Natural Frequency and Displacement Ratio Based Probabilistic Damage Identification for Bridges Using FE Model Update

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ABSTRACT

This paper proposes a natural frequency and displacement ratio-based probabilistic damage identification method for bridges using the finite element (FE) model update. When the damage location is known, it can be detected from an appropriate damage-sensitive feature (DSF). However, damaged components are seldom known before inspections. This makes it difficult to find an appropriate DSF and damage identification is sometimes challenging. This paper aims to propose a method to solve this issue by integrating multiple DSFs, natural frequencies and displacement ratio, as a decision-level data fusion approach. They are complementary in terms of sensitivity to damage. In addition, probability density functions (PDFs) of structural parameters are estimated from PDFs of observed DSFs through the FE model update to consider errors and uncertainties in measurement data. An in-house model bridge experiment is carried out to investigate the feasibility. The results demonstrated that the two kinds of damages in a bearing and girder reproduced in the experiment were successfully identified without false positives even when these damages simultaneously occurred.

INTRODUCTION

In many countries, aged and damaged bridges requiring frequent inspections have been increasing for decades. In contrast, visual inspection is still a common approach although inspectors, especially experts, are insufficient. Therefore, developing laborsaving bridge-monitoring technologies is a keen technical issue.

An appropriate damage-sensitive feature (DSF) enables us to easily detect damages when the location of the damage is known. For example, since the natural frequency of the first bending mode is sensitive to boundary conditions [1,2], they are appropriate for finding bearing defects. On the other hand, it is relatively insensitive to changes in flexural rigidity. Thus, displacement- and strain-related quantities are better features for detecting damage that reduces flexural rigidity, e.g., cracks in a girder [3-5]. However, the location of the damage is seldom known in practice. In addition, damage in multiple

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locations can occur. In such cases, it is challenging to find an appropriate DSF. If damage is overlooked, it can lead to a fatal accident.

To solve these issues above, this paper proposes a probabilistic approach to detect damaged bridge components using the finite element (FE) model update and multiple DSFs with different sensitivities. The FE model update is a technique to update structural parameters in an FE model so that the results of FE simulations match observations [6]. Updated structural parameters make it possible to estimate the integrity of structures. The FE model update adopting Bayesian inference, i.e., Bayesian model update, further estimates posterior probability distributions of structural parameters from observations and prior probability distributions [3,7,8]. It is a powerful method because uncertainties in DSFs can be considered in probability density functions (PDFs). If the change in the posterior probability distribution of a structural parameter is detected, it is possible to identify that the corresponding component is damaged.

This paper adopts natural frequencies and the displacement ratio measured at two positions as DSFs. To utilize displacement in the monitoring of bridges, one needs to know the load. However, taking a ratio of displacement at two positions can eliminate load dependency [3-5]. Hence, displacement ratio and natural frequencies are independent of external forces and can be used to estimate the physical conditions of bridges. The displacement ratio is sensitive to changes in flexural rigidity [3]. On the other hand, the natural frequency of the first bending mode is sensitive to boundary conditions and less sensitive to flexural rigidity. Hence, the sensitivities of these two DSFs are complementary. Taking advantage of this property, this paper attempts to identify cracks in the girder and the bearing defect as representative bridge damage.

DAMAGE IDENTIFICATION METHOD

Damage components in bridges are identified from estimated PDFs of structural parameters. PDFs of the structural parameters are estimated as follows. The Markov Chain Monte Carlo (MCMC) method is adopted as a sampling method in the structural parameters space. The parameter space is defined as prior probability distributions. Each step of MCMC samples a set of structural parameters based on a random walk in the parameter space and judges whether the sampled set is acceptable using observed DSFs such, as the natural frequency of the first bending mode and the probability acceptance α. For instance, in MCMC steps for the *i*-th DSF x_i, α is derived as

$$\alpha = \min\left\{1, \frac{p_i(x_{i2}|\boldsymbol{\theta}_2)}{p_i(x_{i1}|\boldsymbol{\theta}_1)}\right\}, \tag{1}$$

