On the Use of Model-Based Versus Data-Based Approaches for Virtual Sensing in SHM

EVANGELOS PAPATHEOU, KONSTANTINOS TATSIS, KONSTANTINOS AGATHOS, MARCUS HAYWOOD-ALEXANDER, NIKOLAOS DERVILIS and KEITH WORDEN

ABSTRACT

Structural Health Monitoring (SHM) typically aims at producing methodologies for the periodic, and often online, assessment of structures. While there are different approaches to SHM, there is one common necessary element if applied in practice - data acquired from sensors. In an ideal scenario, sensors would be deployed anywhere on a structure of interest and could be added retrospectively, if not included in the original design. In practice, there are various limitations to the availability and installation of sensors, such as physical access and cost. Virtual sensing has thus been proposed as a solution to the problem of sensor availability and different approaches have been applied in various fields. This paper applies and compares two approaches for virtual sensing in structural dynamics: modal expansion which is model-based and Gaussian Process (GP) regression which is data-based. The approaches are demonstrated on data from a Piper PA-28 tailplane structure, which was tested with the help of a scanning-laser vibrometer under laboratory conditions.

INTRODUCTION

Monitoring the state of structures is vital for their continued safe operation, and also for the efficient management of their maintenance. Structural Health Monitoring (SHM), refers essentially to any process that involves a periodic or on-line assessment of a structure, and is a very active research field; comprehensive reviews can be found in [1]. While there are many different approaches to SHM, there is one common necessary element if applied in practice – data acquired from sensors. Without reliable sensors to provide data, no SHM approach can work, regardless of whether it is physics-based or data-driven. In an ideal scenario, sensors could be installed anywhere on a structure of interest and could be added retrospectively if required. In practice this is not usually possible, as there are limitations in terms of cost and physical access. Retrospective

E Papatheou, Email: e.papatheou@exeter.ac.uk, K. Agathos: Dept. of Engineering, University of Exeter, North Park Road, EX4 4QF, Exeter, UK, K. Tatsis and M. Haywood Alexander: ETH Zurich, Institute of Structural Engineering, Stefano-Franscini-Platz 5, 8093, Zurich, Switzerland, N. Dervilis and K. Worden: Dynamics Research Group, Dept. of Mechanical Engineering, University of Sheffield, Mappin Street, Sheffield, S1 3JD, UK

fitting can be expensive or practically unfeasible, while inclusion in the original design is costly or perhaps impossible without *a priori* knowledge. *Virtual sensing* has been proposed as a solution to the problem of sensor availability and is also an active research area.

In structural dynamics, virtual sensing has made use of Kalman filters [2,3], dynamic substructuring [4, 5] and modal expansion [6]. The latter includes applications in wind turbines [7] and has been compared with Kalman filters [8]. Usually in structural dynamics, the aim is to either infer measurements into unknown (unmeasured) locations and/or to estimate full-field deformation (and strain) of the entire structure of interest. Although Gaussian Process (GP) regression has been used in some form for virtual sensing in other fields, it has not been applied widely in structural dynamics. The purpose of this paper is to apply virtual sensing on a complex real aerospace structure by comparing Modal Expansion and GP regression.

The layout of this paper is as follows: the next section presents the concept of virtual sensing in the two approaches used here: modal expansion and GP regression. The third section describes the structure, the experimental setup and the data processing. Section four presents the results and compares the two approaches. Finally, a short conclusion recaps the work in order to round off the paper.

