Structural Performance Monitoring Employing Linear Observer

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ABSTRACT

Structural performances are heavily relying on the overall health conditions (damaged or healthy) of any dynamical system. Hence, it is essential to keep them monitored to avoid any partial/fully damaged situation. Due to the availability of the modern technologies, the monitoring tasks are done by employing sensors to reduce manual effort. However, the structural health monitoring via sensors deployment comes with a cost even considering all the merits of modern monitoring approach. Therefore, it is realistic to have a reasonable number of sensors on the structures in order to avoid financial hurdle or to make things more feasible. In order to minimize the sensors number, in this study, the investigations have been done via employing finite number of sensors. As a result, it might be tricky to obtain the missing states of the degree-of-freedom where sensor was not placed. Herein, the missing states are estimated by adopting a linear type observer e.g. Kalman filter as all of the states have not been observed. The numerical simulations have been performed by considering a 7-storey structure in a nearly real-time scenario via the use of MATLAB and SIMULINK. To achieve the optimal performance, the Kalman Gain was also estimated real-time by solving the Riccati equation of the investigated system. The performance of the investigated problem has been evaluated under healthy and different equivalent damaged conditions by adding external noise quantities to the healthy signals. In a nutshell, it is observed that the observer is capable of rendering the original behavior of the structure quite accurately under both healthy and damaged conditions. However, it has been observed that with the significant level of noise the observer struggles a lot attain optimal performances.

INTRODUCTION

In the past decades, it has been quite common that the modern structures are getting accompanied with various types of sensors for their monitoring purpose as well as for controlling them under extreme vibration. The sensor applications onto the structures are gaining attention in the area of the structural health monitoring (SHM) to deal with

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unknown uncertainties and complexity. For instance, a non physics-based structural model is the hope to monitor and understand the fundamental phenomena of the structure where a physics-based structural may not be available. In the aforementioned context, having sensors on or in the structure may be beneficial to keep them in smooth operation or access their overall health conditions. However, due to the feasibility issue, it is not possible to have as many sensors as it might be necessary. Therefore, in order to have the missing degree-of-freedom's (DOF) information it is essential to adopt an observer to predict and estimate the missing states. As for example, if there is a structure that has 10 DOFs, it would be great if sensors are placed at every DOF, however, as it was mentioned that this might not be feasible. Hence, the sensors can be placed only where it is crucial (e.g. top floor if it is a building type structure) and the rest of the DOFs states information can be then estimate via the use of the observer (e.g. Kalman filter).

There are many types and variation the of the Kalman filter (KF) due to the versatility of the original algorithm that was introduced by Rudolf E. Kalman in 1960 [1]. Among many few can be listed such as Alpha beta filter [2], Data assimilation [3], Ensemble Kalman filter [3, 4], Extended Kalman filter, Fast Kalman filter [5], Switching Kalman filter [6], Schmidt–Kalman filter [7] and many more. Due to the assumption of the linear Gaussian state-space model the KF can be treated as a sequential solvers for the Gaussian process regression [8]. Broadly, KF can be divided into linear and non-linear filter, and there are many modified version of both linear and non-linear type filters are already available. The linear filters are such as Wiener filter and KF provides optimal performance when the state variables are assumed to Gaussian random variables. In contrary, the nonlinear filter are based on heuristic or better statistical approximation to render states such as extended Kalman filter [9], unscented Kalman filter [10], Assumed Density Filters [11], Projection Filters [12]. The application of nonlinear type of filters are quite common for system identification in addition to state estimated and prediction [13–15] . Regardless of the type of the filter the goal is obvious to estimate the unknown states as accurate as possible. Hence, due to many reasons the adjustment or modification might works better due the individual complexity (e.g. badly defined problem due to simplification via reduced order) of the problem as well as the filter parameters e.g. noise level.

The adaptation and application of the KF are numerous in terms of disciplines e.g. engineering, science. In engineering applications can be found in mechanical, civil, electrical, robotics including many other branches of engineering. Among many applications, few can be listed such as tracking and navigation [3, 5], reservoir parameters estimation [4], vibration mitigation and control [14, 15], system identification [13–16], computational geophysics [17], data assimilation [18, 19], time-series modeling and interpolation of noisy signals [20], detail applications in SHM [21], real-time monitoring with GPS [22].

