# Structural Parameter Identification with a Physics-Informed Neural Networks-Based Framework

WEIJIA ZHANG, YI-QING NI, LEI YUAN, SHUO HAO and SU-MEI WANG

**ABSTRACT**

Structural parameter identification is a critical aspect of structural health monitoring and maintenance. It belongs to inverse problems, which aim to discover mechanical parameters, such as the stiffness of the concerned system from collected measurement data. Then, the identified parameters are used to evaluate the status of the structure and detect potential structural damage in advance. As a promising machine learning method that effectively combines domain knowledge and deep neural networks, physics-informed neural networks (PINNs) have been widely applied in various fields, including structural parameter identification and structural health monitoring. In this study, we propose a novel framework based on PINNs for parameter identification in structural systems. The framework contains two main components, one is called the physical term that employs PINNs to learn the prior physical knowledge of the structural system; the other is called the discrepancy term that involves another PINNs or even a simple feedforward neural network to present the differences between the observed data and the physical term. In a nonlinear structural system, the physical term can be regarded as the linear part of the structural response, while the discrepancy term represents the nonlinear response. In the PINNs configuration, the residuals of the governing equation, which are calculated by substituting collocation points that are randomly sampled within the domain into the governing equation, are directly incorporated into the total loss function. In addition, the boundary conditions, as well as the initial conditions, are soft-embedded as essential parts in the total loss function. The measurement data is also required by an inverse problem, and the differences between the measurement data and the prediction produced by the PINNs-based physical term will be captured by the discrepancy term. Through the discrepancy term, the nonlinear structural parameter can be figured out. Finally, a two-degree of freedom mass-spring system with nonlinear spring is investigated to illustrate the ability of the proposed framework in structural parameter identification.

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## INTRODUCTION

Structural parameter identification is a fundamental process employed in engineering and physics to infer key properties of physical systems, including their mass, stiffness, and damping, based on measurements taken from the system [1-2]. It involves developing mathematical models of the system and then comparing the predicted behaviour of the model with the actual behaviour of the system, as measured by sensors or other instruments. By adjusting the parameters of the model, the structural properties of the system can be estimated more accurately.

Structural parameter identification has applications in many fields, including civil engineering [3] and mechanical engineering [4]. In structural engineering, structural parameter identification can be used to detect damage or degradation in a structure, such as cracks, corrosion, or fatigue, by comparing the measured response of the structure to a reference model. Furthermore, it helps to monitor the health of a structure over time, by tracking changes in its behaviour and properties, and alerting operators to potential problems before they become critical. By accurately estimating the structural properties of a system, structural parameter identification can improve its performance, increase its reliability, and reduce the risk of failure or catastrophic events. Structural parameter identification is also essential for ensuring the safety and sustainability of critical infrastructure, such as buildings, bridges, dams, and transportation systems [5].

The commonly used methods for structural parameter identification can be broadly classified into two categories. One is model-based methods such as finite element method, modal analysis, and frequency domain decomposition. The other is data-driven methods including machine learning methods [6-9]. In recent years, physics-informed neural networks (PINNs) have become an emerging field of machine learning, which fills the gap between model-based methods and data-driven methods by incorporating physical laws into neural networks [10]. The principle of PINNs is directly embedding the governing equation and related conditions into loss functions. This well-designed framework has been proven effective in solving a diverse range of differential problems including ordinary differential equations [11] and partial differential equations [12-13]. PINNs are a powerful tool for modelling nonlinear behaviour because of its prior ability in capturing the underlying relationship between inputs and outputs. In addition, PINNs are also robust for dealing with inverse problems even with noisy data by defining unknowns as trainable parameters in the deep neural networks.

In this study, PINNs are utilized to identify nonlinear structural parameters from dynamical data. However, PINNs sometimes fail to train when solving dynamical problems. Therefore, we propose a framework to enhance the performance of original PINNs by reconstructing it into two components including a physical term and a discrepancy term, and introducing auxiliary outputs to learn the derivative of the solution. A two degree of freedom mass-spring system is used as the numerical example to demonstrate the effectiveness of the proposed framework.

## METHODS

In this section, physics-informed neural networks (PINNs) which is a newly emerged machine learning method for solving differential equations will be introduced first. After briefing the basic knowledge of PINNs, the framework for structural parameter identification, which reconstructs PINNs into a physical term and a discrepancy term will be proposed. In addition, the auxiliary outputs will also be used to facilitate computation.

