Optimization of Damage Features Contaminated by Nonstationary Colored Noise Algorithms using Johansen Cointegration

SAHAR HASSANI, ULRIKE DACKERMANN
and AMIR H. GANDOMI

ABSTRACT

For structural health monitoring (SHM), researchers have primarily investigated the effects of signals contaminated with stationary white noise, with very few studies examining pollution caused by highly correlated nonstationary colored noise, such as Brown noise. To address this issue, this paper proposes an optimization-based damage detection technique for composite structures exposed to nonstationary colored noise using condensed frequency response functions (CFRF) as damage-sensitive features (DSF). Two different signal-to-noise ratios (SNR) of Brownian motions, i.e., 20 and 10, are used in the investigation to contaminate CFRFs. Contamination produces nonstationary patterns, making it difficult to detect damage with vibration-driven methods. In this study, we propose a new goal function based on Johansen cointegration, an econometric concept. The proposed objective function converts nonstationary CFRF signals into stationary representations, subsequently fed into optimization-based model updating algorithms. The Reptile Search Algorithm (RSA) is employed to update unknown structural damage indices based on the constructed objective function. The new method is validated on a finite element (FE) model simulating composite laminates with different ply orientations. By comparing the proposed method to a damage detection approach in the literature, the superiority of the proposed method is demonstrated.

INTRODUCTION

Model updating is a model-based SHM technique for improving the accuracy of numerical models simulating the behavior of structures. These methods aim to modify the unknown parameters of a structure's FE model through iteration until the measured responses match the results obtained from the updated FE model. In optimization-based model updating methods, a function based on the difference between measured and analytical structural responses is optimized to update the unknown physical parameters. Using computational techniques, an optimization algorithm provides the best solution to

Sahar Hassani, Centre for Infrastructure Engineering and Safety, School of Civil and Environmental Engineering, University of New South Wales, Sydney, NSW, Australia, Ulrike Dackermann, Centre for Infrastructure Engineering and Safety, School of Civil and Environmental Engineering, University of New South Wales, Sydney, NSW, Australia, Amir H. Gandomi, Faculty of Engineering and IT, University of Technology Sydney, Ultimo, NSW 2007, Australia, University Research and Innovation Center (EKIK), Óbuda University, 1034 Budapest, Hungary.

these objective functions. Problems can be solved by various optimization algorithms, including nonlinear programming, linear programming, metaheuristic algorithms, and integer programming. Dinh-Cong et al. [1] proposed a novel damage detection approach based on FE model updating using two sub-objectives function, modal assurance criterion (MAC) and flexibility matrix change (FMC). An algorithm based on multi-objective cuckoo search (MOCS) was applied to solve a function-based multi-objective optimization, producing a set of Pareto-optimum solutions.

Various structural responses have been used to update models, including modal information, Frequency Response Functions (FRFs), strain responses, time histories, dynamic responses, and static-modal data combinations. Complex structures, for example, three-dimensional trusses and composite structures, exhibit closely spaced modal characteristics. In the presence of such a phenomenon, it is difficult to detect damage since a significant amount of uncertainty exists in their response. In these structures, small variations in stiffness or mass can have a large effect on the modal data. As a result, such data provides suboptimal DSFs for SHM. As a superior feature, many researchers use FRFs to detect damage on systems with closely-spaced eigenvalues; however, these are highly sensitive to noise in measurements. Therefore, detecting damage to such systems through noisy FRF data is challenging. A more detailed discussion of closely-situated eigenvalues can be found in Hassani et al. [2].

White noise and colored noise are noise types that can contaminate vibration data, such as FRFs. Gaussian distributions are usually used for the former, which have a flat power spectral density (PSD). MATLAB-generated and theoretical PSD plots of noise signals of different colors are shown in Figure 1. Based on the log-log plots of the figure, it can be seen that the noise energy varies according to noise type and frequency range. Brown noise introduces the most severe nonstationarity to the signal since it primarily affects lower frequencies.

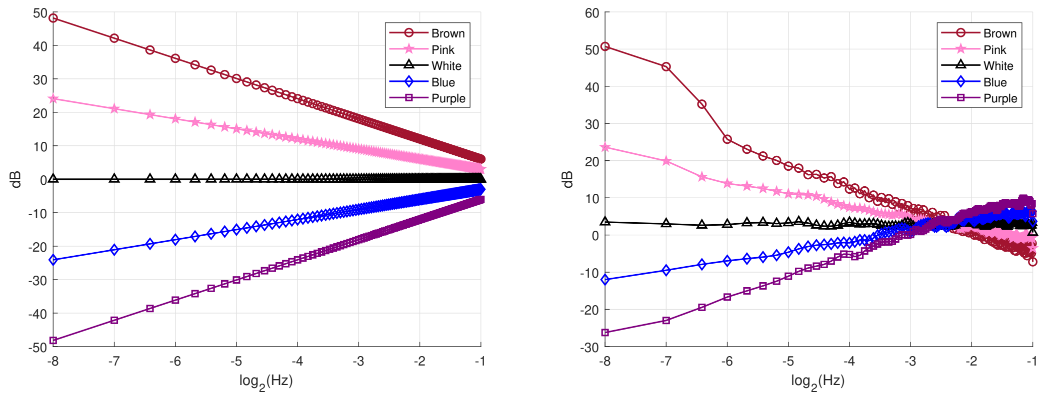
This study investigates the effects of brown noise on damage detection using CFRF signals as a nonstationary colored noise that is the most severe. Various methods are available for modeling colored noise. In this case, white noise can be converted to colored noise by passing it through a causal linear-time invariant filter [3]. For SHM applications, it is usually assumed that noise follows a stationary pattern [4]. This is a consequence of the assumption that all system dynamics are time-invariant. Therefore, a stationary colored Gaussian noise distribution can be assumed for the generated colored noise.

