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Tracking of Variable Structure Model for Generalized Nonlinear Systems

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ABSTRACT

Based on the concept of generalized nonlinear system model tracking, the tracking problem is studied by using variable structure control method. In nonlinear system, based on the classification system, the design of generalized tracking system switching function S, makes the switching surface S=0, realize the model tracking, and the design of variable structure control law, realization of generalized nonlinear systems with sliding mode.

INTRODUCTION

In the practice of human production, a large number of practical problems are difficult to use simple linear model as the model of the real system, and it must be described by a nonlinear model. For example, the circuit system, the restricted robot system, etc. The research on nonlinear singular control system has both theoretical value and practical significance. Document[1] has studied the problem of variable structure control for descriptor systems. Document[2] has discussed the control problem of sliding mode of nonlinear system. Document[3][4] has studied the model tracking problem, This article through the case classification for generalized nonlinear systems, the tracking system design under various conditions of the switching function of S, makes the switching surface S=0, realize the model tracking, and the design of variable structure control law, realization of generalized nonlinear systems with sliding mode.

SYSTEM DESCRIPTION

General nonlinear system

$$\begin{cases} E\dot{x}(t) = A(x) + B(x)u \\ y = Cx \end{cases} \quad (1)$$

Here, $E \in R^{n\times n}$ is singular definite matrix, $rankE = r < n$, $A(x) \in R^n$ is n dimension function vector, $B(x)$ is $n \times m$ function matrix, $x \in R^n$ is State of the

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system, $y \in R^p$ is output of the system, C is $p \times n$ constant matrix, $u \in R^m$ is System control input .

Design system (1) reference model

$$\begin{cases} E\dot{\omega} = A_m \omega \\ y_m = L\omega \end{cases} \qquad (2)$$

Here, $\omega \in R^n$ is model state, $A_m \in R^{n\times n}, L \in R^{p\times n}$ is constant matrix, y_m is model of output. The purpose is under certain conditions, design u, system（1）output tracking the output of system（2）, also $\lim_{t\to\infty}(y(t) - y_m(t)) = 0$

Consider the following conditions：

Situation（1）: system (1) , $m = n$, $B(x)$ reversible .

Situation（2）: system (1) , $m < n$, has the following form of decomposition

$$\begin{cases} E\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} A_{11}x_1 + A_{12}x_2 \\ A_2(x) + B_2(x)u \end{bmatrix} \\ y = C_1x_1 + C_2x_2 \end{cases}$$

$B_2(x)$ reversible （3）Situation(3)：system （3）,output $y = C_1 x_1$, system follow a normal reference model .

$$\dot{\omega} = A_m \omega \quad , \quad y_m = L\omega \qquad (4)$$

Lemma if (E, A) stable, and existing matrix R meet the following equation

$$\begin{cases} AR + F - RA_m = 0 \\ CR - L = 0 \\ RE = ER \end{cases} \qquad (5)$$

so

$$\begin{cases} E\dot{x}(t) = Ax(t) + F\omega(t) \\ E\dot{\omega}(t) = A_m\omega(t) \\ e(t) = Cx(t) - L\omega(t) \end{cases} \qquad (6)$$

Output $e(t) \to 0 (t \to \infty)$, here the coefficient matrix is constant matrix.

Prove：as（5）、（6）know

$$e = Cx - L\omega = Cx - CR\omega = C(x - R\omega)$$

$$E\dot{x} = Ax + F\omega = Ax + (RA_m - AR)\omega$$

$$RE\dot{\omega} = RA_m\omega$$

As $RE = ER$ so $E(\dot{x} - R\dot{\omega}) = A(x - R\omega)$

Because (E, A) stable

So $x - R\omega \to 0 \quad (t \to \infty)$

and $e(t) \to 0 (t \to \infty)$

GENERALIZED NONLINEAR SYSTEMS MODEL TRACKING

By using the variable structure control method to discuss the tracking problem, first to design generalized tracking system switching function S, makes the switching surface S=0, model tracking; after the design of the variable structure control law, implementation of sliding mode motion.

