Underwater Target Ranging and Sound Speed Profile Inversion via Eigenray Propagation Time-Delay Matching

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Abstract. An underwater target ranging and sound speed profile (SSP) inversion algorithm for synchronous beacon is proposed. In the condition of actual SSP unavailable, based on the multipath propagation time-delay estimation, a genetic algorithm is employed to estimate the target range and SSP by eigenray propagation time-delay matching. Lake trial results show the algorithm could achieve target ranging and SSP inversion simultaneously.

1 Introduction

Target ranging is the key step for underwater acoustic positioning and navigation system. Because of the multipath effect, it is difficult to fix a single precise receiving time of underwater acoustic signal. Usually, the earliest received signal from the fastest path among all the multiple paths was chosen as the representation of receiving time [1,2]. However, the multipath time-delay could reveal the main characteristic of the underwater acoustic channel, it is possible to achieve accurate target ranging and further utilities by exploring the potential usage of multipath time-delay.

The sound propagation time-delay depends on the actual SSP, but it is a demanding job in many cases to get the actual SSP. The multipath effect has negative influence on time delay estimation, but it would be of help to achieve both target ranging and SSP inversion in uncertain underwater environments.

2 Ranging Algorithm with certain SSP [3,4,5,6]

Supposing that an underwater vehicle receives navigation signal from a time-synchronized beacon, the receiver could get a cluster of signal transmitting time delay \( \{ \tau_i \} \), \( \tau_i \) stands for the time delay of \( i \)-th path. Meanwhile, the receiver, with sound speed profile, water column depth and other environmental factors acquired, could calculate the signal propagation time delay of different eigenrays at different distance \( r \) and depth \( z \): \( \{ I_k^{r,z} \} \). The depth of beacon could be appointed in advance or be notified to the receiver by underwater acoustic communication. Thus, \( \{ I_k^{r,z} \} \) could be simplified as \( \{ I_k^r \} \).

Defining \( \{ \tau_i \} \) as measure vector, and \( \{ I_k^{r,z} \} \) as copy vector, the time-delay matching algorithm is as follow:
\[
\hat{r} = \arg \min_r \, D(r) = \sum_{i=1}^{\omega_k} \omega_k \min (\tau_i - L_{r}^{r})^2
\]  

\(\omega_k\) is the weight for eigenray time delay. It is of great importance to estimate the time delay and choose proper weights value \(\omega_k\) for time-delay matching algorithm. There are already lot of time delay estimating algorithms [11, 12, 13]. Here, the discussion is mainly focused on the optimizing method of \(\omega_k\).

3 Ranging and SSP inversion in uncertain environment

Now since the SSP is unknown to the receiver, calculating the time delay under actual SSP is impossible, but the set of time-delay \(\{L_{r}^{r,\theta}\}\) at distance \(r\) and SSP is available. Here \(\theta\) is the parameters by which the SSP could be calculated precisely, further discussed in chapter 2.1. Hence, the question becomes how to find proper value of \(r\) and \(\theta\) to minimize the time delay error:

\[
\arg \min_{r, \theta} \, D(r, \theta) = \sum_{i=1}^{\omega_k} \omega_k \min (\tau_i - L_{r}^{r,\theta})^2
\]

3.1 The parameterized expression of SSP

There are mainly 2 ways for parameterized expression of SSP. First, The Empirical Orthogonal Function (EOF) are widely used for SSP expression and SSP inversion, but it is necessary to get a set of priori SSPs in the target sea area [8, 9]. The other way is SSP model with structural parameters, such as Munk model, GDEM, etc. The SSP data can be calculated precisely by the structure parameters. Intuitively, there is little difference between EOF and SSP structure model for SSP inversion for they are all about optimizing a series of numbers(coefficients of EOF or the structural parameters).

Considering that there is few priori SSP data of the lake water column where the trial in chapter 3 is conducted, a Layered Sound Speed Profile Model (LSSPM) proposed in [10] is employed for SSP inversion. The sound speed function is expressed in equation (3).

\[
F(z) = \frac{1}{1+(z/A)^2B} \quad 0 < z < H
\]

\(F(z)\) is related to 2 parameters, \(A\) and \(B\). \(A\) is the key position of the thermocline, and \(B\) represents the gradient of the thermocline. The relation between the sound speed profile \(c(x)\) and \(F(z)\) is shown in equation (4).

\[
F(z) = \frac{1}{1+(z/A)^2B} \quad 0 < z < H
\]

3.2 Boundary conditions

Usually, the boundary conditions are important for sound propagation calculation. But according to the references[3, 11], the acoustic properties of sediments have little influence on the multipath time-delay, which means the mismatch of properties of sediments has little influence on the ranging and SSP inversion algorithm of time-delay matching. It is worthy to notice that, when the glancing angle of the eigenray is relatively big, the mismatch of properties of sediments would induce significant amplitude mismatch, especially in low SNR cases.