VIRTUAL SENSING

Modal Expansion

Modal expansion as a form of virtual sensing is usually based on model reduction techniques and makes use of the mode shapes of a structure (see [6]). For most reduction/expansion approaches there can be a relationship between the full and reduced spaces by,

$$\mathbf{X_f} = \mathbf{T}\mathbf{X_r} \tag{1}$$

where $\mathbf{X_f}$ is the displacement of the structure in the full-space and $\mathbf{X_r}$ the displacement in the reduced space: $\mathbf{T}$ is a transformation matrix. A similar relationship can be established for different types of response, and also for the mass and stiffness matrices of a model of a structure,

$$\mathbf{M_f} = \mathbf{T^T}\mathbf{M_r}\mathbf{T} \tag{2}$$

$$\mathbf{K_f} = \mathbf{T^T}\mathbf{K_r}\mathbf{T} \tag{3}$$

The SEREP transformation matrix [6] uses the mode shapes of the structure which can be derived either from an FE model or experimentally measured (see [9]) by using the following [6],

$$\mathbf{T} = \mathbf{U_f}(\mathbf{U_r^T}\mathbf{U_r})^{-1}\mathbf{U_r^T} \tag{4}$$

where $\mathbf{U_f}$ and $\mathbf{U_r}$ are the mode shapes of the structure in full and reduced space respectively. $\mathbf{U_r}$ does not need to be a square matrix, which means that the number of mode shapes chosen for the expansion, and the number of the chosen reduced degrees of freedom do not need to be the same. In this paper, experimentally measured mode shapes are used and no FE model of the structure of interest is developed.

GP Regression

Rasmussen and Williams [10] define a Gaussian process (GP) as "a collection of random variables, any finite number of which have a joint Gaussian distribution". A GP is thus completely defined by a mean and a covariance function. Here, a zero-mean and a squared-exponential covariance function are applied. Covariance functions are typically accompanied by some hyperparameters in order to obtain a better control over the types of functions that are considered for the inference. Optimal hyperparameters for GP regression can be chosen with the help of the maximum marginal likelihood of the predictions with respect to the hyperparameters. For more details on GP regression, and the exact solution of the maximisation of the marginal log likelihood via its partial derivatives, the reader is referred again to [10].

For the purposes of virtual sensing, GP regression is used here when the input vector consists of the time data of one or more sensors, and the target data-vector is the time response of a sensor of interest. Only single-output GPs are used here in the classical multiple-input-single-output (MISO) format, therefore if one needs to predict the response in different locations of the structure, multiple GP models will be required. The software used for the implementation of GP regression was provided by [10].

Performance Assessment

When comparing time traces, it may be typical to use the Time Response Assurance Criterion (TRAC) [6]. Similar to the Modal Assurance Criterion (MAC) [11], TRAC is defined by,

$$TRAC_{ij} = \frac{(RTO_i^T RTO_j^T)^2}{(RTO_i^T RTO_i)(RTO_j^T)RTO_j} \tag{5}$$

where RTO_i and RTO_j represent the time signals at points i and j. TRAC takes values between 0 and 1.

The TRAC does not take into account scaling, so in order to check the quality of estimates, the normalised mean-square error (MSE) is also used,

$$MSE(\hat{y}) = \frac{100}{N\sigma_y^2} \sum_{i=1}^{n} (y_i - \hat{y}_i)^2 \tag{6}$$

where the caret denotes an estimated quantity, y_i is the actual observation, N the total number of observations and σ_y the standard deviation. In general, an MSE error below 5 is considered a good fit and below 1, excellent.

EXPERIMENTAL DATA

The structure of interest was one half of a tailplane from a Piper PA-28 of the 'Cherokee' variant. It can be seen in Figure 1 and it belongs to a small population of structures where population-based SHM (PBSHM) has been applied in [12]. The wing section has a symmetric airfoil (NACA0012) with a chord of 0.76 m and a span of 3.62 m. The tailplane was excited using Gaussian white noise through an electrodynamic shaker

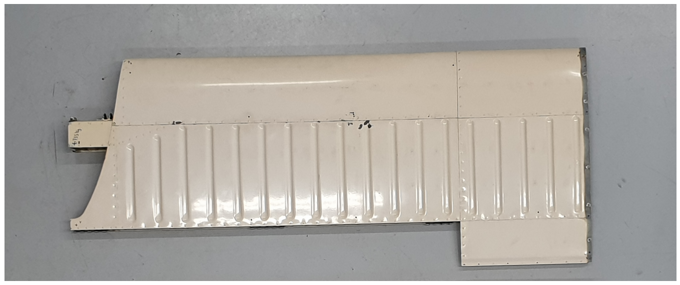

Figure 1. The tailplane structure which was used in this study.

