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Plasma of Magnetic Monopoles in Kagome Spin Ice

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Abstract. The monopole's charge, concentration and the frequency of long-wavelength plasmon at the band-edges for the periodic arrays of 2D magnetic-Dirac plasma layers at low temperature (0.5K~2K) are studied. The charge decreases as applied field becomes strong, and increases with increasing temperature. The concentration decreases as applied field becomes strong. The frequency increases as the temperature rises, and decreases as the strength of applied field increases. The range of the frequency is 3.8 × 10 11 Hz ~ 3.5 × 10 12 Hz

Introduction

Spin ices are frustrated magnets. Applying a magnetic-field pulse to spin-ice crystal atlow temperature, one can obtain magnetic monopoles[1]. Gases of magnetic monopoleshave recently been predicted to exist in spin ice[2]. The description and understanding of spin ice magnetic monopole are becoming increasingly important not only in condensed matter, but also in other branches like field theories. The study of monopoles turns into aproper applied science[3].

In two-dimensional spin ice such as kagome spin ice, ferromagnetic interactions lead to a pair of topological defects forming on the adjoining triangles. One defect has two spins pointing in and one out. Another has two spins pointing out and one in. If the spin of rare-earth ion in kagome spin ice is regarded as a magnetic dipole, its two separated magnetic charges sit on the vertices of the diamond lattice. The 2 in-1 out (2 out -1 in)defect bears a net difference of one positive (negative) charges. Then the total charge carriedby a local excitation is $\pm Q_m = \pm\mu/a_d$. Effective Ising model[4] describes the thermodynamic properties of spin ice with nearest-neighbour exchange coupling and the long-range magnetic dipolar interactions.

In the DSI model, the spins interact through dipolar interactions. So the charges defined on the extremities of these spins must interact through an effective magnetic Coulomb interaction. They behave like massless magnetic monopoles.The defect pair can be viewed as north-south magnetic monopole pair. In spin ice, north magnetic monopole's anti-particle is not its own, but is south magnetic monopole. Hence monopoles in spin ice can be considered as Dirac fermion [5].

Flipping many triangles' spins gives rise to a two-dimensional magnetic monopoles gas that interact through hybrid magnetic Coulomb potential. One of the hybrid magnetic Coulomb potential $V_C(r)$ is the usual $1/r$ interaction between magnetic charges, where as the other is a logarithmic confining interaction between the monopoles of entropic origin $V_S(r)$ [6]. Generally, $V_C(r) > V_S(r)$. One can only consider the the usual $V_C(r)$ magnetic Coulomb interaction.

Monopoles perform overdamped motion in diffusive regime, and can separate and move essentially independently. Hence the collection of monopoles in kagome spin ice could be considered as a two-dimensional plasma of magnetic monopoles.

Two-dimensional plasma of magnetic monopoles can be loosely described as a two-dimensional magnetically neutral medium of North magnetic monopoles and South magnetic monopoles. It is analogy to its electric counterparts-two-dimensional plasma composed of electrons and ions. In two-dimensional plasma of magnetic monopoles, monopoles are unbound, but are not 'free'. They are close together so that each monopole influences many nearby monopoles, rather than just interacting with the closest monopole. Since long range magnetic Coulomb interactionsin magnetic monopole plasma have the effect of smoothing net charge fluctuations, the magnetic charge density oscillates rapidly in space. The restoring force causing these oscillationsin spin ice arises from the long-range magnetic Coulomb potential. In fact, the long-rangemagnetic Coulomb interaction of magnetic monopoles is the long- range spin dipolar interaction of defects in spin ice. These oscillations are essentially oscillations of defects density. The plasmon band can describe these oscillations of the magnetic monopoles.

Magnetic monopoles can be created by application of an external magnetic field on spin ice. When the spins of a triangles in two-dimensional kagome spin ice point 2 in-1 out or 1 in-2 out, the charge of magnetic monopole at the triangles is $\pm Q_m$, and $\pm 3Q_m$ when its spins point all in or all out[7].