To reduce the effects of noise, advanced signal processing techniques, such as the Hilbert–Huang transform, can be applied to capture variations in FRF data caused by damage, even when noise levels are high. For damage detection based on vibration data, advanced signal processing is a critical component, as real-world signals tend to be nonlinear and nonstationary, especially in large and complex structures such as bridges. Signal processing approaches that use time-frequency signals extract sub-signals from an original signal based on two assumptions:

1. Decomposed sub-signals are monocomponent, which means they have one single oscillation mode. Here, the frequency of these sub-signals fluctuates around the center frequency within a narrow band.

2. Original signal is constructed from sums of its sub-signals.

Due to their first property, such sub-signals can be defined in terms of instantaneous frequency, phase, and amplitude. Using the sum of such properties of the decomposed signals, the original signal's instantaneous properties can be obtained. A number of time-frequency signal



(a) (b)

Figure 1. PSD plots of (a) theoretic and (b) noise generated by MATLAB of brown, pink, white, blue, and purple color [4].

processing approaches have been used to identify structural damage, such as Wavelet transformations, empirical mode decompositions (EMDs), variational mode decompositions (VMDs), and ensemble empirical mode decompositions (EEMDs). Applying these approaches, denoising can also be achieved. As such, Mousavi et al. [5] analyzed damage detected in steel truss bridge models using a complete ensemble EMD with an adaptive noise algorithm. Several researchers have studied the effects of noisy nonstationary patterns on FRFs. Hanson et al. [6] stated that generating FRF using colored noise is inherently unstable because in-band poles and zeros cannot be used as a reference.

The concept of cointegration was initially derived from econometrics and sought to represent nonstationary signals as stationary. An analysis of cointegration's effectiveness for SHM was published in [7]. Li et al. [8] investigated the long-term monitoring of civil infrastructure, mitigating the effects of nonstationary temperature variations.

In this study, the stationary representation is employed as a mapping and damage feature for CFRF signals contaminated with nonstationary colored noises using the cointegration technique. The use of CFRFs as DSF for damage detection has been widely studied for complex structures. Due to closely-spaced eigenvalues in these complex systems, modal data cannot be used for detecting damage primarily due to these characteristics. This paper presents a new objective function using Johansen cointegration for nonstationary signals contaminated by colored noises. An optimization algorithm inspired by the hunting behavior of crocodiles is used in this study. This algorithm is called Reptile Search Algorithm (RSA) [9]. Testing and validation of the new method are conducted on composite laminates, which are examples of complex structures with closely-spaced eigenvalues. As part of the evaluation of the proposed method, three performance criteria are used: the mean sizing error (MSE), the relative error (RE), and the closeness index (CI). The new method clearly demonstrates its ability to deal with highly correlated nonstationary colored noise. Furthermore, its superiority compared to previous methods in the literature is shown.

PROPOSED METHODOLOGY OVERVIEW

Three steps are involved in the proposed method - (1) simulating noisy CFRFs using the CFRF matrix and contaminating them based on colored brown noise, (2) formulating the DSF based on the Johansen cointegration approach, and (3) developing a novel objective function and detecting damage using the derived DSF by applying an optimization-based damage detection method.

Simulation and contamination of CFRFs with colored noise

Assuming that a n-DoF system is excited at its translational DOFs [1] by a vector of dynamic forces $\bar{\mathbf{f}}$, we can write the corresponding differential equation, based on the assumption that damage only affects stiffness:

$$\bar{\mathbf{M}}\ddot{\bar{\mathbf{x}}} + \bar{\mathbf{C}}\dot{\bar{\mathbf{x}}} + \bar{\mathbf{K}}^{\mathrm{d}}\bar{\mathbf{x}} = \bar{\mathbf{f}} \tag{1}$$

where

$$\bar{\mathbf{K}}^{\mathrm{d}} = \sum_{i=1}^{ne} \alpha_i \bar{\mathbf{k}}_i \tag{2}$$

$\bar{\mathbf{K}}^{\mathrm{d}}$, $\bar{C}$, and $\bar{M}$, represent the condensed stiffness matrix, damping matrix, and mass matrix, respectively. This equation uses the Rayleigh damping model of the form $[\bar{\mathbf{C}}] = a[\bar{\mathbf{M}}] + b[\bar{\mathbf{K}}^{\mathrm{d}}]$ in Eq.(1). This was accomplished by considering a damping ratio of 5% for the two lowest modes of the structure, b and a.