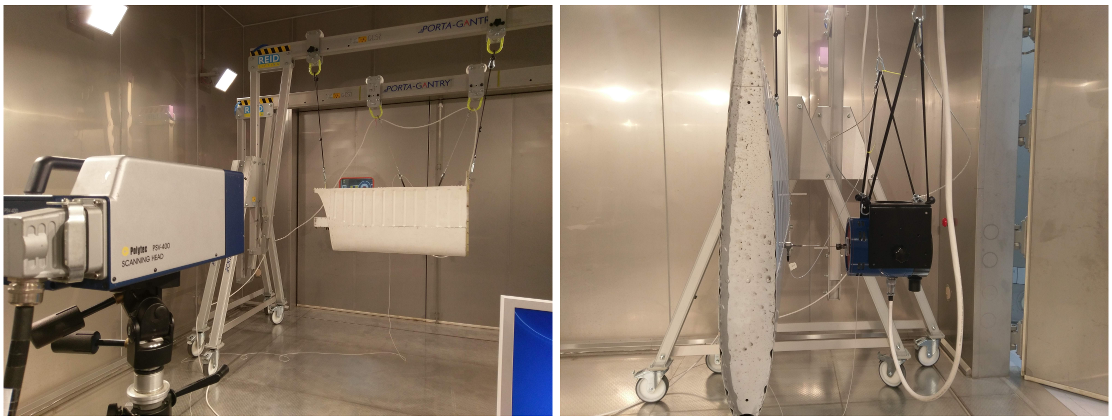

Figure 2. An example of a tailplane structure tested with a scanning laser vibrometer [12] (left). Shaker setup (right).

while it was suspended to approximate free-free boundary conditions (see Figure 2). The responses of the structure were recorded using a scanning-laser Doppler vibrometer, a Polytec PSV-400 scanning head with a PSV-A-420 geometry scan unit controlled by an OFV-5000 controller and a JSV-500 junction box.

All tests were performed with a frequency bandwidth of 1 kHz, a sample rate of 2.56 kHz and a frequency resolution of 0.3125 Hz (3200 spectral points). For the tailplane structure used here, there were in total 295 scan points and these are considered 'sensor locations' throughout this paper. For each response point there were six averages recorded to estimate mobility (velocity based) Frequency Response Functions (FRFs, H1 estimator). Other types of FRFs (receptance, accelerance) were available, but were not used in this study. The first 25 Natural frequencies and mode shapes of the structure were extracted from the FRFs using the SDTools Matlab toolbox (max frequency at 225 Hz). During modal expansion, only the first 15 mode shapes were used here.

Time data were not available from the experimental test, but were estimated and synthesised by using inverse Fourier transforms from the experimentally-recorded FRFs. For the purposes of this work, the time signal which was used for virtual sensing was the structure's response to a random multi-sine (20 random sine wave frequencies) in the bandwidth of 90 to 150 Hz, which is a subset of the frequency bandwidth of the experimentally-acquired mode shapes.

