

# Enhancing Guided-Wave-Based Structural Health Monitoring Using Metamaterial Devices Designed by Topology Optimization

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## ABSTRACT

Lamb waves inside thin-walled structures have received extensive attention due to their great promise in applications such as structural health monitoring (SHM). Applications point at the common need for effective conditioning and manipulation of the wave propagation in terms of both frequency content and mode components. In this work, the concept of metamaterials is exploited to construct functional meta-devices (MDs). The MDs are designed to deliver prescribed functionalities after they are surface-mounted onto a structure conveying Lamb waves. To this end, a unified inverse-design scheme based on topology optimization is proposed and applied to achieve multifold functions such as frequency filtering and single-mode transmission. Typical scenarios with different frequencies and modes are discussed. Functional MDs with broadband working frequencies are obtained by using the established design strategy. A representative MD with a finite number of unit cells is examined through finite element simulations. Numerical simulations show that, through wave modulation of the designed MD, Lamb waves located in pass bands can transmit through the MD, while the waves within bandgaps are prohibited to propagate by the MD, which agrees well with the predicted dispersion features. An experiment is finally carried out to confirm the prescribed wave manipulation functions of the designed MD from the SHM perspective, which is finally validated experimentally using a metal specimen containing local plasticized incipient damage. This work provides a universal approach for topologically customizing MDs for the precise and tactical control of Lamb wave propagation, especially for SHM applications.

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## INTRODUCTION

Guided waves, especially Lamb waves, have received increasing attention due to their appealing advantages like low energy consumption, long propagation distance and high sensitivity to structural damages. These properties triggered vast engineering applications, exemplified by structural health monitoring (SHM) [1, 2]. However, some factors impede the practical feasibility of the detection methods, such as undesired wave distortion caused by transducers, installation adhesives and measurement equipment, etc. [3]. Meanwhile, the signal complexity, due to the co-existence of multiple wave modes and the undesired frequency components, is also seen as another major problem. This poses a harsh demand on the probing signals and hatches out the need for their meticulous control and manipulation.

The aforementioned problems can potentially be tackled by embracing the concept of metamaterials. Metamaterial is a type of artificial composite structure/material that offers abundant possibilities for manipulating wave propagation properties [4, 5]. In the context of SHM, metamaterial components are only allowed to be surface-mounted [6] to ensure the integrity of the structure under inspection, as opposed to the conventional and widely explored metamaterials with perforation or insertion in the structures/materials themselves [7]. This hatched out the design of a few pillared-type metamaterials. For example, Tian et al. [8] utilized the aluminum-lead composite structures to respectively achieve the mitigation for deceptive secondary waves. However, most of existing designs originate from empirical and intuitive designs with predefined structural configurations followed by tedious parameter tuning. In addition, to cope with more specific and higher-level demands, it will be extremely difficult, if not impossible, to follow the empirical metamaterial design approach. These challenges call for a systematic inverse-design strategy to tailor-make the property of metamaterials on demand.

In this study, we propose a unified inverse-design framework based on topology optimization to tactically design functional metamaterial devices, called meta-devices (MDs), for Lamb wave manipulation. The customized single-side and single-phase MDs are expected to deliver broadband wave manipulation functionalities including selective frequency filtering and single-mode transmission. Numerical studies are conducted to confirm the efficacy of a representative MD in terms of expected guided wave manipulation functions. The ultimate efficacy of the MD-assisted wave manipulation in favor of SHM is assessed by experimentally examining plastic deformation damage in an SHM system.