Situation（1）（$m=n$, $B(x)$ Reversible）Tracking Problem

Theorem 1 for system (1) , if $B(x)$ reversible and

①existence constant matrix $\bar{A}$, send $(E,\bar{A})$ stable;
② existence constant matrix R and F, satisfy matrix equation

$$\begin{cases} \bar{A}R+F-RA_m=0 \\ CR-L=0 \\ RE=ER \end{cases}$$

exist switching function:

$$\begin{cases} S=KEx+DQ \\ \dot{Q}=-D^{-1}K(\bar{A}x+F\omega) \end{cases} \quad (7)$$

On the Switching surface $S=0$, realize model tracking $y(t)=y_m(t)(t\to\infty)$.
Prove：Using dynamic compensator , structure switching function

$$\begin{cases} S=KEx+DQ \\ \dot{Q}=h(x,\omega) \end{cases} \quad (8)$$

order $\dot{S}=0$
so

$$KA(x)+KB(x)u+Dh(x,\omega)=0 \quad (9)$$

As $B(x)$ reversible,

$$u_{eq}=-B^{-1}(x)(A(x)-\bar{A}x-F\omega) \quad (10)$$

The ideal sliding mode motion equation is $E\dot{x}=\bar{A}x+F\omega$
as（9）and（10）

so $h(x,\omega)=-D^{-1}K(\bar{A}x+F\omega)$
so dynamic compensator equation is

$\dot{Q}=-D^{-1}K(\bar{A}x+F\omega)$

as switching function（7), on the Switching surface $S=0$

$$E\dot{x}=\bar{A}x+F\omega$$

$$E\dot{\omega}=F\omega$$

$$e=Cx-L\omega$$

Lemma and theorem1 conditions know $e(t)\to 0(t\to\infty)$,
on the switching surface realize the tracking.

order $\dot{S} = -f(s)$, Here $f(s)$ is s continuous function, satisfy $S^T f(s) > 0$.
Along the track of the solutions of (1), S about t derivative is

$$\dot{S} = KE\dot{x} + D\dot{Q}$$

$$= KA(x) + KB(x)u - K(\bar{A}x + F\omega)$$

If the selection of control u

$$u = -[KB(x)]^{-1}[KA(x) - K\bar{A}x - KF\omega + f(s)] \quad (11)$$

There is $S^T \dot{S} = -S^T f(s) < 0$

In the switching function（7） and the variable structure control（11), system (1) output tracking reference model the output of (2) .

Situation（2）（$m < n$, $B_2(x)$ Reversible）Tracking Problem

Theorem2 for system（3）

$$\begin{cases} E\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} A_{11}x_1 + A_{12}x_2 \\ A_2(x) + B_2(x)u \end{bmatrix} \\ y = C_1x_1 + C_2x_2 \end{cases}, B_2(x) \text{ reversible}$$

if：

①existence constant matrix $A_2 \in R^{m\times n}$, make $(E, \bar{A})$ stable, Here $\bar{A} = \begin{bmatrix} A_1 \\ A_2 \end{bmatrix}$, $A_1 = (A_{11} \quad A_{12}) \in R^{(n-m)\times n}$

②existence constant matrix R and F, satisfy matrix equation

$$\begin{cases} \bar{A}R + F - RA_m = 0 \\ CR - L = 0 \\ RE = ER \end{cases}$$

There is switching function

$$\begin{cases} S = KEx + DQ \\ \dot{Q} = -D^{-1}K(\bar{A}x + F\omega) \end{cases}$$

On the Switching surface $S = 0$, realize model racking $y(t) = y_m(t)(t \to \infty)$.
Prove：design switching function like（8）

By $\dot{S} = 0$ so

$$K\begin{bmatrix} A_{11}x_1 + A_{12}x_2 \\ A_2(x) + B_2(x)u \end{bmatrix} = -Dh(x, \omega)$$

As $B_2(x)$ reversible, take the equivalent control for

$$u_{eq} = -B_2^{-1}(x)[A_2(x) - A_2x - \bar{F}\omega]$$

As u_{eq} plug in （$A_2(x)+B_2(x)u$） ,so

$$A(x)+B(x)u_{eq} = \bar{A}x + F\omega, \quad F=\begin{bmatrix}0\\ \bar{F}\end{bmatrix}$$