The magnetic charge of a monopole in kagome spin ice can be tuned continuously. Both the temperture and the applied magnetic-field determine the monopoles' charge Q_m [8]. The plasmon band of magnetic monopoles in two-dimensional kagomespin ice must be influenced by the temperature and the applied field.

In this paper, we investigate the temperature and the applied field dependent behaviour of monopoles gas in kagome spin ice.

Theory

The Charge and Concentration of Magnetic Monopole in Two-Dimensional Spin Ice

Giblin and Bramwell et al studied the charge and concentration of free-magnetic monopole in spin ice[1]. They gave a formalism to calculate the monopole's charge as

$$Q_m = \left| \frac{2^7 \pi^2 K_B^3 T^3 A_d}{\mu_0^2 B} \right|^{1/5}, \tag{1}$$

here A_d is the distance of neighbor interation between monopoles B is the strenght of the applied magnetic field.

Combining this expression with E_{qs}.(1), we can obtain a expression of monopole concentration

$$n_f^{eq}(B,T) = \lambda T^{3/40} B^{-3/20} \exp\left(-T_f / T\right) \exp\left(\alpha T^{-1/10} B^{1/5}\right), \tag{2}$$

where $\lambda = 2^{-53/40} \pi^{-13/40} \mu_0^{3/40} k_B^{3/40} A_d^{-9/40}$, $\alpha = 2^{11/10} \pi^{-1/10} \mu_0^{-1/10} K_B^{-1/10} A_d^{3/10}$.

Plasmon Frequency of Long-Wavelength Plasmon at the Band-Edges for Periodic Arrays of 2D Magnetic-Dirac Plasma Layers

In spin ice, since monopoles are massless, plasma of monopoles is a massless Dirac plasma. Horing studied plasma frequency of two-dimensional (2D) massless electric-Dirac fermions[9]. Sarmaand Hwang developed a theory for plasmon frequency of 2D massless Dirac particles [10].

The periodic arrays of 2D magnetic-Dirac plasma layers can exist in kagome spin ice or other materials. The plasmon bands for periodic 2D magnetic-Dirac plasma layers canbe defined in accordance with the way for 2D electric-Dirac plasma layers. The frequencyof long-wavelength plasmon at the band-edges is analogy to its electric counterparts. Weapply Sarma and Hwang' theory to plasma of magnetic monopoles by replacing electrical quantities with the appropriate magnetic ones. Specifically, $e \to Q_m$, $(\varepsilon_0 \varepsilon_r)^{-1} \to \mu_0 \mu_r$, $n_e \to n(B)$, $r_s \to r_s^{'}$, $\beta \to \beta^{'}$, $\beta_s \to \beta_s^{'}$, $\beta_v \to \beta_v^{'}$. The frequency of long-wavelength plasmonat the band-edges for the periodic arrays of 2D magnetic-Dirac plasma layers is expressedas

$$\omega_m = \sqrt{r_s^{'}} \left(4\pi\beta^{'}\right)^{1/4} \left(n(B)/d^2\right)^{1/4} v_F^{'}, \tag{3}$$

where $r_s^{'} = \mu_0\mu_r Q_m^2/(h\nu_F^{'})$ for notational simplicity, $\nu_F^{'}$ is the Fermi speed of magnetic Driac fermions, the factor "$\beta^{'}$" is the degeneracy factor, $\beta^{'} = \beta_s^{'}\beta_v^{'}$, $\beta_s^{'}$ is the spin degeneracy, and $\beta_v^{'}$ is the valley or pseudo spin degeneracy, $d^{'}$ is the superlattice period forthe periodic arrays of 2D magnetic-Dirac plasma layers, a background permeability $(\mu_0\mu_r)$ which differs from unity in spin ice based magnetic monopole systems in general.