where $p_i(.)$ is the PDF of the *i*-th DSF and $x_{ij}|\boldsymbol{\theta}_j$ (j = 1,2) is the DSF x_i at a given set of structural parameters $\boldsymbol{\theta}_i$. If α is higher than a random number u generated from the uniform distribution $U[0,1]$, $\boldsymbol{\theta}_2$ is accepted and $\boldsymbol{\theta}_1$ is updated to $\boldsymbol{\theta}_2$. In the next step, new $\boldsymbol{\theta}_2$ is sampled by adding noise to $\boldsymbol{\theta}_1$, $\boldsymbol{\theta}_1 + \boldsymbol{w}$, where elements of $\boldsymbol{w}$ are random numbers generated by Gaussian noise. After the MCMC process is completed, the distribution of accepted $\boldsymbol{\theta}_2$ in the parameter space is obtained. PDFs of each structural parameter are estimated from the marginal distribution along each axis. Finally, the posterior probability distribution of structural parameters for the DSF is obtained.

The resultant posterior distribution of the k-th structural parameter θ_k according to all DSFs considered $\boldsymbol{x}$, $q(\theta_k|\boldsymbol{x})$, is derived as the product of the posterior distributions estimated from each DSF and the normalization, which can be considered as a decision-level data fusion approach [9,10],

$$q(\theta_k|\boldsymbol{x}) = \prod_i q(\theta_k|x_i) \Big/ \int \left[\prod_i q(\theta_k|x_i) \right] d\theta_k \, , \qquad (2)$$

where $q(\theta_k|x_i)$ is the posterior distribution of the k-th structural parameter based on the i-th DSF. In this paper, $\boldsymbol{x}$ corresponds to the three DSFs: natural frequencies of the first and second bending modes and displacement ratio. If $q(\theta_k|\boldsymbol{x})$ is significantly different between the reference scenario and the other scenario, the bridge component corresponding to θ_k is expected to be damaged.

EXPERIMENT

To verify the proposed method, an in-house experiment is carried out with a model bridge. The model bridge is a 5.4 m-long "I"-shaped simply supported steel beam shown in Figure 1 (a). Natural frequencies and the displacement ratio are derived under a

