

Kinematic Modeling and Trajectory Tracking Control of a Wheeled Omni-directional Mobile Logistics Platform

Zhenqing Gao, Yuanxin Yang, Yanping Du, Yuan Zhang*,
Zhaohua Wang and Weidong Xu

ABSTRACT

At present, the production workshop generally has the following characteristics: not-standardized layout, Cramped space and more working environment changes, Automated Guided Vehicle (AGV), in logistic, has the limitation of movement flexibility and intelligence, and cannot meet the logistics requirements. In this paper, a circular omni-directional mobile logistics platform with zero turning radius is designed by the four mecanum wheels structure. First of all, we devise the Mobile structure. On the basis, using relevant mathematical knowledge, the kinematic model and dynamic model of the platform are designed. Then, define the trajectory tracking error equation, and choose backstepping method to realize the trajectory tracking control. Finally, we simulate the nonlinear system dynamics using simulink, the graphical matlab workspace. The omni-directional autonomous motion of the robot is realized. It is an effective attempt to improve the intelligent of the logistics system.

INTRODUCTION

With the booming rise of the Industry 4.0 wave in China, the voice of change from Chinese manufacturing to China made is also rising. In the transition period, a large number of industrial robots application gradually becomes a new normal enterprise intelligent [1]. Under the fast development of electronic commerce, logistics industry, as the third party of our country's economic activities, is facing the new demands and challenges. Some of the characteristics of logistics, such as delivery unit miniaturization, more varieties, small quantity, more batches and short cycle, make the traditional logistics difficult to meet the new demand. Then, the automated logistics, which is based on the mobile robot, is emerging [2-3]. The inherent agility of the omni-directional mobile robot makes it widely studied for dynamic environmental applications [4]. As the world's largest foundry Foxconn of

*Corresponding author: Yuan Zhang 441274878@qq.com, Beijing Institute of Graphic Communication, Beijing, 102600

Apple has enabled the robot Foxbots on the product line. Google has acquired of 8 robotic companies for the development of manufacturing and logistics industry. Through sales channels, Google's robots can complete all processes automatically from the supply chain, and sent the goods to the hands of consumers directly[5]. A new kind of logistics robot, designed by a well-known Japanese manufacture, can realize the automatic loading and unloading of the goods through two mechanical arm flexibly[6]. Amazon robot Kiva can move the shelves to the front of staff directly. This logistics mode rains out the displacement action of the transmission line, reduces a lot of unnecessary labor, makes the logistics process more automated, and shows the world's most advanced logistics management operation [7].

This paper designed an omni-directional mobile logistics platform, focusing on the mobile structure, kinematics model, dynamic model and trajectory tracking control.

MOBILE STRUCTURE DESIGNS AND THE KINEMATIC MODEL ANALYSIS

Most conventional driving systems have several structural limitations in vertical, horizontal, and diagonal movements because of the cylindrical wheel structure. For example, moving the vehicle in a particular direction is necessary when driving in narrow spaces or in spaces with many obstacles. In these situations, the existing cylindrical wheel structure cannot drive in a random direction because it requires a

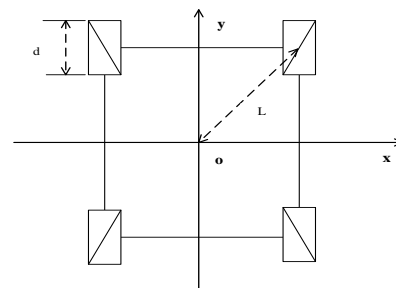


Figure 1. Platform Structure.

lot of space for rotating and turning the vehicle body[8-9]. An omni-directional mobile robot is a type of holonomic robots, it has the ability to move simultaneously and independently in translation and rotation[4,10]. The omni-directional wheel, as an important part of the mobile robot is widely studies by researchers, some typical like mecanum wheel, orthogonal wheel, ball wheel and eccentric direction wheel etc. Since the mecanum wheel's compact structure and flexible movement, it is generally applied in mobile structure design. And, based on the obvious advantages of the four wheeled structure: single direction acceleration, low motor power consumption, good stability, we choose the wheel group layout structure, as shown in Fig. 1.