## INVERSE-DESIGN FRAMEWORK BASED ON TOPOLOGY OPTIMIZATION

As shown in Figure 1(a), an MD comprises an array of periodically arranged elements (called unit cells), to be mounted on the surface of a structure under inspection. The proposed inverse strategy for the design of the unit cell based on topology optimization is established as illustrated in Figures 1(b) and 1(c). The design domain of the MD is discretized into  $m \times m$  binary pixels, where numbers represent different materials. Targeting different frequency bands for different wave manipulation purposes as detailed later, the objective functions are generally formulated as:

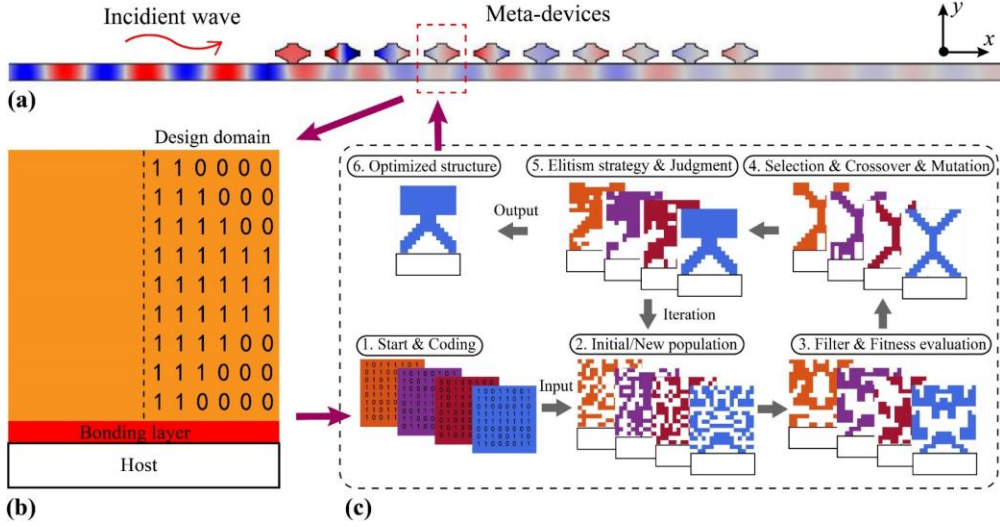


Figure 1. Schematic of the inversely designed meta-devices. (a) the wave filtering phenomenon induced by surface-mounted meta-devices, (b) the unit cell, including the host structure, the adhesive bonding layer and the design domain represented by a ‘0’ (for vacuum) and ‘1’ (for solid) matrix. (c) the topology optimization flowchart.

$$Find : \theta_n = 0 \text{ or } 1, \quad (1)$$

$$Maximize : F_i = \max(\Omega_i^{stop}) - \min(\Omega_i^{stop}) \quad (2)$$

$$Subject \text{ to} : k = [0, \pi / a], \quad (3)$$

$$min : d(\Sigma) \geq d_1, \quad (4)$$

$$N(\Sigma) = N_1. \quad (5)$$

where  $\theta_n$  denotes the value selection of the  $n$ -th pixel in the entire  $m \times m$  matrix, in which “1” represents a solid material and “0” for the vacuum;  $\Omega_i^{stop}$  represents the set of the stop bands associated with the  $i$ -th specific objective;  $a$  is the periodic constant; Eq. (4) states that the minimum size of the geometric feature should be larger than the prescribed value  $d_1$ , which is set to  $a/16$ . Eq. (5) limits the total number of the blocks within the design domain to  $N_1=1$ , so as to facilitate the sample fabrication and implementation.

Two typical cases are discussed to further explain the proposed inverse-design framework for Lamb wave customization.

**Objective 1:** A broadband selective frequency filter can be desired to tactically eliminate undesired frequency components. In this case, the MDs should allow the passing of the waves within working frequencies while stopping the undesired-frequency components. This is mathematically defined as

$$\Omega_i^{stop} = \{ \Omega_i^{stop} \} \cap (f_L^{stop}, f_U^{stop}) \cup \{ \Omega_i^{pass} \} \cap (f_L^{pass}, f_U^{pass}) \quad (6)$$

where  $\Omega_i^{stop}$  denotes the set of the stop bands while  $\Omega_i^{pass}$  for the pass bands;  $f_L^{stop}$  and  $f_U^{stop}$  are the lower and upper edges of the constrained stop band range, respectively;  $f_L^{pass}$  and  $f_U^{pass}$  are the lower and upper edges of the constrained pass band range, respectively.