Plasmon Frequency of Long-Wavelength Plasmon at the Band-Edges for Periodic Arrays of 2D Magnetic-Dirac Plasma Layers in Spin Ice

Since magnetic monopole in spin ice is considered as massless fermion, it's spin can be considered as $1/2$[11]. Thus the factor $\beta_s^{'} = 2$. Spin ice is not a magnetic system consisting of magnetic monopole atoms, the conception about magnetic monopole's pseudospin isuseless. Thereby, the factor $\beta_v^{'} = 1$. Then, $\beta^{'} = \beta_s^{'}\beta_v^{'} = 2$.

Substituting $\beta^{'} = 2$, $\nu_F^{'} = \pi^{-1/2}\omega n(B)^{-1/2}/h$ into Eq.(3) we can write the expression of the frequency of long-wavelength plasmon at the band-edges for the periodicarrays of 2D magnetic-Dirac plasma layers in the form

$$\omega_\pi = \eta T^{3/5} B^{-1/5}, \tag{4}$$

where $\eta = 2^{43/20}\pi^{-2/5}\mu_0^{1/10}\mu_r^{1/2}k_B^{11/10}A_d^{1/5}h^{-1}\omega^{1/2}d^{'-1/2}$

Results and Discussion

In the case of a low temperature $Ho_2Ti_2O_7$ kagome spin ice, we study monopole's charge, the number density of monopoles and the frequency of long-wavelength plasmon at the band-edges. The temperature is in the range of 0.5K ~ 2K, and the strength of field is inthe range of 1mT ~ 0.1T.

The monopole's charge is shown in Fig.1. The charge is in ther ange of $2.4 \times 10^{-13} \sim 2.2 \times 10^{-12}$ $J/\mathrm{T/m}$. It decreases slowly as the applied field be comes strong. But it increases with increasing temperature. This is in agreement with Bramwell'sresult[12].

Fig.2 shows monopole concentration's evolution following applied field and temperature. The maximum value of the concentration is 0.18. The concentration is approximately 0.0002 when $T = 0.5K$ and $B = 0.1T$. At a relatively low temperature, it decreases slightly as the applied field becomes strong. It increases rapidly with increasing temperature.

The number density of monopoles is relatively low ($2 \times 10^{16} \sim 1.8 \times 10^{18}/m^2$). But the semonopoles obey quantum statistics because of zero mass. For the two-dimensional plasma of monopoles, the thermal de-Broglie wavelength($\sim 10^{-3}m$) is much larger than the Debye length($\sim 10^{-12}m$). Hence we can consider the plasma as a quantum plasma (Debye length of monopoles is defined as $\lambda_D = \sqrt{k_B T/\left[\mu_0 n(B) Q_m^2\right]}$. For a massless particle, the thermal wavelength may be defined as $\Lambda = ch/\left(2\pi^{1/3}k_B T\right)$, where c is the speed of light).

In Fig.3 we show the way that the frequency of long-wavelength plasmon at the band-edges for the periodic arrays of 2D magnetic-Dirac plasma layers changes as applied field and temperature. When the applied field is strong, the frequency increases slightly with increasing temperature. Otherwise, it increases rapidly with increasing temperature. And it decreases as the strength of applied field increases. The range of the frequency is $3.8 \times 10^{11} Hz \sim 3.5 \times 10^{12} Hz$. If oscillations of massless magnetic monopoles are considered as harmonic oscillations which frequency ω satisfies the relation of $E = h\omega$(where themonopole's energy $E \sim \omega \approx 8.9K$), the frequency has a approximate value of $\omega \sim 1.1 \times 10^{12} Hz$. This estimate backs up our theoretical results for the plasma frequency.