Due to many advantages herein the KF has adopted as an observer to estimate the missing information of 7 degree of freedom system. In order to run in a almost real-time scenario the numerical simulations have been performed by employing the SIMULINK. The optimal Kalman Gain has been estimated in real-time via SIMULINK by solving the discrete Riccati equation.

OVERVIEW OF THE OBSERVER MODEL

The Klaman filter was introduced in 1960 and since then the original algorithm has been adopted and modified by many researchers in different area of science and engineering applications. The modification is essential due to the underlying sensitivity of the filter's parameters that can alter the overall performance. Hence, depending on the individual problem characteristics it is crucial that ones must take care of the noise quantities as well as the initialization of the states.

In order to employ any observer it is essential to formulate any dynamical system using state-space formulation. The reason of using state-space formulation is that the aforementioned strategy allows to deal with two main equations which makes thing much easier to handle with. The equations of state-space formulation are system or process equation (given by X_{k+1}) and the second one is called the observation or measurement equation (given by Y_k).

$$X_{k+1} = AX_k + BU_k \qquad Y_k = CX_k + DU_k \tag{1}$$

where X means the states (displacements and velocities), A is the system matrix, B is the input matrix, U is the input vector, Y is the measured quantities, C output matrix, D feed-through matrix.

Both process and measurement equations needs to be adjusted based on the structural information such as inputs (single or multiple inputs and if control force exist), desired measured quantities (displacements, velocities or accelerations). After finalizing the measured quantities, those information needs to be passed through the observer to estimate missing states as well as all other desired information. Therefore, in order to apply the KF, both of those equation needs to be modified as given below,

$$X_{k+1} = AX_k + BU_k + W_k \qquad Y_k = CX_k + DU_k + V_k \tag{2}$$

where W indicates the process noise vector, V is the measurement noise vector.

In case of KF, both the process and measurement noise assumed to be zero mean Gaussian white noise. The process of the KF linked to two main steps known as *Predict* and *Update*. In the prediction step, *a priori* states and the covariances are estimated. Later in the update step, the Kalman gain is estimated and the measurements of the *a posteriori* states and covariances are performed. And the aforementioned process is done recursively until the operation is stopped. The Predict equations of states ($\hat{X}$) and covariance ($\hat{P}$) are given as,

$$\hat{X}_{k|k-1} = AX_{k-1|k-1} + BU_k \qquad \hat{P}_{k|k-1} = AP_{k|k-1}A^T + Q_k \tag{3}$$

The optimal Kalman gain (K_{gain}) is given by,

$$K_{gain_k} = \hat{P}_{k|k-1}B^T[B\hat{P}_{k|k-1}B^T + R_k] \tag{4}$$

The Update equations of states ($X_{k|k}$) and covariance ($P_{k|k}$) are given by,

$$X_{k|k} = \hat{X}_{k|k-1} + K_{gain_k}[Y_k - B\hat{X}_{k|k-1}] \qquad P_{k|k} = [I - K_{gain_k}B]P_{k|k-1}\hat{P}_{k|k-1} \tag{5}$$