Rearranging the Fourier transform of Eq.(1) with the excitation frequency ω_k gives us:

$$\bar{\mathbf{X}}_k = \left(-\omega_k^2\bar{\mathbf{M}} + j\omega_k\ \bar{\mathbf{C}} + \bar{\mathbf{K}}^{\mathrm{d}}\right)^{-1}\bar{\mathbf{F}}_k \tag{3}$$

where

$$\bar{\mathbf{H}}_k = \left(-\omega_k^2\bar{\mathbf{M}} + j\omega_k\bar{\mathbf{C}} + \bar{\mathbf{K}}^{\mathrm{d}}\right)^{-1} \tag{4}$$

Following that, the obtained columns $\bar{H}$ are polluted by colored noise with spectral properties $|f|^{-\beta}$ where f and β correspond to cyclic frequencies and real numbers between 2 and -2.

This paper uses the following procedures to contaminate CFRF signals with colored brown noise. We first calculate the power of the signal and the power of the noise as follows:

$$P_{\bar{\mathbf{H}}(:,i)} = \frac{1}{N}\sum_{n=1}^{n=N} \bar{\mathbf{H}}(n,i)^2 \tag{5}$$

$$P_{\text{noise}} = \frac{1}{N}\sum_{n=1}^{n=N} \epsilon(n)^2 \tag{6}$$

ϵ and $\bar{\mathbf{H}}(:,i)$ represent simulated noise and the i^{th} column of $\bar{\mathbf{H}}$, respectively. To achieve the specified SNR values in db, we normalize simulated noise using λ as follows:

$$\text{SNR}_{\text{db}} = 10\,\log_{10}\left(\frac{P_{\bar{\mathbf{H}}(:,i)}}{\lambda^2\ P_{\text{noise}}}\right) \tag{7}$$

So, CFRF's i^{th} noisy column can be calculated according to this equation:

$$\bar{\mathbf{H}}_{\text{noisy}}(:,i) = \bar{\mathbf{H}}(:,i) + \lambda\ \epsilon^{\text{t}} \tag{8}$$

where $\bar{\mathbf{H}}_{\text{noisy}}(:,i)$ is considered as the i^{th} noisy column of the $\bar{\mathbf{H}}$, and the superscript t denoting the transpose operator.

[1] Master DOFs in a condensed model.

Applying Johansen cointegration to generate a noise-polluted DSF

Through the cointegration of the columns of the contaminated CFRF, a unique signal is obtained that is clear of nonstationary colored noise:

$$\mathbf{\Psi} = \sum_{j=1}^{j=p} a_j \ \bar{\mathbf{H}}_{\text{noisy}}(:, j) \tag{9}$$

where $\mathbf{\Psi}$ represents the residual of the Johansen cointegration of the brown noisy CFRF matrix columns $\bar{\mathbf{H}}_{\text{noisy}}$, assigned as CICFRF; p represents the number of excitation points in $\bar{\mathbf{H}}_{\text{noisy}}$; and a_j corresponds to the cointegration coefficient for the j^{th} row based on Eq.(9). Accordingly, the first eigenvector, which corresponds to the largest eigenvalue, produces CICFRF_1, the most stationary combination of CFRF columns. Consequently, CICFRF_2 shows a less stationary CFRF column combination because of its second eigenvector. The results obtained from CICFRF_1 and CICFRF_2 are further compared in the section below, so CICFRF_1 is considered to be a DSF in this study.

Proposed objective function

The sum of the partial derivatives of stiffness resulting from damage is expressed as follows:

$$\delta\bar{\mathbf{K}} = \sum_{i=1}^{n} \frac{\partial\bar{\mathbf{K}}}{\partial\hat{\alpha}_{\text{i}}}\delta\hat{\alpha}_{\text{i}} \tag{10}$$

In addition, the variations in structural response can be expressed as follows:

$$\delta\bar{\mathbf{X}}^{\text{c}} \simeq -\bar{\mathbf{H}}^{\text{m}} \times \delta\bar{\mathbf{K}} \times \bar{\mathbf{X}}^{\text{c}} \tag{11}$$