RESULTS

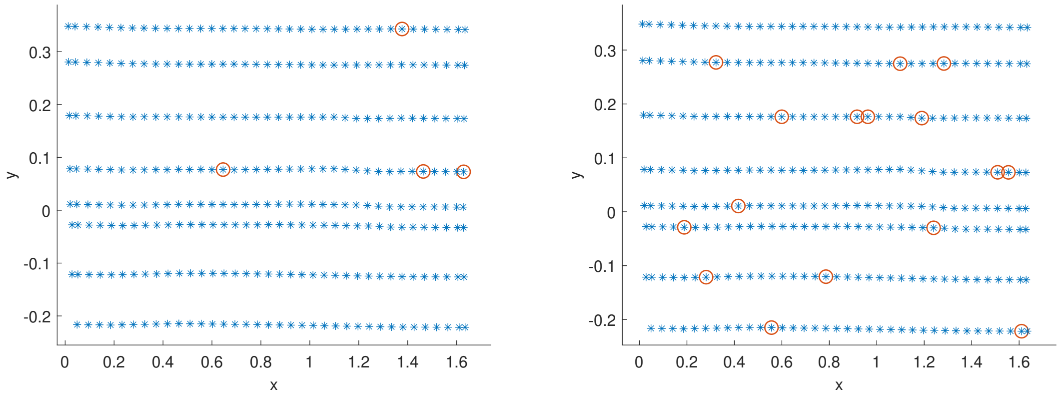

Figure 3. Optimised sensor locations (using a Genetic Algorithm) for modal expansion using 4 sensors (left) and 16 sensors (right).

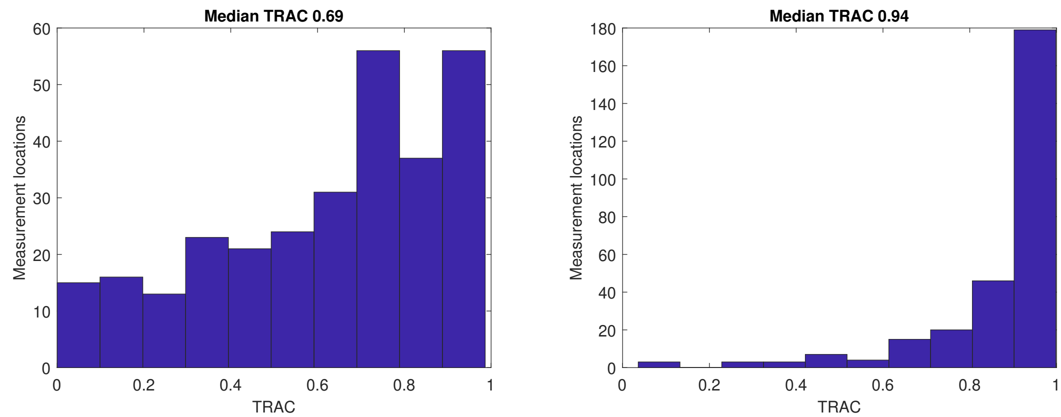

Figure 4. Modal expansion: histogram of TRAC performance for response prediction over all possible measurement locations when using 4 sensors (left) and 16 sensors (right). Solution optimised for modal expansion.

Modal Expansion

Using modal expansion and the experimentally-measured mode shapes, excellent results were possible both in terms of TRAC and MSE. However, results varied based on the measurement locations used for prediction. There were in total 295 locations, so there were too many possible combinations. An optimisation scheme was then used based on Genetic Algorithms (GA) which selected the best locations (sensors) to predict all the rest of the sensors of the tailplane. The objective function was based on the average TRAC value and the number of possible sensors used for the prediction was limited to 4 and 16. Figure 3 shows the finally selected locations along the scanning grid of the tailplane. Figure 4 shows the performance of the selected configurations (in histograms) for all the possible locations: average TRAC was 0.62 for the 4-sensor case and 0.88 for the 16 sensors. It is clear overall that the 16-sensor case performed significantly better. Figures 5 and 6 show examples of excellent (high TRAC and low MSE), and also higher MSE response prediction with the modal-expansion approach.