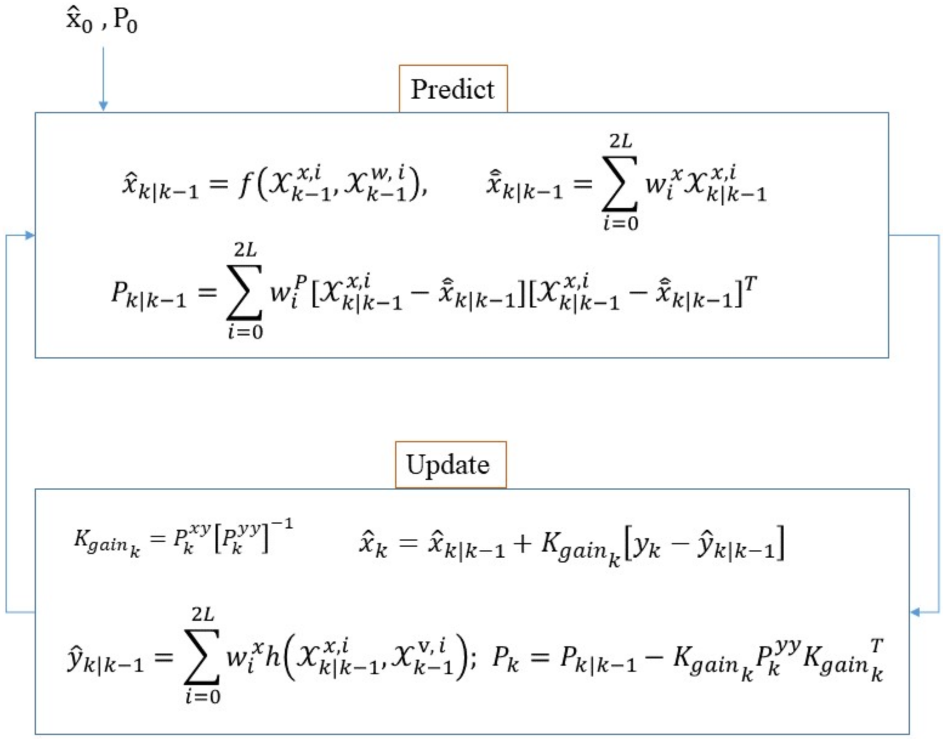

Figure 1. The closed-loop overview of the Kalman filter.

Finally, the measurement ($\hat{Y}_{k|k}$) is done via the following equation,

$$\hat{Y}_{k|k} = Y_k - BX_{k|k} \tag{6}$$

The whole process of KF runs recursively until the desire stopping time. An overview of the KF in closed-loop form has been depicted in Figure 1.

NUMERICAL INVESTIGATIONS AND DISCUSSION

The numerical investigations are performed by using a 7 degree-of-freedom (DOF) dynamical system. Where it is assumed that 4 accelerometers are placed at 1st, 3rd, 5th, and 7th DOF. The aforementioned placement of the sensors render a finite senors scenario that is why an observer is essential to predict all the missing DOFs information. The simulations are performed for a duration of 240 sec with a sampling frequency of 1000 Hz. Further, the floor masses are assumed to be same weighing about 50000Kg each and the floor stiffness coefficients are set to 70000 kN/m. The damping coefficients are estimated based on the modal properties.

In order to evaluate the outcome, initially, the acceleration data of the 1st, 3rd, 5th, and 7th DOF are compared in Figure 2. And the comparison shows the estimated data agreed with the original signals quite well. Additionally, in order to show the noise level of the corrupted signals a full-time history and zoomed view of the 5th and 7th DOFs are presented in Figures 3 and 4, respectively. As the noise level almost always needs to be adjusted depending on the problems nature, hence, it is quite difficult to decide the level of noise that is optimal for the problem. And the KF itself is highly sensitive to the noise

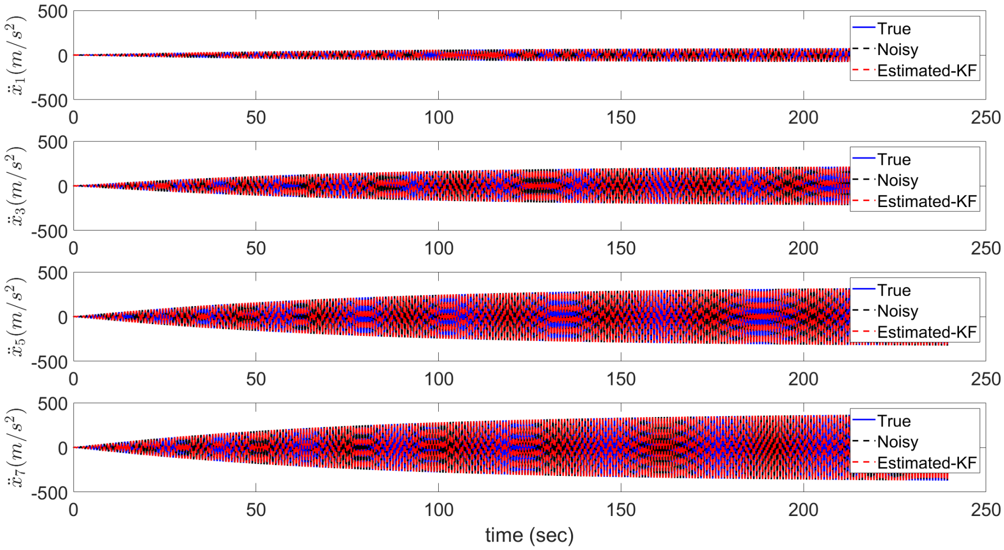

Figure 2. Comparison of original, noisy and predicted accelerations.