The initial pose of the platform is showed in Fig. 1. Assume that the local coordinate system origin O is placed in the center of the robot and coincides with the center of gravity. Set parameters as follows:

- (1) Drive roller's offset angle is 45° ;
- (2) The initial angles between four wheels and the X axis are 60° , 120° , 240° and 300° ;
- (3) xoy—Local rectangular coordinate system with the center point O of the platform;
- (4) L—Distance from the center point O of the platform to the center of the wheel rotary shaft;
- (5) r—Wheel action radius;
- (6) $[v_x \ v_y \ \omega]^T$ —Generalized velocity of the point O;

The inverse kinematic analysis result of four wheeled omni-directional mobile structure is showed as formula (1).

$$\begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \\ \omega_4 \end{bmatrix} = -\frac{1}{r} \begin{bmatrix} -1 & -1 & \frac{\sqrt{3}-1}{2}l \\ 1 & -1 & \frac{\sqrt{3}-1}{2}l \\ -1 & -1 & \frac{\sqrt{3}-1}{2}l \\ 1 & -1 & \frac{\sqrt{3}-1}{2}l \end{bmatrix} \begin{bmatrix} v_x \\ v_y \\ \omega \end{bmatrix} \quad (1)$$

The formula (1) shows the relationship between the speed of four omni-directional wheels and the overall motion pose of the platform. It is the mathematical basis of the motion control.

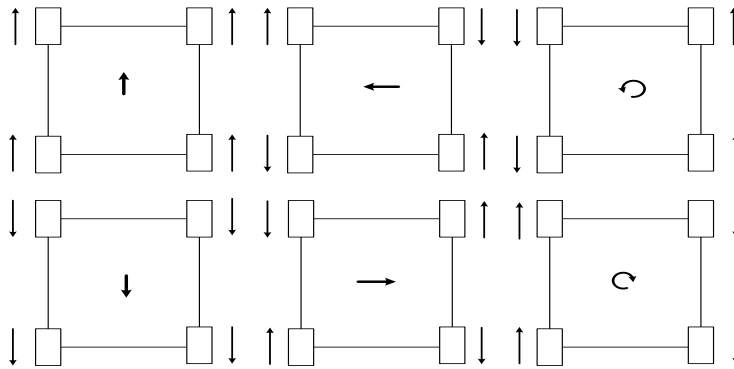


Figure 2. Principle of the Platform Omni-directional Motion.

The principle of the platform omni-directional motion is shown in **Fig. 2**. Through the combination movement of each wheel, the platform can realize the omni-directional movement, not only the linear motion along the x axis or y axis, but also the fixed rotation movement around the center point. The omni-directional mobile robot can move along any trajectory under the premise of the body posture unchanged, and realize the omni-directional mobility.

DYNAMIC MODEL ANALYSES

The kinematics model of the robot reflects the mapping relationship between the wheel speed and the body speed, while, the kinetic model reflects the relationship between the motion performance and the driving force of the robot.

In practice, the platform will be subjected to a variety of external forces, making it difficult to analyze the dynamics model. And, since the study is about the robot indoor motion, we do the following assumptions:

- (1)Ignore the influence of the air resistance;
- (2)The platform quality distribution is uniform;
- (3)The platform moves on a flat surface;
- (4)The motion of the wheel is rolling without slipping;
- (5)The pressure of each wheel set produced by the high center of gravity is neglected when the platform is in the acceleration movement.

About the effect of external forces, we just consider the friction factor caused by the contact with the ground, which is the reason platform moving forward.

The total energy of the mobile robot includes: the translational energy and the rotational kinetic energy of the platform, each wheel rotational kinetic energy, as shown in formula (2):

$$K = \frac{1}{2}m (V_x^2 + V_y^2) + \frac{1}{2}I_z\omega^2 + \frac{1}{2}I_w(\omega_1^2 + \omega_2^2 + \omega_3^2 + \omega_4^2) \quad (2)$$

In addition, since the existence of friction, the energy consumption of each wheel can be expressed as the formula (3):

$$F = \frac{1}{2}F_\omega(\omega_1^2 + \omega_2^2 + \omega_3^2 + \omega_4^2) \quad (3)$$

Where m is the total weight of the platform; I_z is the r moment of inertia of the platform; I_w is the moment of inertia of the wheel assemblies; F is the wheel friction coefficient of friction.