Have to $h(x,\omega) = -D^{-1}K(\bar{A}x + F\omega)$

Along the orbit of the solutions of (3), $S(t)$ about t derivative is

$$\dot{S}(t) = KE\dot{x} + D\dot{Q}$$

$$= (K_1 \quad K_2)\begin{bmatrix}A_{11}x_1 + A_{12}x_2\\ A_2(x)+B_2(x)u\end{bmatrix} - (K_1 \quad K_2)\begin{bmatrix}A_{11}x_1 + A_{12}x_2\\ A_2x + \bar{F}\omega\end{bmatrix}$$

$$= K_2\left[A_2(x)+B_2(x)u - A_2x - \bar{F}\omega\right]$$

As $\dot{S} = -f(s)$

so $u = -[K_2B_2(x)]^{-1}[K_2A_2(x) - K_2A_2x - K_2\bar{F}\omega + f(s)]$ （12）

At the switching function（7）and variable structure control（12), system (1) output tracking reference model the output of (3).

Situation（3）（$B_2(x)$Reversible, $y=C_1x_1$） Tracking Problem

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Consider a special case

$$E=\begin{bmatrix}I_{n-m} & 0\\ 0 & 0_{m\times m}\end{bmatrix}$$

there are

$$\begin{cases}\dot{x}_1 = A_{11}x_1 + A_{12}x_2\\ [0]\dot{x}_2 = A_2(x)+B_2(x)u\\ y = Cx_1\end{cases} \quad B_2(x) \text{ reversible} \qquad (13)$$

Theorem3 for system （13), if

①existing matrix K_1,make $\sigma(A_{11}+A_{12}K_1)\subset c^-$;

② existing matrix R satisfy algebra equation

$$(A_{11}+A_{12}K_1)R + A_{12}F - RA_m = 0$$

$$CR - L = 0$$

There are switching function

$$S = -K_1x_1 + x_2 - F\omega \qquad (14)$$

On the Switching surface $S=0$, (13) the output tracking of the output of the system.

$\dot{\omega} = A_m \omega$, $y_m = L\omega$

Prove：design switching function

$S = -K_1 x_1 + x_2 - DQ$, $\dot{Q} = h(x,\omega)$

On the Switching surface $S = 0$,

$$x_2 = K_1 x_1 + DQ$$

$$\dot{S}(t) = -K_1 A_{11} x_1 - K_1 A_{12} x_2 + \dot{x}_2 - Dh(x,\omega) \qquad (15)$$

As $B_2(x)$ reversible, order

$$u = -\dot{x}_2 + v$$

Plug in（13） have to

$$\dot{x}_2 = [B_2(x)]^{-1}[A_2(x) + B_2(x)v]$$

Plug in（15）have to

$\dot{S} = -K_1 A_{11} x_1 - K_1 A_{12} x_2 + [B_2(x)]^{-1} A_2(x) + v - Dh(x,\omega)$ Taking the equivalent control V_{eq} is

$$V_{eq} = K_1 A_{11} x_1 + K_1 A_{12} x_2 - [B_2(x)]^{-1} A_2(x) + FA_m \omega$$

There are $h(x,\omega) = -D^{-1} FA_m \omega$

As switching function（14）, on the Switching surface $S = 0$,

$$\dot{x}_1 = (A_{11} + A_{12} K_1) x_1 + A_{12} F\omega$$

$$\dot{\omega} = A_m \omega$$

$$e = Cx_1 - L\omega$$

order $\dot{S} = -f(s)$

so $u = -\dot{x}_2 + V_{eq} - f(s)$

Under the conditions of lemma and theorem3,

so $e(t) \to 0 (t \to \infty)$.

CONCLUSION

This artic has Based on the situation classification of Generalized nonlinear systems, the design of various situation tracking under system switching function S, the switching surface S=0, model tracking, and the design of variable structure control law, the realization of Generalized nonlinear systems with sliding mode motion.

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