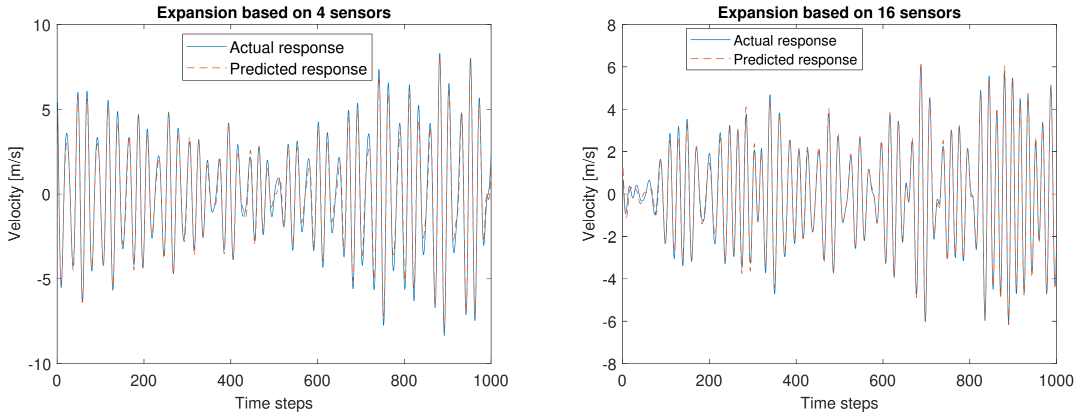

Figure 5. Modal expansion: examples of excellent response prediction when using 4 sensors TRAC 0.99 and MSE 1.73 (left) and 16 sensors TRAC 0.99 MSE 0.78 (right).

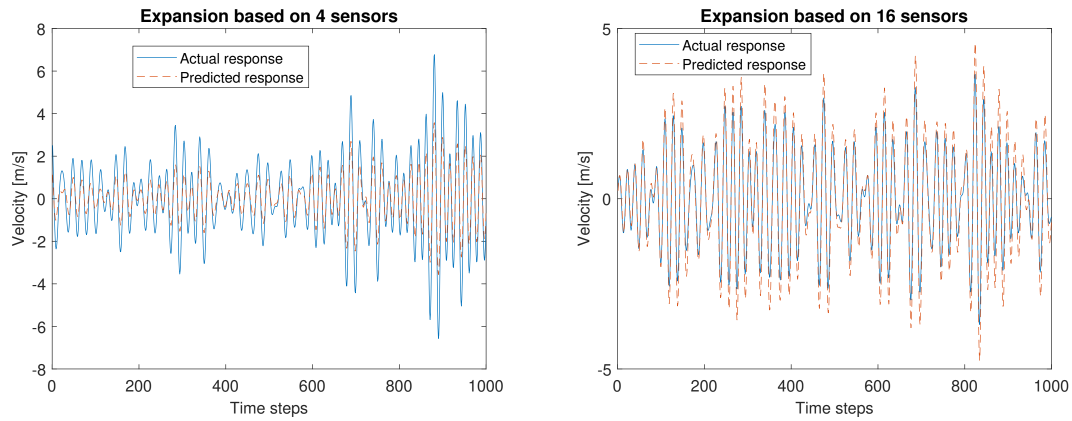

Figure 6. Modal expansion: examples of response prediction with high TRAC values, but lower performance MSE values when using 4 sensors (left) and 16 sensors (right).

GP Regression

Direct comparison of the GP regression with the modal expansion is not exactly possible, as the single-output nature of the GP models used here, requires multiple models for the full assessment of the scanning grid. Nevertheless, multiple models were trained and it was found that very good prediction was possible, but results also varied based on the locations. The optimised locations for the modal expansion did not perform equally well in the GPs, with average TRAC values around 0.52; however it was possible to optimise locations for the GPs only. Figure 7 shows the performance of the optimised locations for the GPs and Figure 8 shows response prediction using GP regression. It can be seen that in the 4-sensor case, the GPs outperform the modal expansion with an average TRAC of 0.77 compared to 0.62, but have lower TRAC values for the 16-sensor case.