**Objective 2:** A broadband single-mode (symmetric (S) or antisymmetric (A) Lamb wave mode) transmission filter is desired to create a pass band containing a single mode whilst stopping the other. This can be formulated as

$$\Omega_2^{stop} = \{\{\Omega_{single}^{pass}\} \cap (f_L^{pass}, f_U^{pass})\} \quad (7)$$

where  $\Omega_{single}^{stop}$  denotes the set of the single-mode pass bands (A or S mode).

Genetic algorithm (GA) [9] is employed as the searching method for updating design variables. The GA-based topological inverse-design scheme, as illustrated in Figure 1(c), is readily available in the literature [10, 11].

## RESULTS AND DISCUSSIONS

### Topologically Designed Meta-Devices

The host structure is a 2mm-thick aluminum plate. The surface-mounted MDs within a 6mm×6mm design domain use aluminum. The adhesive bonding layer is 0.05mm thick. Lead and aluminum are used as the MD materials in objective 1 and objective 2, respectively. The material parameters are listed in Table 1.

In the first case, the MDs are designed to deliver selective frequency wave transmission and prohibition. As in a typical nonlinear-guided-wave-based SHM system, the designed MDs are expected to allow the passing of the fundamental frequency wave whilst simultaneously filtering out its undesired second harmonic waves. Deploying the established optimization procedure, the optimization results are shown in Figure 2. The evolution process is illustrated in Figure 2(a) with some representative intermediary configurations listed. It is observed that the evolution curve of the objective values increases dramatically at the early stage of the iteration and keeps almost flat at the late stage. This indicates that the proposed scheme can quickly discern the suitable structures and maintain stable convergence eventually. The finally optimized MD is presented in Figure 2(b) with its corresponding dispersion relations shown in Figure 2(c).

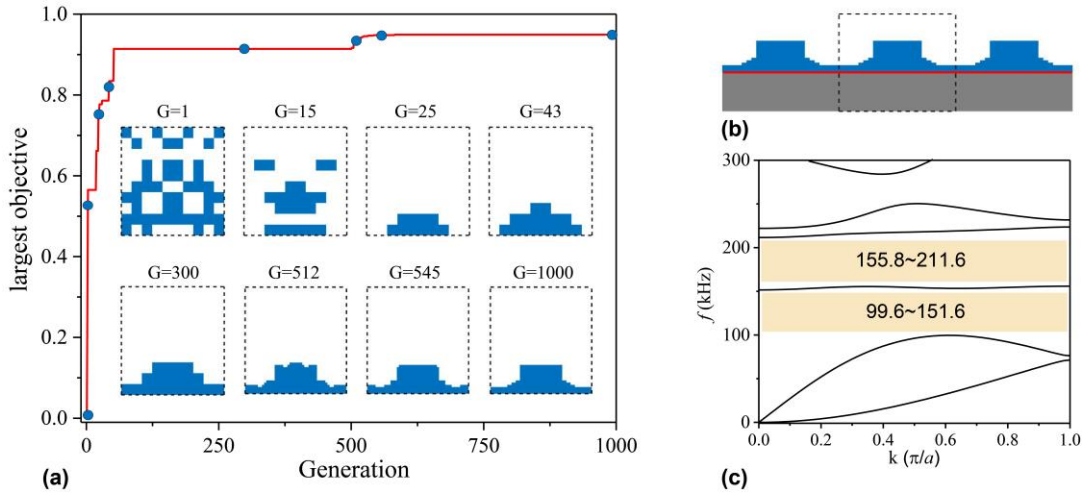


Figure 2. The optimization results. (a) the evolution history, (b) the optimized structure, and (c) the corresponding dispersion relations. The bandgap regions are highlighted with shadows.