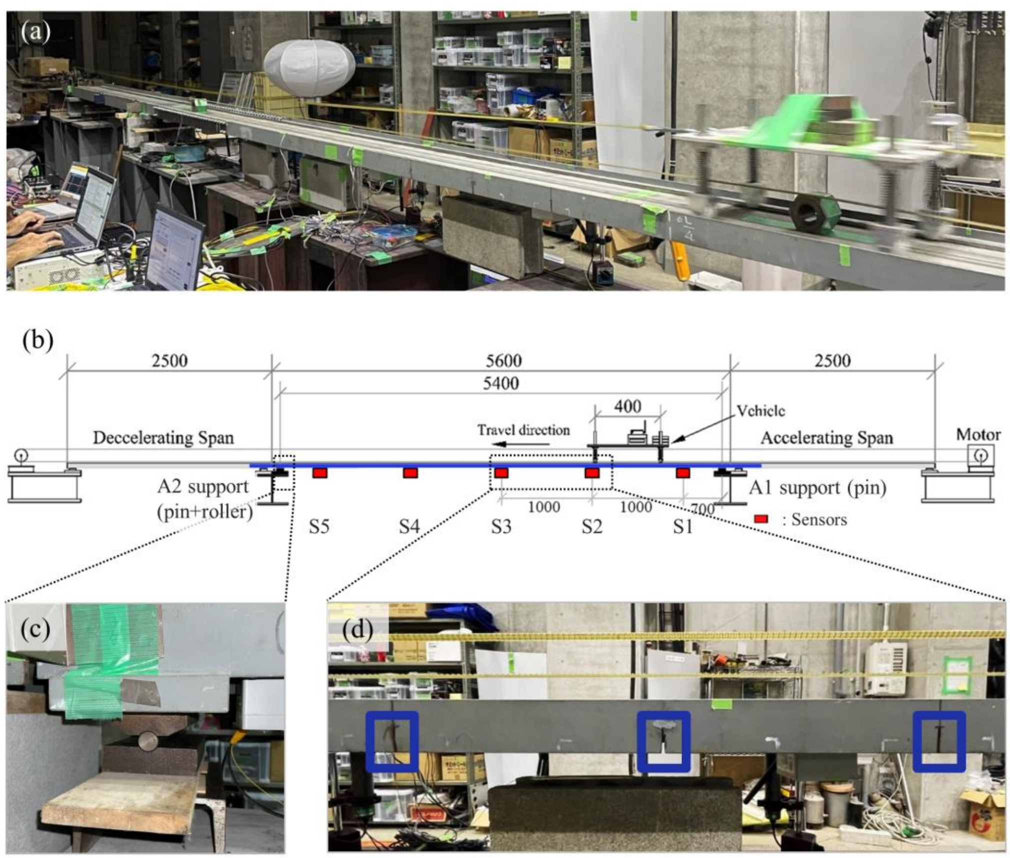

Figure 1. (a) General view of the model bridge, (b) elevation view, (c) roller bearing defect of A2 support reproduced by inserting a wooden board instead of the roller, (d) three saw cuts on the girder as modeled cracks.

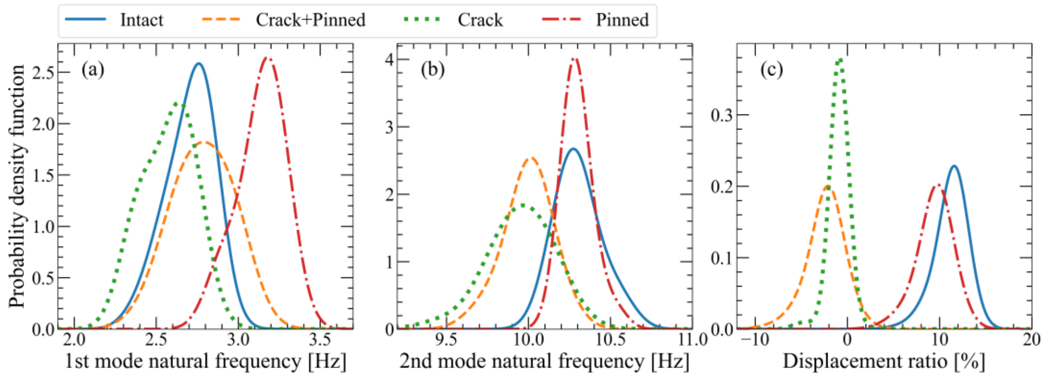

Figure 2. PDFs of observed quantities (a) first bending mode (b) second bending mode natural frequency, and (c) displacement ratio at S2 and S4 in each scenario.

traveling model vehicle. The mass of the model vehicle was fixed at 25 kg to increase the efficiency of the experiment. Accelerometers (ACCs) and displacement transducers were installed at positions denoted as S1, S2, S3, S4, and S5, as shown in Figure 1 (b).

Artificial damage, such as cracks in beams and roller bearing defects, are introduced into the model bridge. The A2 support is originally the pin and roller. The roller bearing is removable. In the experiment, as shown in Figure (c), the roller bearing of the A2 support is removed and a wooden plate is inserted instead to increase friction in the bridge axial direction, thereby reproducing the roller bearing failure. Cracks in the beam are modeled by three saw cuts to reduce the flexural rigidity as shown in Figure 1 (d). The saw cuts were introduced between S2 and S3 which resulted in approximately 20% reduction of the flexural rigidity of the beam. The intact state of the bridge is reproduced by reinforcing the saw cuts with steel plates. This paper refers to the bridge condition with the roller bearing defect of A2 support and saw cuts as the “crack+pinned scenario” and with the intact A2 support and reinforced girder as the “intact scenario”. Similarly, the “crack scenario” and “pinned scenario” are defined when either saw cuts or the roller bearing defect is adopted.

Natural frequencies were identified from ACC data in the free vibration using the Frequency Domain Decomposition method [11,12]. Free vibrations were identified from the spectrogram of ACC data. Frequencies of the first and second bending modes were identified. The displacement ratio is derived from data around the maximum displacement $\pm$0.5 seconds observed at S2 and S4 based on the method in [3].