From the Lagrange equation:

$$\frac{d}{dt} \left(\frac{\partial K}{\partial \dot{q}} \right) - \frac{\partial K}{\partial q} + \frac{\partial F}{\partial \dot{q}} = \tau \quad (4)$$

The dynamic model of the omnidirectional mobile platform can be expressed as:

$$\begin{pmatrix} m + \frac{4I_w}{r^2} & 0 & 0 \\ 0 & m + \frac{4I_w}{r^2} & 4l(1-\sqrt{3}) \\ 0 & 4l(1-\sqrt{3}) & \frac{2l^2(\sqrt{3}-1)^2}{r^2} + I_z \end{pmatrix} \begin{pmatrix} \dot{V}_x \\ \dot{V}_y \\ \dot{\omega} \end{pmatrix} - \begin{pmatrix} \frac{4I_w}{r^2} & 0 & 0 \\ 0 & \frac{4I_w}{r^2} & 4l(1-\sqrt{3}) \\ 0 & 4l(1-\sqrt{3}) & \frac{2l^2(\sqrt{3}-1)^2}{r^2} \end{pmatrix} \begin{pmatrix} V_x \\ V_y \\ \omega \end{pmatrix} = \begin{pmatrix} F_x \\ F_y \\ F_\omega \end{pmatrix} \quad (5)$$

TRAJECTORY TRACKING CONTROL

The Error Equation

Assume that there is a virtual mobile platform, its trajectory is under the ideal conditions, $(x_r(t), y_r(t), \theta_r(t))$ refers to the virtual track position vector in the local coordinates. And, $(x(t), y(t), \theta(t))$ refers to the mobile platform position vector in the local coordinate system.

The trajectory tracking error is defined as:

$$e = \begin{bmatrix} x_r(t) - x(t) \\ y_r(t) - y(t) \\ \theta_r(t) - \theta(t) \end{bmatrix} \quad (6)$$

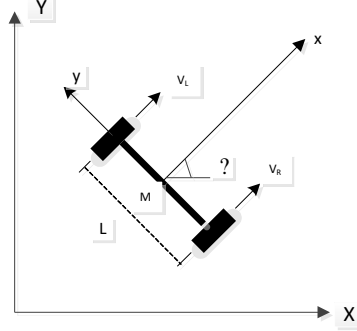


Figure 3. Global Coordinate of Platform.

From the Fig. 3, we can get that the transformation matrix of the global coordinate system and local coordinate system is:

$$R = \begin{bmatrix} \cos\theta(t) & \sin\theta(t) & 0 \\ -\sin\theta(t) & \cos\theta(t) & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (7)$$

Hence:

$$\begin{bmatrix} x_e(t) \\ y_e(t) \\ \theta_e(t) \end{bmatrix} = \begin{bmatrix} \cos\theta(t) & \sin\theta(t) & 0 \\ -\sin\theta(t) & \cos\theta(t) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_r(t) - x(t) \\ y_r(t) - y(t) \\ \theta_r(t) - \theta(t) \end{bmatrix} \quad (8)$$

Formula (8) indicates the trajectory tracking error in local coordinate system. In order to make the mobile logistics platform move without error, (x_e, y_e, θ_e) needs to meet the consistent and bounded and satisfy the formula (9):

$$\lim_{t \rightarrow \infty} \|[x_e \quad y_e \quad \theta_e]^T\| = 0 \quad (9)$$

Control Law

A precise trajectory tracking control is a key component for applications of omni-directional robots. This paper chooses backstepping method to realize the platform control, it decomposes a complex system into a number of simple small system. By the control method design of single small systems, the whole system control law will be introduced.

First, reference Lyapunov function:

$$V_1 = \frac{1}{2}x_e^2 + \frac{1}{2}y_e^2 \quad (10)$$

Hence:

$$\dot{V}_1 = x_e \dot{x}_e + y_e \dot{y}_e = x_e (\dot{x}_r - \dot{x}) + y_e (\dot{y}_r - \dot{y}) \quad (11)$$

Refer formula (3), introduce a virtual parameters α_1 , and define:

$$\begin{cases} \dot{x} = v \cos \alpha_1 = \dot{x}_r + c_1 x_e \\ \dot{y} = v \sin \alpha_1 = \dot{y}_r + c_2 y_e \end{cases} \quad (12)$$

Where c_1, c_2 are positive constants.

Thus: $\dot{V}_1 = -c_1 x_e^2 - c_2 y_e^2 < 0$,

And:

$$\begin{cases} v = \sqrt{\left(\dot{x}_r + c_1 x_e\right)^2 + \left(\dot{y}_r + c_2 y_e\right)^2} \\ \alpha = \arctan \frac{\dot{x}_r + c_1 x_e}{\dot{y}_r + c_2 y_e} \end{cases} \quad (13)$$

Define the Lyapunov function as:

$$V_2 = V_1 + \frac{1}{2}(\alpha - \theta)^2 \quad (14)$$

So:

$$\dot{V}_2 = -c_1 x_e^2 - c_2 y_e^2 + (\alpha - \theta) (\dot{\alpha} - \dot{\omega}) \quad (15)$$

Define :

$$\dot{\omega} = \dot{\alpha} + c_3(\alpha - \theta) \quad (16)$$

Where c_3 is positive constant.