To further explore the functions of the MDs on the basis of the properties previously achieved (passing fundamental frequency waves and filtering double frequency waves), the single-mode transmission function of Lamb waves, exemplified by the passing of single S0-mode wave, is added to reduce the signal complexity. The optimized structure is obtained and shown in Figure 3(a). It delivers a broad S0-mode pass band in the range of 76.4~117.9 kHz in Figure 3(b). To illustrate the single-mode function of the designed MD, the mode at 100 kHz within this specific region is extracted and shown in Figure 3(c), in which the colors and arrows denote the particle displacement value and direction, respectively. It is clear that the displacement in the plate is mainly polarized at the  $x$  direction, which corresponds to the S0-mode wave.

### Finite Element Simulation of Transmission Responses

The performance of the optimized MDs is then assessed in numerical simulations, as shown in the model in Figure 4. The designed MD in Figure 2(b) is taken as a representative example to be discussed in detail hereafter. To examine its selective wave filtering performance, frequency-domain response analysis is conducted. The frequency response function, defined as  $FR=20\log(u_1/u_0)$  with  $u_1$  and  $u_0$  being the displacement amplitudes of the receivers with and without the MD respectively, of the system with/without the designed MD is shown in Figure 5(a), where a negative FR value implies an attenuation of the wave within the corresponding frequency range. It can be found that the attenuation zones agree well with the previously predicted bandgap ranges (Figure 3(c)) marked with shadow, thus confirming the broadband filtering function of the MD. Moreover, the wave fields under the 60 kHz excitation (fundamental wave in the pass band) and the 120 kHz excitation (secondary wave in the stop band) are extracted and shown in Figure 5(b). It is explicit that the 60 kHz wave can pass the MD and continue to propagate along the plate while the 120 kHz wave component is prohibited.

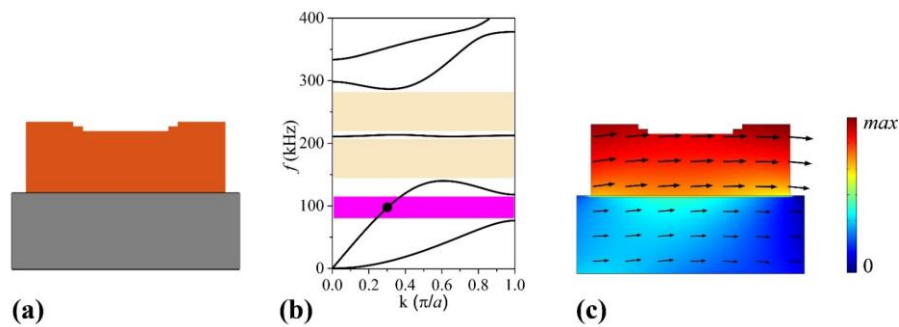


Figure 3. Optimization results for the widest single S0-mode band (objective 2). (a) the optimized structure, (b) the corresponding dispersion relations. The pink shadow area denotes the single-mode region. (c) a typical mode (at 100 kHz) within the single-mode band. The arrows indicate the displacement value and direction.

TABLE I. THE MATERIAL PARAMETERS

	Density (kg/m <sup>3</sup> )	Poisson's ratio	Young's modulus (GPa)
Aluminum	2700	0.33	70
Adhesive	1080	0.4	1.31
Lead	11600	0.369	40.8

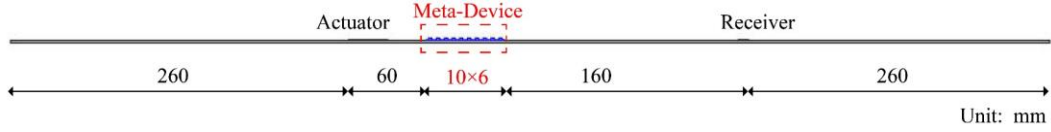


Figure 4. Simulation models.