## Physics-Informed Neural Networks (PINNs)

PINNs are a physics-based machine learning method designed for generating surrogate model of ordinary or partial differential equations by integrating the governing equation into the neural network architecture as a loss function. This paradigm works efficiently in solving differential equations mainly because its critical component and a useful embedded function, namely deep neural networks (DNNs) and automatic differentiation (AD), respectively.

DNNs are also called multi-layer perceptron (MLP), which is a fully connected neural network consists of an input layer, an output layer, and several hidden layers with neurons in each layer. A DNN takes one or more inputs in, then generate outputs through a linear transformation and an activation function in each hidden layer. This process is called forward propagation that can be explained in the following formula:

$$z(x) = \sigma(w_i x + b_i) \quad (1)$$

where $x$ and $z$ are the input and output of a hidden layer, respectively; $w_i$ and $b_i$ are hyperparameters weight and bias in the hidden layer, respectively; $\sigma(\cdot)$ is an activation function, which can be Tanh, ReLU, or others [14]. Specially, there is no activation function in the last hidden layer. Thanks to the multi-layer structure of DNNs imitating human neural networks, DNNs possess a strong ability in capturing the underlying relationship between inputs and outputs, even the relationship contains complex factors or is not easy to be expressed by a theoretical model.

To achieve a more accurate prediction, the hyperparameters of hidden layers will be optimized by gradient descent algorithms according to the loss between network outputs and labelled data during the training process. This procedure is called backward propagation, which involves commonly used optimizers including L-BFGS and ADAM.

In the backward propagation, the gradient of each output is required for optimization. Therefore, the derivatives of outputs with respect to inputs will be calculated according to the forward propagation chain by the AD function [15], which is based on the chain rule and pre-embedded in many machine learning frameworks such as TensorFlow and PyTorch. AD is a useful function because it can compute the derivatives in the computational graph automatically and explicitly, which makes the updating of hyperparameters in the backward propagation feasible.

With two key components, DNNs and AD, the basic framework of PINNs is established. Then, the most important step is to encode the previously known physical laws into DNNs. The physical information is usually described by a governing equation with certain boundary and initial conditions:

$$D^n[u(x,t);\sigma] + f(x,t) = 0, \quad x \in \Omega, \ \ t \in \mathrm{T} \quad (2)$$

$$B[u(0,t), x = 0, t] = 0 \ \ on \ \ \Omega \quad (3)$$

$$I[u(x,0), x, t=0] = 0 \quad on \ \mathrm{T} \tag{4}$$

where $u(x,t)$ is the solution to the governing equation; $x$ and $t$ denote spatial and time variables, respectively; $D^n(\cdot)$ denotes a differential operator taking nth order derivatives; $\sigma$ denotes a parameter in $D^n(\cdot)$; $B(\cdot)$ and $I(\cdot)$ denote the boundary and initial conditions, respectively.

The way of PINNs to make the DNNs obey the physical law is to translate the governing equation to a loss function and minimize it. In an inverse problem, the parameter $\sigma$ is unknown. The total loss function can be formulated as:

$$MSE_{total} = w_f \cdot MSE_f + w_b \cdot MSE_b + w_i \cdot MSE_i + w_m \cdot MSE_m \tag{5}$$

in which $MSE$ means the mean square error of residuals between true values and predicted values; $MSE_f$, $MSE_b$, $MSE_i$, and $MSE_m$ denote the penalty terms for the governing equation, boundary conditions, initial conditions, and measurement data, respectively; $\theta$ denotes hyperparameters, i.e. weights and biases, of DNN.

Because of the presence of the unknown parameter, we have to rely on measurement data. Fortunately, only a few measurement data is required by PINNs, which is also an outstanding merit of PINNs. By minimizing the total loss function to approach zero, PINNs will generate the solution to the governing equation and discover the unknown parameter simultaneously without discretization and truncation errors because this is a mesh-free method. A framework of PINNs for inverse problems is shown in Figure 1.

## The Framework for Structural Parameter Identification

Although PINNs are suitable for solving nonlinear problems due to deep neural networks' superior ability to approximate nonlinear functions, PINNs may face difficulties in the case of high order governing equations. To improve the efficiency of computation, we proposed a novel framework in this subsection.

First, we reconstruct the original PINNs into two parts: one is a physical term, the other is a discrepancy term. Both of them can be represented by individual PINNs or even DNNs. This is a flexible representation of original PINNs. For example, if there is a nonlinear structural system, the physical term can learn the linear component, while the discrepancy term can capture the nonlinear behaviour. By adopting this method, the computational burden of original PINNs will be reduced, and the efficiency will be accordingly improved.