Figure 2 shows the PDFs of the natural frequencies of the first and second bending modes and the displacement ratio in each scenario. These PDFs are estimated from histograms and the Kernel Density Estimation. In the natural frequency of the first bending mode for the “crack+pinned scenario”, the effects of the bearing defects that increase the natural frequency and saw cuts that decrease it are eliminated. Since the natural frequency of the second bending mode is weakly dependent on the boundary condition, natural frequencies in the crack and crack+pinned scenarios decrease. PDFs of displacement ratio shows significant difference depending on saw cuts. Hence, the displacement ratio is sensitive to changes in flexural rigidity, as expected. In contrast, it is not insensitive to the boundary condition. Hereafter, this paper focuses on the intact and crack+pinned scenarios.

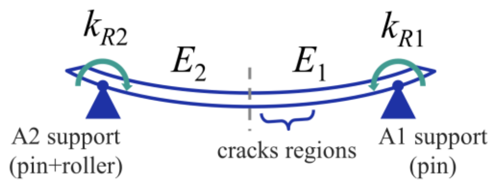

Figure 3. Structural parameters in the FE model.

FE MODEL UPDATE

For the FE model update, the bridge is modeled with beam elements using the software "Midas Civil". As structural parameters, rotational spring stiffness at A1 and A2 support (k_{R1} and k_{R2}, respectively) and Young's modulus for each half span of the bridge (E_1 and E_2) instead of the flexural rigidity are adopted, as shown in Figure 3.

The range of each structural parameter for simulations is defined as follows. According to pilot simulations, it is validated that natural frequencies are independent of k_{R1} and k_{R2} when either below 10^{-1} kN m/rad or above 10^6 kN m/rad. Therefore, the lower and upper limits of k_{R1} and k_{R2} were set as 10^{-1} and 10^6 kN m/rad, respectively. For E_1 and E_2, the upper and lower limit of the ranges are set as +10% and -25% of the nominal value of the steel, respectively. No damage increases flexural rigidity; thus, the narrower margin is adopted for the upper limit. The lower limit is based on the property of the model bridge: i.e., the flexural rigidity of the model bridge decreased by 25% of the nominal value due to the damage [13].

In the parameter space defined as the ranges above, sets of structural parameters are sampled 500 times using the Latin hypercube sampling. At each set of structural parameters, the natural frequencies of the first and second bending modes and the displacement ratio are simulated. Surrogate models for these DSFs are developed using 500 results of simulations and the Gaussian Process Regression. The performance of the surrogate models is validated from the maximum percentage error of about 1%.

From experimental results and the surrogate models, PDFs of the structural parameters are estimated through FE model update. To minimize additional uncertainties in estimated PDFs of structural parameters, prior probability distributions should be set as narrowly as possible considering information in structures and materials [7]. Hence, regarding k_{R1}, which is not the target of detecting damage here, a narrow distribution is adopted as the prior distribution for k_{R1}. The targets for detecting damages are k_{R2}, E_1, and E_2, then this paper estimates only PDFs of these three parameters. Identifying roller-bearing defects is a reasonable approach because roller-bearing defects can affect the bridge more than pin-bearing defects.

For the prior distribution of k_{R1}, a uniform distribution that ranges from 10^1 to 10^2 kN m/rad is adopted; $\log_{10} k_{R1}$ is sampled from $U[1, 2]$. The prior distribution of k_{R2} is set as follows. According to simulations, observed natural frequencies are never reproduced in simulations when k_{R2} is higher than 10^2 kN m/rad. Therefore, the prior distribution of $\log_{10} k_{R2}$ is set as $U[-1, 2]$. Prior distributions of E_1 and E_2 are assumed as $U[0.75E_0, 1.1E_0]$, where E_0 is the nominal value of Young's modulus for steel.