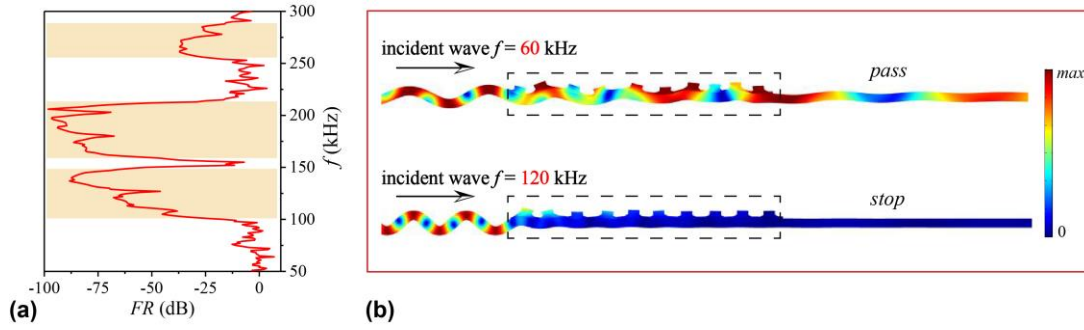


Figure 5. Numerically simulated wave transmission. (a) the frequency-domain responses, (b) the displacement fields at 60 kHz in the pass band and 120 kHz in the stop band.

## Experimental Study

A typical pitch-catch ultrasonic experiment in Figure 6(a) is finally conducted to further evaluate the proposed wave manipulation technique in views of SHM using nonlinear-guided waves. Herein, a repetitive bending process, as shown in Figure 6(b), is applied to an aluminum beam to generate the material nonlinearity (to mimic incipient damage inside materials) which is mainly caused by the dislocation effect. Bending cycles are gradually increased to mimic the evolution of material degradation. Figures 7(a) and 7(b) show that when deploying MF, the linear signal undergoes no obvious changes in terms of energy level while the secondary wave is significantly filtered out. With increasing bending cycles, as shown in Figure 7(c), the nonlinear wave amplitudes first dramatically go up, then gradually decrease before finally reaching a rather stable level, which reflects the typical variation trend of the nonlinear intensity. After removing the MF, the difference between the intact and the damage cases is quite small, as shown in Figure 7(d), which testifies the enhanced identification capability of the system arising from the use of the MF.

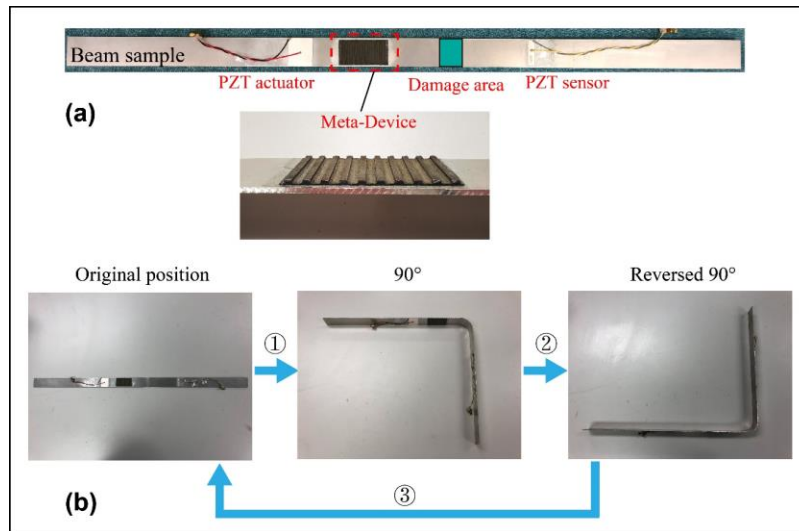


Figure 6. Experimental set-up (a) experimental samples, (b) one complete bending cycle for the beam sample.