Hence:

$$\dot{V}_2 = -c_1 x_e^2 - c_2 y_e^2 - c_3(\alpha - \theta)^2 < 0 \quad (17)$$

From the above, we can see that the platform can move along the curve without error under the appropriate feedback factors v and ω .

Simulation

Given equations, it was possible to simulate the nonlinear system dynamics using simulink, the graphical matlab workspace. The controller used is a Wheel independent proportional-integral-derivative controller, as shown in Fig.4.

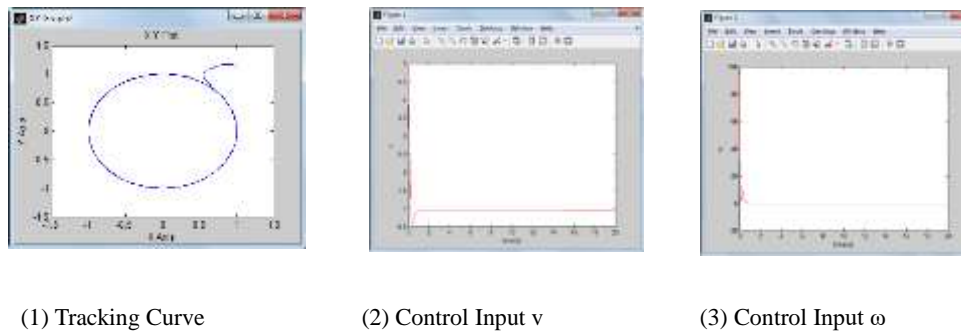


Figure 4. Tracking Curve and Control Input v, ω .

In this experiment, the proposed mobile robot moved along a circular path with or without the guidance of the indoor localization system, setting the references as: $c_1 = c_2 = 5$, $c_3 = 30$ and state=[0 0 0]. From this experimental result, it can be seen that the localization system can successfully maintain the robot heading angle along a circular path.

CONCLUSIONS

The research and development of the omnidirectional mobile robot without logistics track oriented, highly maneuverable, greatly improved the flexibility of logistics transport equipment, improve the operational efficiency of the logistics system and intelligent degree, with the traditional way of wheeled mobile can not replace the function, the related research for robot applications and logistics equipment has a certain value. Further research work needs to be carried out in the aspects of positioning accuracy and adaptability to the environment.

ACKNOWLEDGMENTS

The authors would like to express their thanks to the editor and anonymous reviewers for their help in revising the manuscript. This research is sponsored by BIGC Project (No. Eb201605).

REFERENCES

- [1] Q.Q. Liu, W. Liang. *The focus of intelligent logistics to celebrate the "Robot Revolution"* [J]. GDSTC, (2015), 17:26-29.
- [2] B.W. Shen, N.B. Yu, J.T. Liu. *Intelligent scheduling and path planning for the intelligent scheduling and path planning of warehousing logistics robot cluster* [J]. SCI, (2014), 06:659-664.
- [3] Q. Wang. *Research on the development of logistics industry in China* [J]. SCI, (2016),05:37-38.
- [4] Y. Liu, J. Zhu, R.L. Williams II, J Wu. *Omni-directional mobile robot controller based on trajectory linearization.*[J].SCI,(2008),56.
- [5] F. Tobey, C.Q. Zhang, *Apple, Amazon and Google to the robot Era* [J]. SCI, (2014),03:53-54.
- [6] *Japan launched a walking logistics robot* [J]. CMB, (2015),09:30-31.
- [7] J.P. Wu, *Application and Prospect Analysis of the Amazon warehouse Kiva robot technique and application prospect of*[J]. Logistics, (2015),10:159-164.
- [8] H. J. Kim and B. K. Kim, *Minimum-energy trajectory planning and control on a straight line with rotation for three-wheeled omni-directional mobile robots, Intelligent Robots and Systems (IROS)*, IEEE/RSJ, (2012) 3119-3124.
- [9] H. Lee, Y. Na, T. Park, Y. Cha and J. Park, *Fabrication of omni-directional driving system using unconstrained steel ball*,MedSci,29 (1) (2015) 93~97.
- [10] Pin, F.G. and Killough, S.M. *A new family of omnidirectional and holonomic wheeled platforms for mobile robot*, IEEE, trans. Robotics Automat 10(2)(1994),480~489.