The total steps of MCMC were 50,000 and burn-in is initial 5,000 steps. The stationarity and Markov properties in the MCMC steps are validated from autocorrelation functions of parameters' paths. The acceptance rate was around 30%. After the MCMC process, PDFs of k_{R2}, E_1, and E_2 are estimated according to Equation

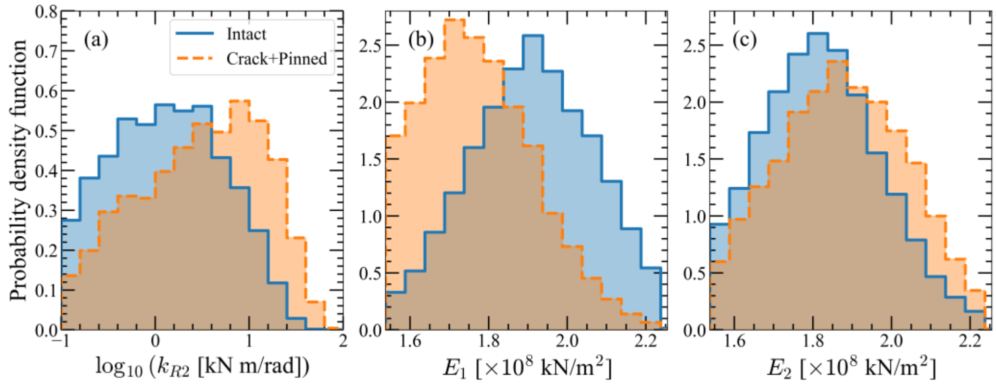

Figure 4. Estimated PDFs of structural parameters (a) k_{R2}, (b) E_1, and (c) E_2 in each scenario.

TABLE I. *P*-VALUES OF THE KOLMOGOROV–SMIRNOV TEST FOR THE RESULTS OF FIGURE 4.

	k_{R2} PDFs	E_1 PDFs	E_2 PDFs
p-value	0.0013	3.6×10^{-7}	0.078

TABLE II. SUMMARY OF THE DAMAGE IDENTIFICATION IN EACH SCENARIO DEPENDING ON DSFS.

Scenario	Natural freqs.	Disp. ratio	Natural freqs. & Disp. ratio
Crack+Pinned	N	Y/FP	Y
Crack	N	Y/FP	Y
Pinned	Y/FP	N	Y

"Y" INDICATES THAT ALL DAMAGED COMPONENTS ARE SUCCESSFULLY IDENTIFIED WITHOUT FALSE POSITIVES. "Y/FP" INDICATES THAT DAMAGED COMPONENTS ARE IDENTIFIED BUT INCLUDING FALSE POSITIVES. "N" INDICATES THAT THE IDENTIFICATION IS FAILED.

(2). Figure 4 shows these results. Detailed discussion for them is described in the next section.

RESULTS AND DISCUSSION

Figure 4 indicates that structural parameters corresponding to damaged parts (k_{R2} and E_1) show a clear difference between the intact and crack+pinned scenarios. In contrast, PDFs of E_2 are similar. To evaluate these differences quantitatively, the Kolmogorov–Smirnov test is applied. The null/alternative hypotheses are that populations of the two distributions are different/the same. In other words, the difference between the two distributions is statistically significant when the alternative hypothesis is accepted. The significance level is adopted as 5%.

The *p*-values of the Kolmogorov–Smirnov test for each structural parameter PDF are listed in Table I. According to these results, differences in PDFs of k_{R2} and E_1 between the intact and crack+pinned scenarios are statistically significant. The bridge components corresponding to these parameters are damaged in the model bridge. On the other hand, PDFs of E_2 are not statistically significant. Hence, all damaged components are successfully identified without false positives or negatives. Moreover, we confirmed that all damaged components are identified even in other scenarios. The