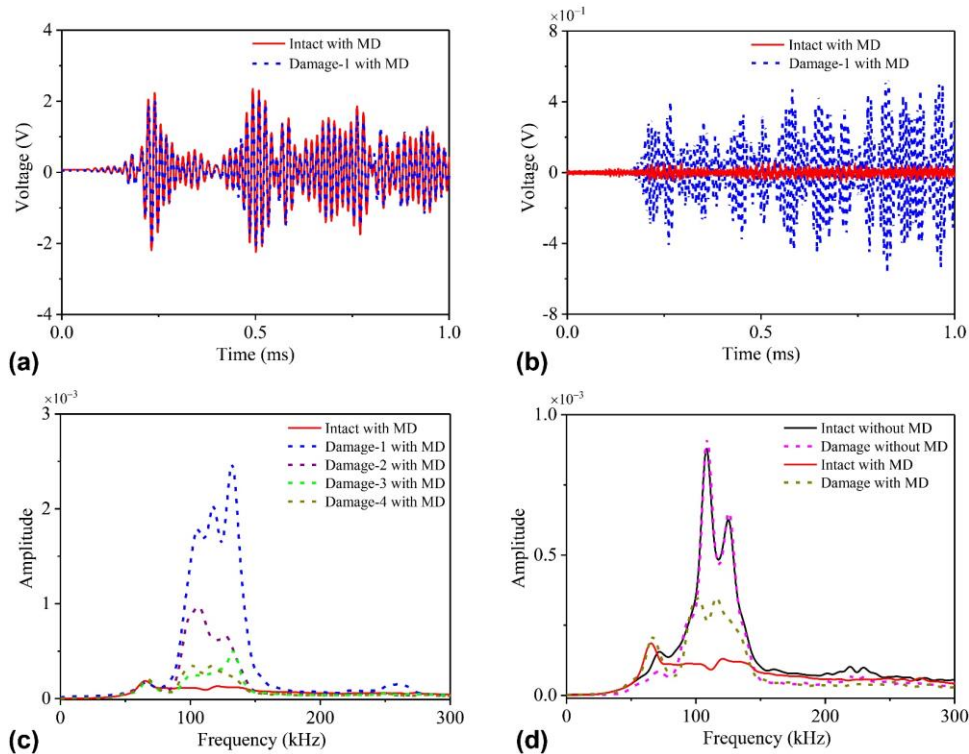


Figure 7. Experimental results. (a) linear signals. (b) nonlinear signals with or without damage. (c) frequency spectra of the nonlinear signals corresponding to different-degrees damages. (d) frequency spectra of the nonlinear signals with or without damage and meta-device (MD).

## CONCLUSIONS

Aiming at effective Lamb wave manipulation, especially for SHM applications, this work proposes a unified inverse strategy for meta-devices (MDs) design. The MDs, taking the form of topologically designed lattices, are to be surface-mounted

onto the host structure for the manipulation of guided Lamb waves. The unified inverse-design framework is based on topology optimization which delivers customized and broadband functional MDs in terms of selective filtering of the wave frequency and single-mode transmission. Broadband MDs are obtained by using the established design strategy. With a typical MD comprising a finite number of unit cells, numerical simulations demonstrate its broadband and selective wave filtering functionality, in the frequency regions well correlated with the bandgap regions predicted by dispersion analyses. The structural damage used in experiments is created through a well-controlled bending operation on a metallic beam sample. Bending cycles are varied to mimic the evolution of plasticized damages induced by the density dislocation effect. Damage detection results show that, with the embodiment of the MD in the system, non-monotonous variation of the captured nonlinear energy level with the bending cycle are observed clearly.

## ACKNOWLEDGMENTS

The work was supported by grants from the Research Grants Council of Hong Kong Special Administrative Region (PolyU 152013/21E) and the Innovation and Technology Commission of the HKSAR Government to the Hong Kong Branch of National Rail Transit Electrification and Automation Engineering Technology Research Center.

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