Then, we use auxiliary outputs to reduce the order of governing equations, which will significantly improve the accuracy because only automatic differentiation for lower order is required. This method is inspired by the idea of auxiliary physics-informed neural network [16] and neural ordinary differential equation [17]. The former one uses auxiliary outputs to calculate integral in integro-differential equations (IDEs), while the latter one learns the derivative of the hidden states. This method can be implemented by defining auxiliary outputs such as the first-order derivative of the solution:

$$v(x) = u'(x) \tag{6}$$

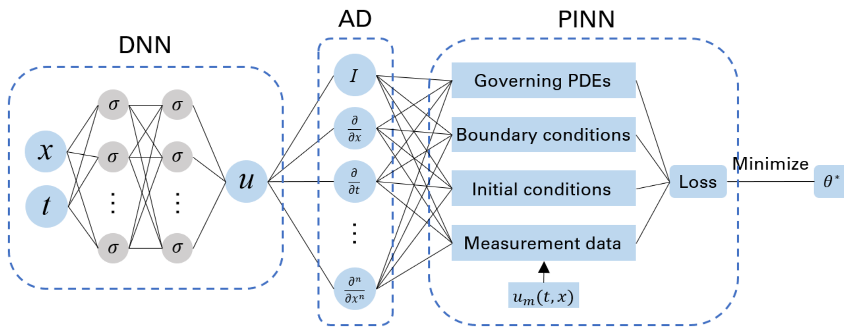


Figure 1. A typical framework of PINNs for inverse problems.

where $u(x)$ is the solution of the structural system, and $v(x)$ is the derivative of $u(x)$ with respect to $x$. Therefore, an extra penalty term should be added to the total loss function:

$$MSE_a = \frac{1}{N_f}\sum_{i=1}^{N_f}\left|\frac{du_{pred}(x_i^f;\theta)}{dx} - v_{pred}(x_i^f;\theta)\right|^2 \tag{7}$$

To summarize, the proposed framework is illustrated in Figure 2.

## NUMERICAL EXAMPLE

In this section, we use a numerical example to demonstrate the ability of our proposed framework in identifying the unknown parameter in the dynamical structural system. It is a mass-spring system with two degrees of freedom (DOF), which is shown in Figure 3. The first mass is connected to a nonlinear spring with squared nonlinearity $k_n u_1^2$.

The equations of motion under free vibration can be expressed as:

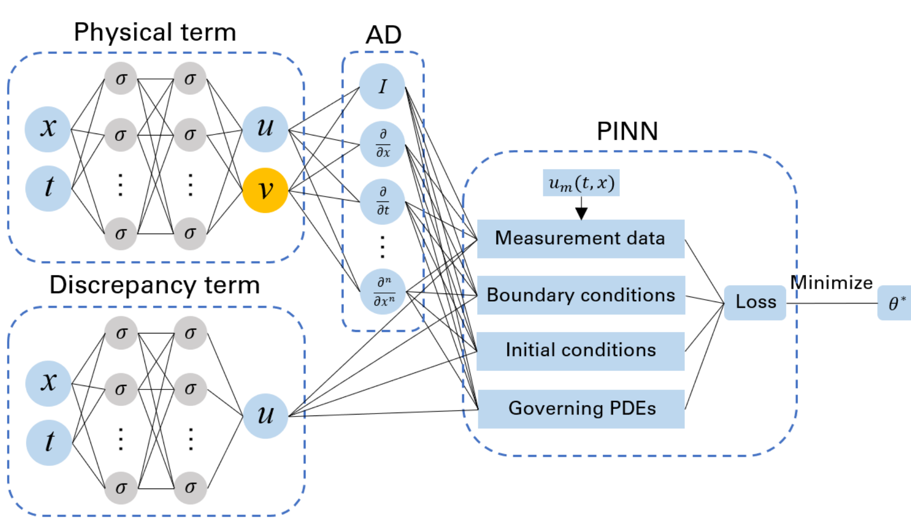


Figure 2. The proposed PINNs-based framework.