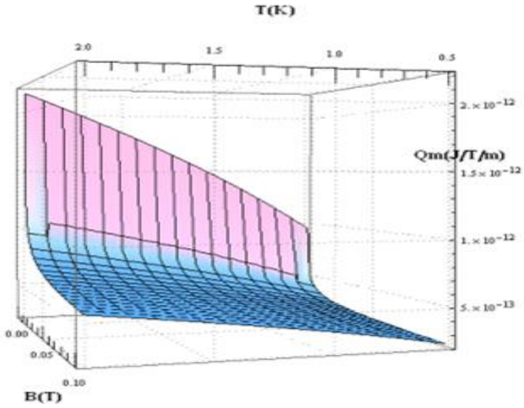

Figure 1. Temperature and applied magnetic field dependence of the monopole’charge. The temperature $T \in (0.5K, 2K)$, and the strength of field $B \in (1mT, 0.1T)$. The range of monopole’s charge is $2.4 \times 10^{-13} \sim 2.2 \times 10^{-12}\ J/T\ /m$.

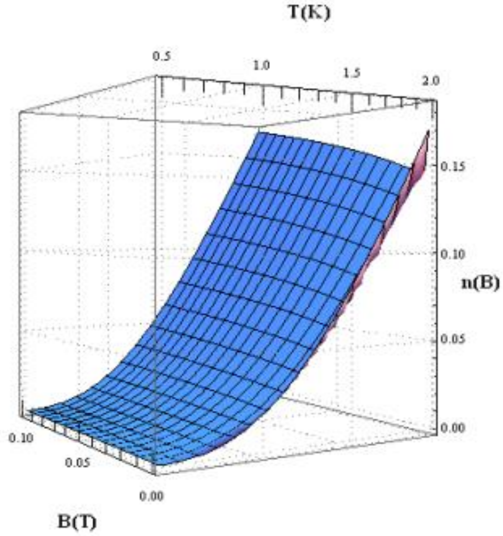

Figure 2. Temperature and applied magnetic field dependence of the monopole con-centration. The temperature $T \in (0.5K, 2K)$, and the strength of field $B \in (1mT, 0.1T)$. The range of the monopole concentration is 0.0002 ~ 0.18.

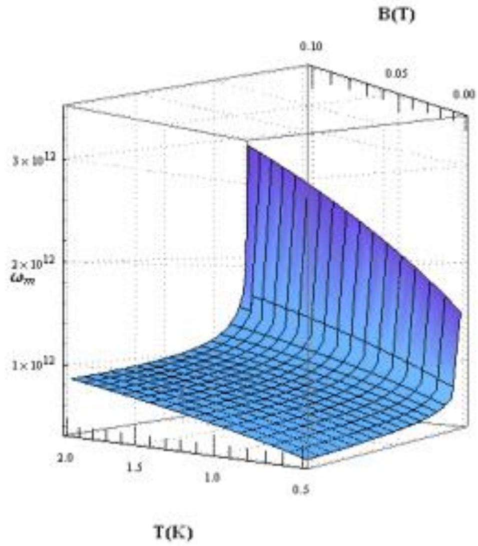

Figure 3. Temperature and applied magnetic field dependence of the plasma frequency. The temperature $T \in (0.5K, 2K)$, and the strength of field $B \in (1mT, 0.1T)$. The range of the plasma frequency is $3.8 \times 10^{11} Hz \sim 3.5 \times 10^{12} Hz$.

Summary

We study the charge and concentration of magnetic monopole and the frequency of long-wavelength plasmon at the band-edges for the periodic arrays of 2D magnetic-Dirac plasmalayers, and get a conclusion as following. The charge decreases as applied field be comes strong, and increases with increasing temperature. The concentration decreases as applied field becomes strong. The frequency increases slightly with increasing temperature, and decreases as the strength of applied field increases. It is not relatied to the number density of monopoles in kagome spin ice. The range of the frequency is $3.8 \times 10^{11} Hz \sim 3.5 \times 10^{12} Hz$.The plasma oscillation of magnetic monopoles in kagome spin ice is one type of the collective oscillations of the electric particles system in external magnetic field.

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