Measured and computed quantities are indicated by the superscripts m and c, respectively. Eq.(10) can be substituted into Eq.(11) by rewriting the result as:

$$\delta\bar{\mathbf{X}} \simeq \bar{\mathbf{S}} \times \delta\hat{\alpha} \tag{12}$$

where

$$\bar{\mathbf{S}} = \big[-\bar{\mathbf{H}}^{\text{m}} \left(\frac{\partial\bar{\mathbf{K}}}{\partial\hat{\alpha}_1}\right) \bar{\mathbf{X}}^{\text{c}}, \ldots, -\bar{\mathbf{H}}^{\text{m}} \left(\frac{\partial\bar{\mathbf{K}}}{\partial\hat{\alpha}_n}\right) \bar{\mathbf{X}}^{\text{c}}\big] \tag{13}$$

The following can be written based on Eqs.(3) and (4):

$$\delta\bar{\mathbf{X}} = \underbrace{\bar{\mathbf{H}}^{\text{m}} \times \bar{\mathbf{F}}}_{\bar{\mathbf{X}}^{\text{m}}} - \underbrace{\bar{\mathbf{H}}^{\text{c}} \times \bar{\mathbf{F}}}_{\bar{\mathbf{X}}^{\text{c}}} \tag{14}$$

Accordingly, at the t^{th} iteration, $\delta\hat{\alpha}$ is expressed as follows:

$$\delta\hat{\alpha}_t \simeq \left(\bar{\mathbf{S}}\right)^+ \ \left(\bar{\mathbf{H}}^{\text{m}} \times \bar{\mathbf{F}} - \bar{\mathbf{H}}_t^{\text{c}} \times \bar{\mathbf{F}}\right) \tag{15}$$

where $\bar{\mathbf{H}}_t^{\text{c}}$ corresponds to the CFRF computed at time t using an updated damage vector at time $t-1$, i.e. $\hat{\alpha}_{t-1}$, with $\hat{\alpha}_0 = \mathbf{0}$. Inverses of non-square matrices are obtained using the Moore–Penrose inverse with a superscript $+$.

By replacing $\bar{\mathbf{H}}^{\text{m}}$ and $\bar{\mathbf{H}}^{\text{c}}$ by $\mathbf{\Psi}^{\text{c}}$ and $\mathbf{\Psi}^{\text{m}}$, the following equation is obtained:

$$\delta\alpha_t^{\mathbf{\Psi}} \simeq \left(\mathbf{S}^{\mathbf{\Psi}}\right)^+ \ \left(\mathbf{\Psi}^{\text{m}} \times \bar{\mathbf{F}} - \Psi_t^{\text{c}} \times \bar{\mathbf{F}}\right) \tag{16}$$

Eq. 17 can be expressed as an objective function as follows:

$$G(\alpha_t^{\boldsymbol{\Psi}}) = \delta\alpha_t^{\boldsymbol{\Psi}} - \left(\mathbf{S}^{\boldsymbol{\Psi}}\right)^+ \left(\boldsymbol{\Psi}^{\mathrm{m}} \times \bar{\mathbf{F}} - \Psi_t^{\mathrm{c}} \times \bar{\mathbf{F}}\right) \tag{17}$$

In this paper, Eq.(17) is proposed to detect damage in structures with close eigenvalues. During iteration, Eq.(17) is optimized using the RSA, where $\hat{\alpha}_t$, the value of $\alpha^{\boldsymbol{\Psi}}$ at t^{th} iteration, is updated as $\alpha_t^{\boldsymbol{\Psi}} = \alpha_{t-1}^{\boldsymbol{\Psi}} + \delta\alpha_t^{\boldsymbol{\Psi}}$, where $\alpha_0^{\boldsymbol{\Psi}} = \mathbf{0}$. The flowchart of the proposed algorithm can be found in Figure 2.