Boundary roughness indicates the sea surface waves and the bottom undulation of small scale. The roughness of sea surface and bottom has similar influence on the propagation of eigenray. The following discussion focuses on sea surface waves.

According to Kirchhoff Approximation, the reflection coefficient on the mirror reflecting direction is[12, 13, 14]:

\[
R_c = \int_{-\infty}^{+\infty} \exp(-2jk\xi\sin\theta_{in}) \, p(\xi) \, d\xi
\]
In equation (3), \( k \) is wave number, \( p(\xi) \) is the distribution of sea surface height, which is approximately Gaussian distribution. For P-M sea wave spectrum, the RMSE of surface height \( \sigma_\xi = \langle \xi^2 \rangle^{1/2} \) is related to the wind speed \( u \) [13]:

\[
\sigma_\xi = 5.3 \times 10^{-2} \cdot u
\] (4)

The surface reflection coefficient under different windspeed and glancing angle is shown in Fig.1.

![Figure 1. Surface reflection coefficient under different windspeed and glancing angle.](image)

As the glancing angle, wind speed and surface roughness increase, the surface reflection coefficient decreases. Therefore, in a low sea state circumstance, sea surface reflection coefficient is approximately 1, the received signal usually contains numerous multipaths. While in high sea state, sea surface reflection coefficient is small, the energy of eigenray with big glancing angle dissipate very quickly after several surface reflections, the received signal usually has simple multipath pattern. Bottom roughness has a similar influence.

According the discussion above, in order to promote the algorithm's robustness and reduce the error while conducting time-delay matching algorithm, it is necessary to lessen the time-delay weights of eigenrays with big glancing angle and big number of boundary reflection times in equation(2).

### 3.3 Region of feasible solution

The combination of EOFs is scale-restrained by historical data, that means the SSP inversion process by EOF start from a small region of feasible solution. In contrast, the SSP structure model provides little information of certain sea area, as a result, the original region of feasible solution is much more wider. In order to achieve fast and robust result, more information of the SSP shall be acquired.

For the lake trial in chapter 3, the approximate sound speed at the surface and bottom of water column are supposed known to the receiver.

### 4 Lake Trial and Results

#### 4.1 Environment conditions

Lake trial is conducted on September 21th in 2018 on the Qiandaohu Lake, an artificial lake in Zhejiang Province, to examine the performance of the algorithm. In the trial, two boats were employed to carry the underwater acoustic signal transmitting system and receiving system respectively. GNSS systems were also installed on the two boats for
position recording and time synchronization. The depth of the water column in Qiandaohu Lake is 61.5 meters, and the depth of transducer is 2 meters and the hydrophone 30 meters. The actual sound speed profile during the trial is shown in Fig.2, the solid line with circles, the others are the initial herd of SSP for genetic algorithm.

![Sound Speed Profile](image.png)

**Figure 2.** The actual SSP during the trial and the initial SSP herds.

The actual sediments properties are unknown, but according to the local recordation, the bottom of the lake used to be corpland and riverbed 60 years ago when the artificial lake was not constructed yet. Therefore, sandy silt sediment are applied in bellhop simulation. The properties of the sediment are shown in Table.1.

<table>
<thead>
<tr>
<th>Sediment type</th>
<th>Density (g/cm³)</th>
<th>Velocity (m/s)</th>
<th>Attenuation (dB/m/kHz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sandy silt</td>
<td>1.787</td>
<td>1664</td>
<td>0.756</td>
</tr>
</tbody>
</table>

**Table 1.** Properties of the sediment.

**4.2 Results**

In the trial, LFM signal is used for time-delay estimating and the estimating result of one frame is shown in Fig.3. [15,16]

![Eigenray Propagation Time-Delay](image.png)

**Figure 3.** The eigenray propagation time-delay.

The distance between the transmitting boat and the receiving boat on the time of this frame of signal is 1491.6 meters according to GNSS record. The initial herd of SSP is shown in Fig.2. The minimum objective function of each generation and distance
estimation is shown in Fig.4 (a). The SSP inversion result is shown in Fig.4 (b). Generally speaking, the ranging and SSP inversion are accomplished simultaneously, the ranging error is about 1%, and maximum sound speed error is 7m.

(a)The distance estimation result                          (b)The SSP inversion result

Figure 4. Results of ranging and SSP inversion.

5 Conclusion

A target ranging joint with SSP inversion algorithm via eigenray propagation time-delay matching is presented. Lake trial proved that the algorithm could achieve target ranging and SSP inversion simultaneously in uncertain underwater environment. Further researches will be focused on optimizing the region of feasible solution for fast and robust results.

References

7. Shen Yuanhai, Ma Yuanliang, Tu Qingping. On expression of ocean sound profile by layered empirical orthogonal function (EOF)[J]. Journal of Northwestern Polytechnical University, 18, 90-93, (2000).