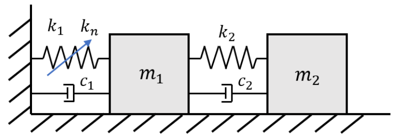


Figure 3. The mass-spring system.

$$\begin{cases} m_1\ddot{u}_1(t) + c_1\dot{u}_1(t) + (k_1 + k_2)u_1(t) - k_2u_2(t) + k_nu_1^2(t) = 0 \\ m_2\ddot{u}_2(t) + c_2\dot{u}_2(t) + k_2u_2(t) - k_2u_1(t) = 0 \end{cases} \tag{8}$$

where $u_i$, $\dot{u}_i$, and $\ddot{u}_i$ are displacement, velocity, and acceleration, respectively. For generalization, $m_i$ are set to be 1, $c_i$ are set to be 0.5, and $k_i$ are set to be 10. The coefficient of the nonlinear spring $k_n$ is assumed to be 3. Equation (8) can be written in the matrix form as:

$$\begin{bmatrix} \dot{u}_1 \\ \dot{u}_2 \\ \ddot{u}_1 \\ \ddot{u}_2 \end{bmatrix} = \begin{bmatrix} 0 & I \\ -K & -C \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ \dot{u}_1 \\ \dot{u}_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -k_nu_1^2 \\ 0 \end{bmatrix} \tag{9}$$

where

$$K = \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix}, C = \begin{bmatrix} c_1 & 0 \\ 0 & c_2 \end{bmatrix} \tag{10}$$

In this numerical example, the aim is to identify the coefficient $k_n$. The analytical solution is difficult to obtain due to the nonlinearity. Therefore, we use ANSYS to get the numerical solution. In the numerical simulation, MASS element is used to model the mass, the COMBIN7 element is used to model the linear spring with stiffness and damping, and COMBIN39 element is used to model the nonlinear spring. The numerical solution of 10 seconds is used as training set.

In the framework, two DNNs with 3 hidden layers with 20 neurons in each layer are used. The input of both DNNs is time *t*, the outputs of the first DNN are displacement $u_i$ and velocity $\dot{u}_i$ of each DOF, while the output of the second DNN is $-k_nu_1^2$. The first DNN is designed as the physical term modelling the linear response of the system, which is the first term on the right-hand side of Eq. (9). The second DNN is the discrepancy term capturing the nonlinear response of the system, which is the second term in Eq. (9). The optimizer used is L-BFGS with a learning rate of 0.1. The unknown coefficient $k_n$ can be identified after the training process. The initial conditions in this case are $u_1^0 = 0.0997$, $u_2^0 = 0.1997$, $\dot{u}_1^0 = 0.0997$, and $\dot{u}_2^0 = 0.1997$.

The results of learning the nonlinear behaviour of the structural system by the proposed framework is shown in Figure 4. It can be seen that the predicted solutions of $u_i$ match the measured values well, which illustrates the ability of the proposed framework to learn nonlinear behaviour and make accurate prediction. However, the nonlinearity in this case is not very significant, thus, the next objective in our future work is to solve problems with higher nonlinearity. The unknown parameter $k_n$ is identified to be 2.98 (the exact value is 3.00) with an error of 0.67%, which also proves that the proposed PINNs-based framework can identify unknown structural parameters accurately with limited measurement data.

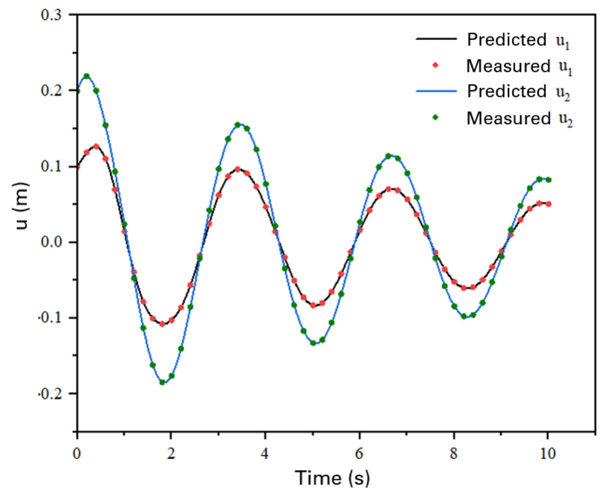


Figure 4. The comparison of the predicted solution and the numerical solution.

## CONCLUSION

In this paper, we introduce a novel two-component framework that employs physics-informed neural networks (PINNs) to accurately and efficiently identify structural parameters. The framework consists of two components: the physical term and the discrepancy term. The physical term utilizes PINNs to learn the underlying physical knowledge of the structural system, while the discrepancy term employs another PINN or a simple feedforward neural network to capture the differences between the observed data and the physical term. Together, these two components enable the identification of unknown structural parameters with high accuracy.

We demonstrate the effectiveness of our proposed framework through a numerical example that models nonlinear behaviour. The results show that our approach is capable of identifying unknown structural parameters with high accuracy. We believe that this framework has significant potential for applications in other fields and plan to explore its use in solving complex engineering problems.

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