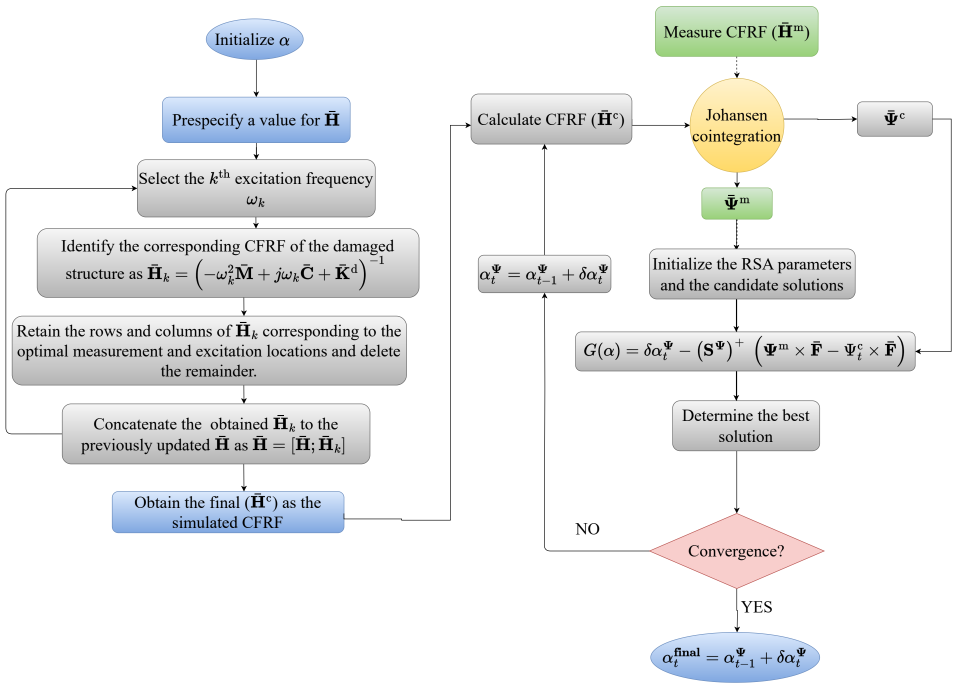


Figure 2. Proposed algorithmic flowchart for damage detection.

NUMERICAL EXAMPLE

Numerical models of composite laminate plates are used to demonstrate the new damage detection method. For this purpose, we adopted the structure and mechanical properties of the composite plate presented by Reddy [10]. The following two configurations of the plate are considered:

- Laminate composite plate with three layers (NoL=3) with ply orientation LA=$(0°/90°/0°)$.
- Laminate composite plate with six layers (NoL=6) with ply orientation LA=$(0°/45°/0°)$.

Each plate has 36 elements with two rotational and three translational DoFs per node, totaling 245 DOFs. Following the imposition of boundary conditions, 125 DOFs remain active. The proposed method is evaluated in relation to two different damage scenarios as listed below:

- Scenario 1: In this case, stiffness reduction is present in elements 4, 16, 24, and 31 with amounts of 0.20, 0.25, 0.30, and 0.15, respectively.

TABLE I. NATURAL FREQUENCIES OF COMPOSITE LAMINATE PLATES WITH VARIOUS CONFIGURATIONS (FIRST TEN MODES).

Lamination scheme		Mode Number 1	2	3	4	5	6	7	8	9	10
Intact	NoL = 3, LA = $(0°/90°/0°)$	7.40	11.14	14.32	16.23	18.74	21.42	23.32	23.90	25.74	26.29
	NoL = 6, LA = $(0°/45°/0°)$	7.64	11.53	14.74	16.82	19.07	21.99	23.78	24.90	25.78	26.60
Case 1	NoL = 3, LA = $(0°/90°/0°)$	7.22	11.00	14.09	16.10	18.50	21.15	22.80	23.68	25.30	26.04
	NoL = 6, LA = $(0°/45°/0°)$	7.45	11.35	14.50	16.65	18.80	21.73	23.37	24.66	25.38	26.20
Case 2	NoL = 3, LA = $(0°/90°/0°)$	7.36	10.96	14.20	15.95	18.50	21.07	23.08	23.50	25.40	25.80
	NoL = 6, LA = $(0°/45°/0°)$	7.57	11.30	14.60	16.50	18.70	21.69	23.48	24.40	25.45	26.10

- Scenario 2: In this case, stiffness reduction is present in elements 3, 10, 12, and 36 with amounts of 0.2, 0.25, 0.3, and 0.15, respectively.

Table I shows ten natural frequencies for both intact and damaged composite laminate plates. The plates' optimal excitation location and frequency range was previously identified by Hassani et al. [4]. As described above, CFRFs are synthesized using the proposed method and contaminated by brown noise–a colored noise that is non-stationary. Figure 3 illustrates the noise-free and different noisy CFRFs, noise-contaminated with the colors brown, pink, and purple. As can be seen from the figures, the CFRF that is contaminated with brown noise at SNR=10 is more distorted than the CFRF that is contaminated with pink or purple noise at SNR=20. Hence, our analysis focuses on CFRFs contaminated with nonstationary brown noise with both SNRs (10 and 20).

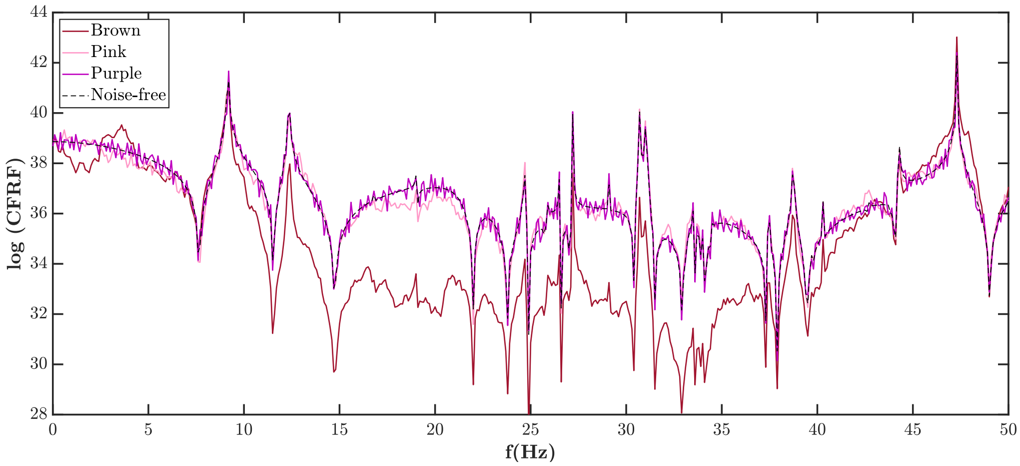
Figure 4 presents the fitness results obtained by solving Eq. (17) and applying the RSA optimization algorithm to all damage scenarios. According to the figure, our proposed objective function converges after only a few iterations, indicating that the chosen optimization algorithm is appropriate. Table II displays the damage identification accuracy indices (MSE, CI, and RE) for the investigated laminated composite models using the original CFRF and our proposed CICFRF signals. It is noted that we adopted the damage indices of $|MSE|$, $|RE|$, and $|CI|$ from Dos Santos et al. [11].