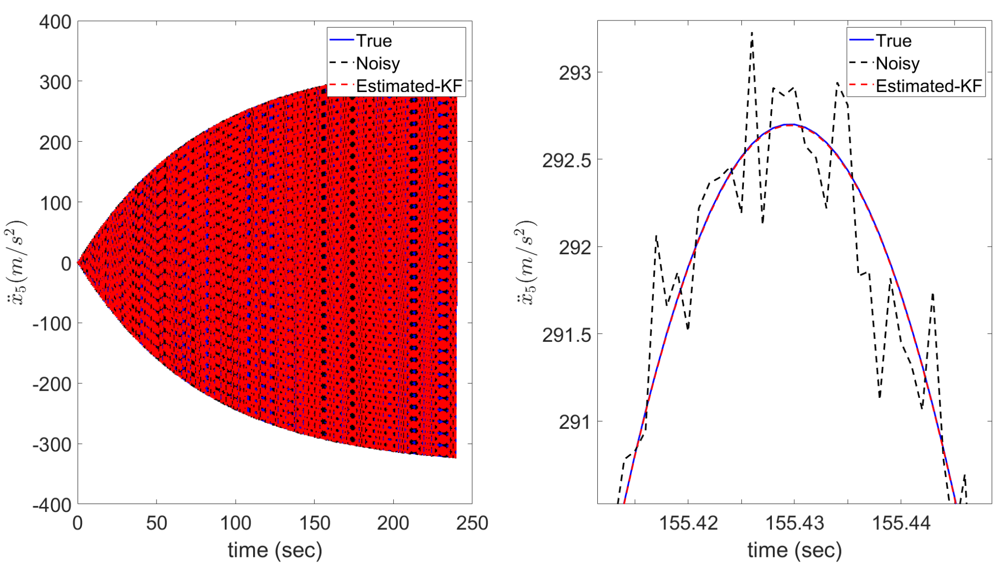

Figure 3. Comparison of original, noisy and predicted accelerations of the 5th DOF.

that has been reported by many current research works [2, 9, 10, 14]. However, herein the noise level was set at a level where the simulations in SIMULINK runs smoothly without any interruption.

In addition to the early mentioned Figure 2, a zoomed view (see the right sub-figure) of the acceleration data of the 5th DOF has been depicted in Figure 3.While the 7th DOF's noisy acceleration data has been compared in Figure 4, for better visualization, see the right sub-figure. And it can be seen from the aforementioned zoomed figures that the added noise to the original signals are reasonable.

Further, all states (displacements and velocities) of the 1st-7th DOFs are evaluated and compared in two groups namely observed (1st, 3rd, 5th, and 7th) and unobserved (2nd, 4th, and 6th) DOFs. The observed DOFs displacements in Figure 5 while the unobserved DOFs displacements information are presented in Figure 6. It can be noted from the aforementioned figures that the estimated results via KF has rendered the orig-