CONCLUSION

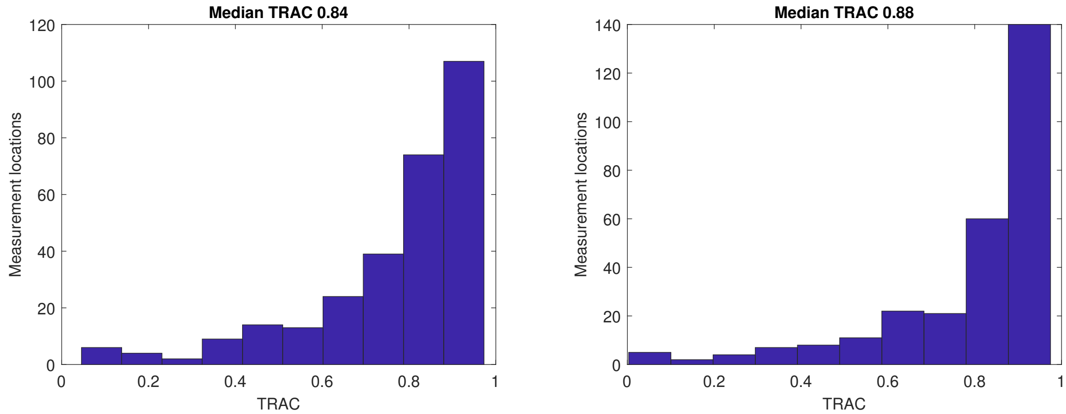

Figure 7. GP regression: histogram of TRAC performance for response prediction over all possible measurement locations when using 4 sensors (left) and 16 sensors (right). Both cases were optimised for GPs.

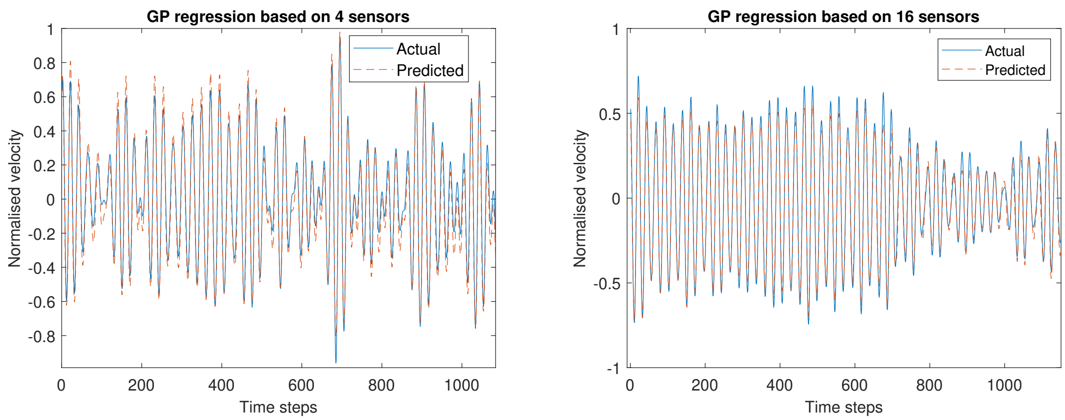

Figure 8. GP regression: examples of response prediction with TRAC values of 0.97 and MSE 3.4 when using 4 sensors (left) MSE 2.7 when using 16 sensors (right).