The accuracy indices in Table II clearly show a significantly improved proposed method performance using CICFRFs compared to using the original CFRFs or the method proposed by Vo-Duy [12]. It is important to note that acceptable damage indices lie within the following range: $|MSE|$ close to 0, $|RE|$ close to 0, $|CI|$ close to 1.

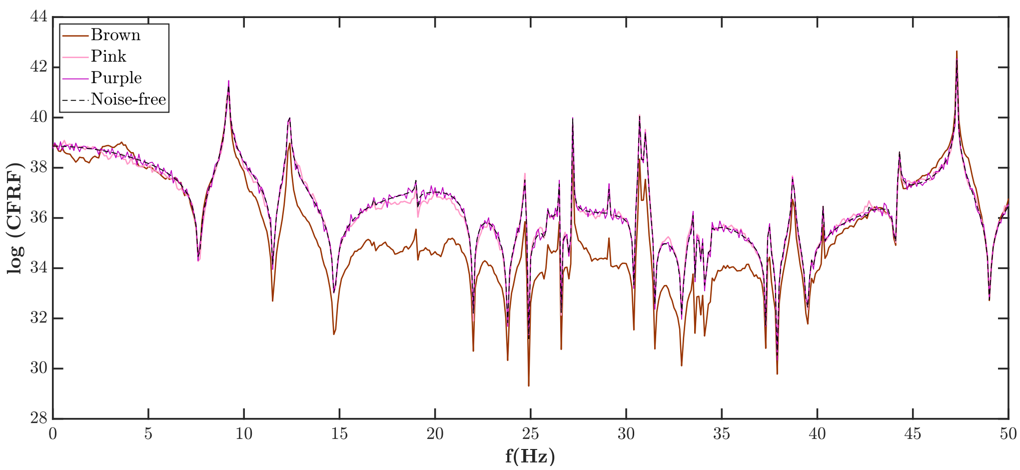
In Figure 5, we display the calculated CICFRF_1 and CICFRF_2 vectors from Eq.(9) of our new method. As discussed earlier, in the proposed objective function, CICFRF_1 and CICFRF_2 are used as input signals for RSA optimization. According to the results, the proposed algorithm performs much better when CICFRF_1 is used instead of CICFRF_2. The superior performance of CICFRF_1 compared with CICFRF_2 results from its improved stationary nature.

CONCLUDING REMARKS

In this paper, we proposed a new objective function to deal with nonstationary colored noise contamination in laminated composite structures. For damage detection, RSA was used to optimize the objective function. The new method used the Johansen cointegration algorithm to cointegrate CFRFs contaminated with brown noise characterized by nonstationarity following the