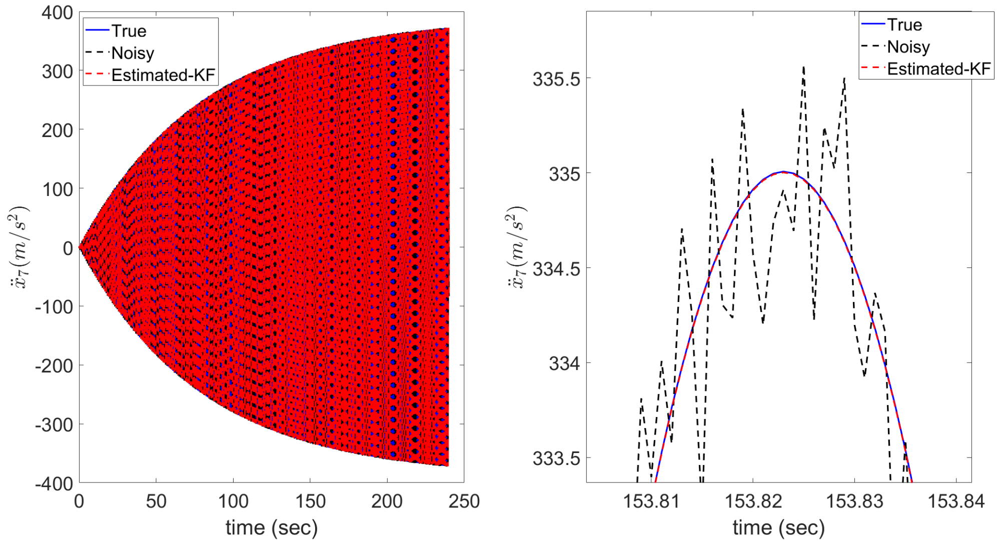

Figure 4. The comparison of the 7th DOF's original, noisy and predicted accelerations.

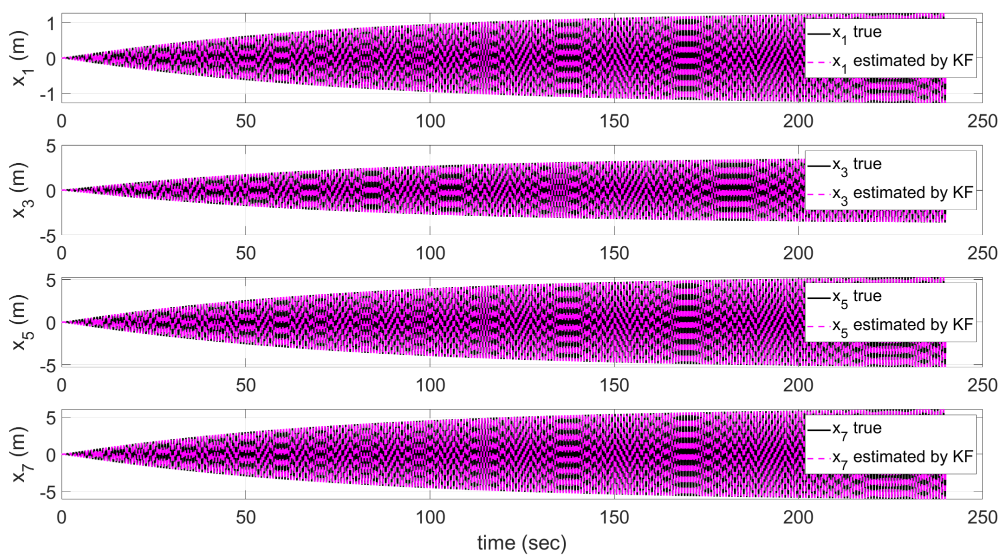

Figure 5. The observed DOF's original and estimated displacements.

inal results quite accurately. Similar results have been observed for the velocities and the observed (see the left sub-figure) and the unobserved (see the right sub-figure) DOFs velocities time-histories are depicted in Figure 7.

CONCLUDING REMARKS

This study has numerically investigated the prediction efficacy of the missing information by employing the Kalman filter. Herein, the investigations are performed by employing a nearly real-time environment called SIMULINK. The simulations are performed by considering a 7-DOF dynamical systems. The outcome of this study can be summarized such as in a real-time environment optimal performance can be achieved by updating the noise covariance in contrary to off-line simulations. More specifically,