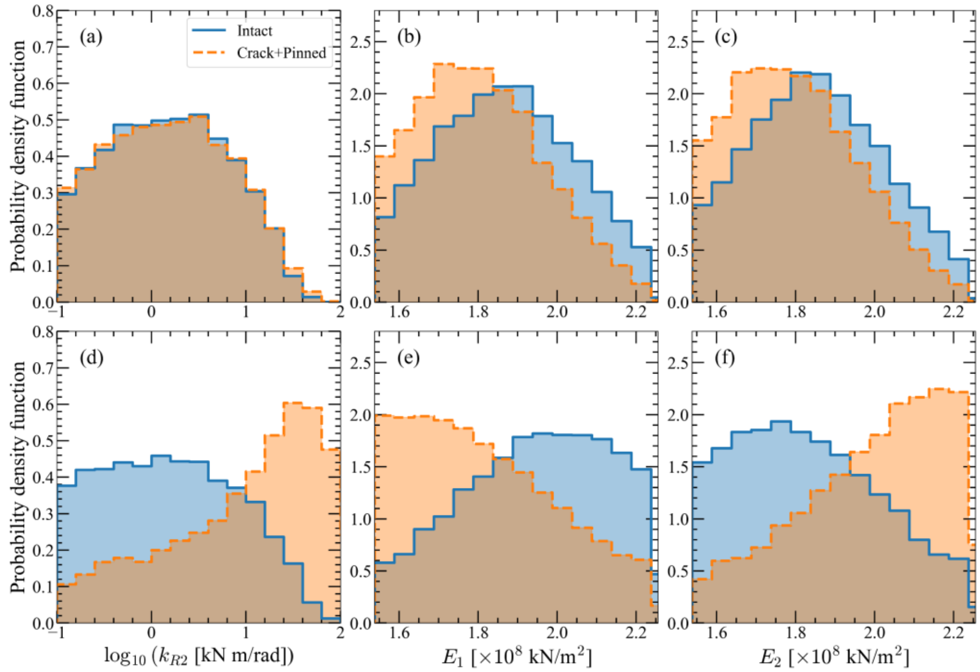

Figure 5. Estimated PDFs of structural parameters in the intact and crack+pinned scenarios derived from (top row, (a)–(c)) natural frequencies of the first and second bending modes and (bottom row, (d)–(f)) displacement ratio.

proposed method works successfully and detects any of damage scenarios composed of cracks and bearing defects in the experiment.

The sensitivity of each DSF to damage detection is also investigated. Table II summarizes damage identification results in each scenario depending on each DSF. Figure 5 shows estimated PDFs of structural parameters, k_{R2}, E_1, E_2, in the intact and crack+pinned scenarios. Figures 5 (a), (b), and (c) show estimated PDFs of the structural parameters considering natural frequencies as a target physical response in the FE model update, while Figures 5 (d), (e), and (f) show those PDFs considering the displacement ratio as a target physical response. It failed to identify the roller bearing defect in the crack+pinned scenario when natural frequencies alone are used as can be seen in Figure 5 (a). Damage identification in the pinned scenario also failed when displacement ratio alone is used. Figure 5 (f) demonstrates that E_2 increases in the crack+pinned scenario whereas the left half girder corresponding to E_2 is intact. This result does not coincide with the actual bridge condition, i.e., false positive. Moreover, scatters of the PDFs in Figure 5, except for Figure 5 (a), are higher than those in Figure 4. It indicates that uncertainties in estimated structural parameters PDFs can be reduced by integrating multiple DSFs.

CONCLUSIONS

This study investigates the feasibility of the probabilistic damage identification method for bridges by integrating multiple damage-sensitive features (DSFs). Natural frequencies and the displacement ratio are used as features. Using these DSFs, PDFs of structural parameters are estimated based on the FE model update in the in-house experiment using the model bridge. The proposed method successfully identifies all damaged components by detecting changes in the estimated PDFs. The main results are as follows.

1. PDFs of features in each scenario show their sensitivities to damage. The natural frequency of the first bending mode is sensitive to the boundary condition. In contrast, the natural frequency of the second bending mode is relatively, and the displacement ratio is strongly sensitive to flexural rigidity.
2. PDFs of structural parameters in the intact and crack+pinned scenarios are estimated from natural frequencies and the displacement ratio. The Kolmogorov–Smirnov test is applied to check the statistical significance of the differences in the PDFs. At the significance level of 5%, all structural parameters corresponding to damaged components in the crack+pinned scenario are identified without false positives. The same results were obtained in other damage scenarios.
3. When structural parameters PDFs are estimated from either natural frequencies or displacement ratio alone, it failed to detect damage.
4. Uncertainties in the estimated PDFs can be reduced when utilizing multiple DSFs.

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