(a) SNR=10

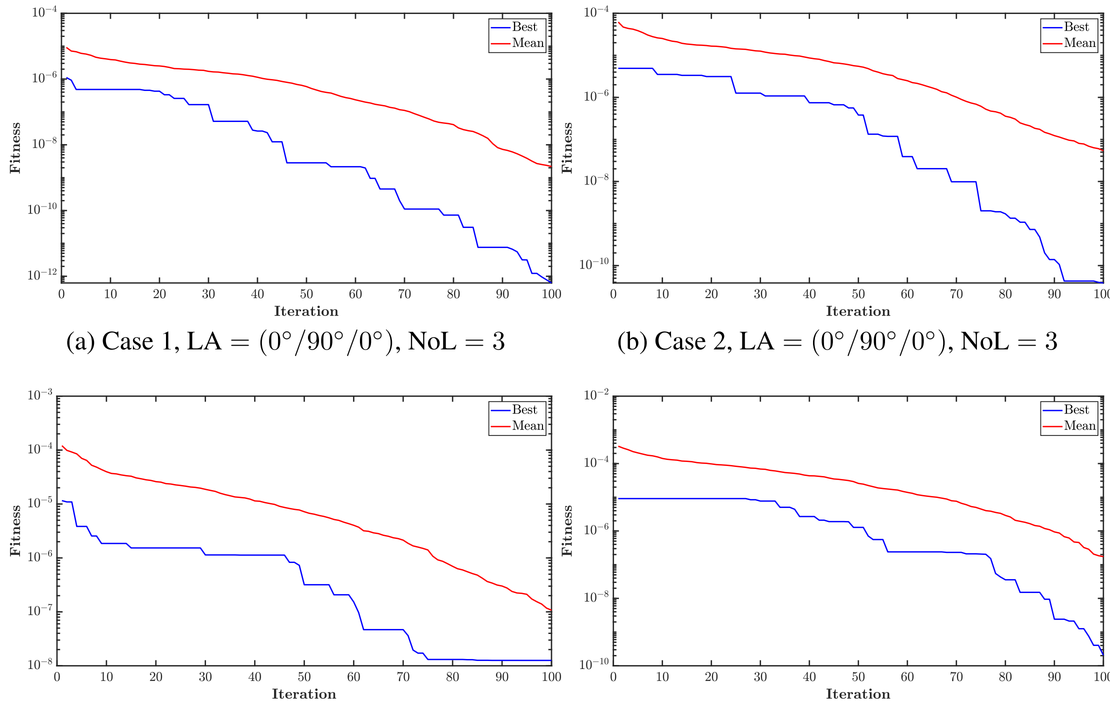


(b) SNR=20

Figure 3. Comparison of CFRFs contaminated with different levels of colored noise and noise-free CFRFs. Based on the plate model, data is obtained for NoL $= 6$, LA $= (0°/45°/0°)$, excitation at DOF 21 and measurement at DOF 62.

Brownian process. For the purpose of demonstrating the effectiveness of the proposed damage detection method, CFRFs highly polluted with colored noise, i.e., SNR=10, were used. This method is based on the assumption that it can reduce the effects of nonstationary colored noise in CFRF signals. In order to test this hypothesis, two cointegration residual vectors were fed into the damage detection problem, namely the CICFRF_1, and CICFRF_2. Based on the fact that CICFRF_1 is more stationary than CICFRF_2, each vector was evaluated for its effectiveness in detecting damage in the investigated laminated composite plate. The obtained results confirmed the aforementioned hypothesis as the method using CICFRF_1 performed significantly better. To further illustrate the method's capabilities, a comparison with recent methods from the literature was carried out. Results showed that the use of CICFRF_1 yielded significantly better damage detection outcomes when the input data (columns of CFRF) were heavily influenced by brown noise.

REFERENCES



(a) Case 1, LA $= (0°/90°/0°)$, NoL $= 3$ (b) Case 2, LA $= (0°/90°/0°)$, NoL $= 3$

(c) Case 1, LA $= (0°/45°/0°)$, NoL $= 6$ (d) Case 2, LA $= (0°/45°/0°)$, NoL $= 6$

Figure 4. Convergence trace of the RSA for 100 iterations for both damage cases and with SNR=10.

TABLE II. SUMMARY OF ERROR INDICES FOR DIFFERENT DAMAGE CASES IN COMPOSITE LAMINATE PLATES WITH VARIOUS CONFIGURATIONS AND SNRS.

Scenario No.	Applied method	SNR	LA $= (0°/90°/0°)$, NoL $= 3$ MSE	RE	CI	LA $= (0°/45°/0°) \times 2$, NoL $= 6$ MSE	RE	CI
1	$CICFRF_1$	10	0.0039	-0.1200	0.9634	0.0035	-0.0979	0.9657
1	$CICFRF_1$	20	0.0033	-0.1079	0.9723	0.0034	-0.0962	0.9861
1	$CICFRF_2$	10	0.0071	-0.1439	0.7951	0.0068	-0.1359	0.8899
1	$CICFRF_2$	20	0.0068	-0.1345	0.9090	0.0070	-0.1200	0.9089
1	[12]	10	0.2923	-6.9888	-0.5678	0.1999	-7.5999	-0.8569
1	[12]	20	0.2342	-5.9999	-0.8087	0.1789	-6.8900	-0.8000
1	CFRF	10	0.0878	-3.4567	-0.8899	0.0943	-4.2005	-0.8904
1	CFRF	20	0.1300	-5.4345	-0.8176	0.1534	-4.8907	-0.8976
2	$CICFRF_1$	10	0.0040	-0.1172	0.9617	0.0037	-0.0989	0.9658
2	$CICFRF_1$	20	0.0035	-0.1082	0.9717	0.0035	-0.0965	0.9858
2	$CICFRF_2$	10	0.0068	-0.1425	0.8456	0.0059	-0.1199	0.8869
2	$CICFRF_2$	20	0.0045	-0.1225	0.9056	0.0043	-0.1299	0.9069
2	[12]	10	0.1912	-5.9007	-0.5159	0.1989	-7.5007	-0.8569
2	[12]	20	0.1612	-4.9999	-0.8979	0.1589	-5.8976	-0.8907
2	CFRF	10	0.0890	-3.4566	-0.8746	0.0923	-4.5002	-0.8936
2	CFRF	20	0.1290	-4.4566	-0.7756	0.1923	-4.7777	-0.9178