The purpose of this paper was to apply, and compare the use of two approaches for virtual sensing in structural dynamics: modal expansion and Gaussian process regression. The application was done on a real and complex aerospace part: a tailplane structure from a PA-28 which was tested with a scanning laser vibrometer. Both approaches were able to predict well the response, but performance varied based on the possible sensor locations. An optimisation using a Genetic Algorithm was done for both cases. Modal expansion performed better in the case of 16 sensors used, but a lot worse for the case of the 4 sensors. Future work will explore and compare the influence of the number of sensors further. Modal expansion is not computationally expensive, but requires mode shapes either from a numerical model or to be experimentally acquired. Gaussian Processes on the other hand do not need any mode shapes, nor any physics-based model, as they are entirely data-driven, but will require a training stage and training data. The latter may mean that a sensor will have to be installed at a place of interest during a training stage; be removed afterwards and be inferred by the remaining sensors during remaining life or normal operation. Accurate physics-based models can be difficult or costly to develop, especially on real and complex structures, and mode shapes can be hard to obtain experimentally, especially with a high spatial resolution. The choice of the algorithm will thus remain with the user, to decide based on the structure complexity and accessibility.

ACKNOWLEDGMENT

The support of the UK Engineering and Physical Sciences Research Council (EPSRC) from grant reference EP/W005816/1 is gratefully acknowledged.

REFERENCES

1. Sohn, H., C. R. Farrar, F. M. Hemez, D. D. Shunk, D. W. Stinemates, B. R. Nadler, and J. J. Czarnecki. 2004. "A Review of Structural Health Monitoring Literature: 1996-2001," Tech. rep., Los Alamos National Laboratory LA-13976-MS.
2. Papadimitriou, C., C. P. Fritzen, P. Kraemer, and E. Ntotsios. 2011. "Fatigue predictions in entire body of metallic structures from a limited number of vibration sensors using Kalman filtering," *Structural Control and Health Monitoring*, 18(5):554–573.
3. Tatsis, K. E., K. Agathos, E. N. Chatzi, and V. K. Dertimanis. 2022. "A hierarchical output-only Bayesian approach for online vibration-based crack detection using parametric reduced-order models," *Mechanical Systems and Signal Processing*, 167:108558.
4. Kullaa, J. 2016. "Virtual sensing of structural vibrations using dynamic substructuring," *Mechanical Systems and Signal Processing*, 79:203–224, ISSN 10961216.
5. Tatsis, K. E., V. K. Dertimanis, C. Papadimitriou, E. Lourens, and E. N. Chatzi. 2021. "A general substructure-based framework for input-state estimation using limited output measurements," *Mechanical Systems and Signal Processing*, 150:107223.
6. Baqersad, J., C. Niezrecki, and P. Avitabile. 2015. "Extracting full-field dynamic strain on a wind turbine rotor subjected to arbitrary excitations using 3D point tracking and a modal expansion technique," *Journal of Sound and Vibration*, 352:16–29, ISSN 10958568.
7. Iliopoulos, A., W. Weijtjens, D. Van Hemelrijck, and C. Devriendt. 2017. "Fatigue assessment of offshore wind turbines on monopile foundations using multi-band modal expansion," *Wind Energy*, 20(8):1463–1479.
8. Noppe, N., K. Tatsis, E. Chatzi, C. Devriendt, and W. Weijtjens. 2018. "Fatigue stress estimation of offshore wind turbine using a Kalman filter in combination with accelerometers," in *Proceedings of the 28th International Conference on Noise and Vibration Engineering, ISMA2018*, Leuven, Belgium.
9. Chipman, C. and P. Avitabile. 2012. "Expansion of transient operating data," *Mechanical Systems and Signal Processing*, 31:1–12.
10. Rasmussen, C. E. and C. K. I. Williams. 2006. *Gaussian processes for machine learning. 2006*, The MIT Press, Cambridge, MA, USA.
11. Allemang, R. J. and D. L. Brown. 1982. "Correlation coefficient for modalvectoranalysis,," in *Proceedings of the 1st International Modal Analysis Conference & Exhibit*, Orlando, FL, USA, pp. 110–116.
12. Bull, L. A., P. A. Gardner, N. Dervilis, E. Papatheou, M. Haywood-Alexander, R. S. Mills, and K. Worden. 2021. "On the transfer of damage detectors between structures: An experimental case study," *Journal of Sound and Vibration*, 501:116072.