mass and stiffness properties of joints in structures might not be advantageous. This issue is mainly due to the asymptotic nature of semi-rigid connections, making them less sensitive to a system's modal characteristics when accompanied by the mass of joints in the parametrization schemes. The grouping of parameters simplified the structural parameter estimation considerably. Finally, it was found that the model with additional mass in the joint zones with complete fixity could be the most representative model of the grid, if the model updating would be limited to the structural parameters of its connections.

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