1. Dinh-Cong, D. and T. Nguyen-Thoi. 2023. "An effective damage identification procedure using model updating technique and multi-objective optimization algorithm for structures made of functionally graded materials," *Engineering with Computers*, 39(2):1229–1247.
2. Hassani, S., M. Mousavi, and A. H. Gandomi. 2022. "A mode shape sensitivity-based method for damage detection of structures with closely-spaced eigenvalues," *Measurement*:110644.
3. Adhikari, K. and S. Kay. 2021. "Optimal Sparse Sampling for Detection of a Known Signal in Non-white Gaussian Noise," *IEEE Signal Processing Letters*, 28:1908–1912.
4. Hassani, S., M. Mousavi, and U. Dackermann. 2023. "Johansen cointegration of frequency response functions contaminated with nonstationary colored noise for structural damage detection," *Journal of*

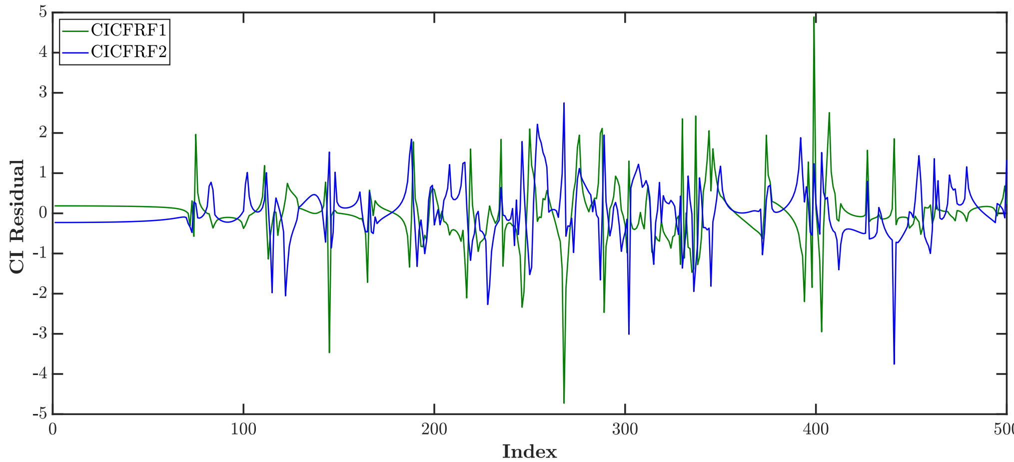


Figure 5. Johansen residual vectors of CI_1 and CI_2 – $CICFRF_1$ and $CICFRF_2$ (SNR=10, with LA $= (0°/90°/0°)$),NoL $= 3$.

Sound and Vibration, 552:117641.

5. Mousavi, A. A., C. Zhang, S. F. Masri, and G. Gholipour. 2022. "Structural damage detection method based on the complete ensemble empirical mode decomposition with adaptive noise: A model steel truss bridge case study," *Structural Health Monitoring*, 21(3):887–912.
6. Hanson, D., R. Randall, J. Antoni, D. Thompson, T. Waters, and R. Ford. 2007. "Cyclostationarity and the cepstrum for operational modal analysis of mimo systems—Part I: Modal parameter identification," *Mechanical systems and signal processing*, 21(6):2441–2458.
7. Cross, E. J. and K. Worden. 2012. "Cointegration and why it works for SHM," in *Journal of Physics: Conference Series*, IOP Publishing, vol. 382, p. 012046.
8. Li, X., W. Qu, L. Xiao, and Y. Lu. 2019. "Removal of temperature effect in impedance-based damage detection using the cointegration method," *Journal of Intelligent Material Systems and Structures*, 30(15):2189–2197.
9. Abualigah, L., M. Abd Elaziz, P. Sumari, Z. W. Geem, and A. H. Gandomi. 2022. "Reptile Search Algorithm (RSA): A nature-inspired meta-heuristic optimizer," *Expert Systems with Applications*, 191:116158.
10. Reddy, J. N. 2003. *Mechanics of laminated composite plates and shells: theory and analysis*, CRC press.
11. Dos Santos, J. A., C. M. Soares, C. M. Soares, and N. Maia. 2005. "Structural damage identification in laminated structures using FRF data," *Composite Structures*, 67(2):239–249, doi: 10.1016/j.compstruct.2004.09.011.
12. Vo-Duy, T., V. Ho-Huu, H. Dang-Trung, and T. Nguyen-Thoi. 2016. "A two-step approach for damage detection in laminated composite structures using modal strain energy method and an improved differential evolution algorithm," *Composite Structures*, 147:42–53.