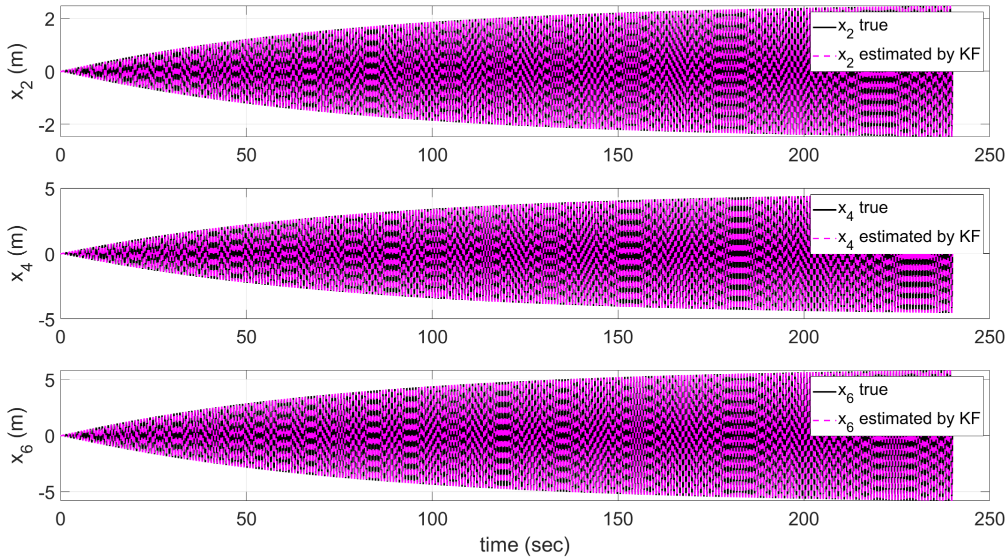

Figure 6. The unobserved DOF's original and estimated displacements.

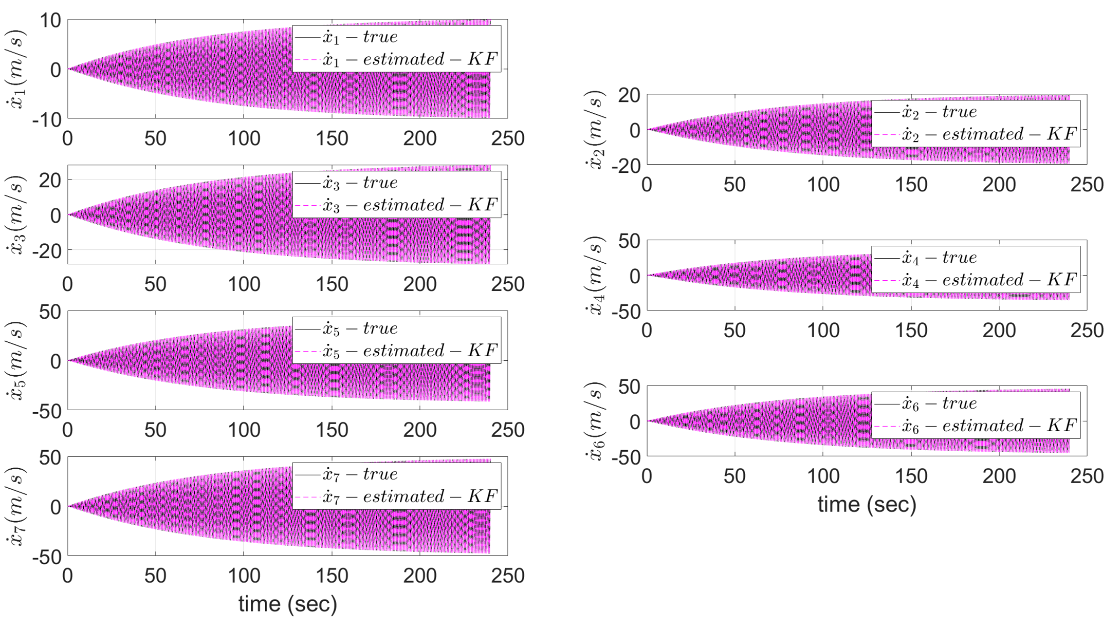

Figure 7. Comparison of original and predicted velocities.

the optimal Kalman gain has been calculated by solving the discrete Riccati equation in SIMULINK. Last but not least, it is observed that the solution of the Riccati equation in SIMULINK may leads better performance than the built-in Kalman filter block (that is availabe in SIMULINK library). Future study is planned to investigate the performance by considering different measurement scenarios

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