Multi-objective and Multi-product Inventory-location Model under Stochastic Demand

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Abstract. This article studies the problem of inventory location for multi-objective and multi-product integrated planning based on existing warehouses. The model constructed in this paper includes two objective functions, minimizing total cost and minimizing transit time, and taking into account three realistic constraints of lead time, minimum order volume, and warehouse capacity. Afterwards, the second-generation genetic algorithm was used to solve the model and the actual case was used to prove that the integrated planning was more cost-effective than the new-build warehouse. And market fluctuations will not have much impact on the results of warehouse planning. Finally, the article validates the constraint conditions and proves that the lead time has a great influence on the warehouse planning results, while the minimum order quantity constraint has little effect on the planning results.

Introduction

The location problem is one of the classical problems in operations research. However, because the supply chain location network is a system problem, more scholars have turned their attention to the study of location-inventory (LIP) issues in recent years. LIP can be divided into single-target LIP and multi-target LIP. Single-target LIP issues, such as Eppen's first comparison of single-target LIP issues with centralized and decentralized inventory systems under stochastic demand. Nozick and Turnquist studied a car case and analyzed the impact of safety inventory costs on DC site selection [1]. Wang Fei et al. compared two-level supply chain LIP issues with designated candidate points and unspecified candidate points [2]. Zhang and Hu Halvorsen-Weare and Kjetil studied similar LIP models with various finite and infinite capabilities [3,4]. Wijk, Adan and van Houtum, Shavandi and Bozorgi studied the LIP model with Poisson distribution and fuzzy demand respectively [5,6]. Multi-objective problems such as Nekooghadirli studied multi-objective and multi-objective LIRP problems for two-stage multiproducts [7]. Azaron et al. The pre-investment multi-targeting-inventory model was studied and GAT (objective negotiation method) was used to solve [8]. Asl-Najafi et al. studied the dynamic closed-loop positioning of Zhao Bobao's inventory problem [9].

First of all, although the multi-objective logistics network planning has attracted attention, the current research results are not many. Second, most models lack case studies and do not consider the actual constraints in the operation of the company, resulting in over-simplification of the model and poor applicability. Finally, most articles put more attention on the solution method of the model, trying to find a more rapid and reliable solution method, and neglecting the discussion and performance analysis of the model itself.

Model Establishment

Problem Description

We examine a three-tier supply chain network problem consisting of a factory, multiple warehouses, and multiple customer areas. In the initial scenario, each customer area has only one
warehouse for its delivery, and each warehouse serves only one customer area. After optimization, it is necessary to realize a warehouse to serve multiple customer areas with a minimum cost and the shortest transit time, and to abandon part of the warehouses.

**Assumptions and Restrictions:**

The assumption include:(1) There is one and only one warehouse in a customer area to provide services; (2) The inventory strategy is continuous inspection \((s, S)\); (3) Demand independence and customer demand obeys a normal distribution with mean \(\mu\) and variance \(\sigma\); (4) The costs of out of stock and ordering are not considered. The restrictions include: (1) Capacity constraints, that is, warehouse does not allow expansion; (2) The lead time of order is determined;

**Symbol Definition**

Indices

- \(i\): Distribution centres, \(i = 1, \ldots, P\)
- \(j\): Customer zones, \(j = 1, \ldots, p\)
- \(p\): Products, \(p = 1, \ldots, P\)

Inputs

- \(F_i\): Fixed costs for DC \(i\)
- \(TC_i^p\): Inbound transportation costs for product \((k)\) from Supplier \((p)\) to DC \((i)\)
- \(DC_{ij}^p\): Outbound transportation costs for product \((p)\) from DC \((i)\) to customer zone \((j)\)
- \(T_{ij}\): Delivery time required from distribution center \((i)\) to customer area \((j)\)
- \(d_{ij}\): Distance between distribution center and customer area \((j)\)
- \(L\): The lead time for the delivery center to place orders with the factory
- \(H\): Annual cost of goods for unit price goods, here is the cost per unit of funds
- \(Z_{\alpha}\): Service level, \(\alpha\) represents the probability that the demand can be satisfied when the customer arrives
- \(\chi\): Days in the year
- \(Cap_i\): Capacity for DC \(i\)
- \(\psi_p\): Storage coefficient of product \(p\)
- \(Order_{p}^{size}\): Lower limit of order limit for product \(p\)
- \(\mu_p\): Average daily demand for product \(p\) in customer region \(j\)
- \(\left(\sigma_p\right)^2\): Variance of daily demand for product \(p\) in customer region \(j\)

Outputs

- \(X_i\): If the warehouse \(i\) is selected as the distribution center, it is 1, otherwise it is 0
- \(Y_{ij}\): If the repository \(i\) serves the client area \(j\), otherwise 0
- \(S_i^p\): Target inventory for each order of \(p\)-items in distribution center \(i\)

**Modelling**

\[
\begin{align*}
\min & \left( \sum_i X_i F_i + \chi \sum_p \sum_i \sum_j TC_i^p \cdot \mu_p \cdot Y_{ij} + \chi \sum_p \sum_i \sum_j DC_{ij}^p \cdot d_{ij} \cdot \mu_p \cdot Y_{ij} + H \sum_p \sum_i \sum_j \frac{X_j}{\mu_p} \cdot \mu_p \cdot Y_{ij}\right) \\
& + \min \sum_p \sum_i T_{ij} \cdot \left(\frac{\psi_p \cdot Y_{ij}}{\sum_j \mu_j}\right) \\
& + \sum_p \sum_i \frac{X_i}{\mu_p} \cdot \psi_p \cdot Y_{ij} + \psi_p \cdot Z_{\alpha} \cdot \sqrt{L \cdot \sum_j \left(\sigma_j^p\right)^2} \cdot Y_{ij} \leq Cap_i \quad \forall i \\
Y_{ij} \leq X_i & \quad \forall i, j
\end{align*}
\]
\[
\sum_i Y_{ij} = 1 \quad \forall j
\] (5)

\[X_i, Y_{ij} \in \{0,1\}
\] (6)

\[S_i^p - z_\alpha \sqrt{L} \geq Order_{size}^p
\] (7)

The objective function Eq.1 represents the minimization cost function, including (1) the fixed annual cost of the warehouse, (2) the annual transportation cost, (3) the increase in the distribution cost, (4) the turnover inventory holding cost, (5) Safety stock holding costs. The objective function Eq.2 denotes a minimization of the delivery time function, which is determined by the weight of each warehouse's annual demand and the delivery time from the warehouse to the customer area.

Problem Definition for a Real–World Scenario

A company has 17 existing warehouses and needs to integrate existing warehouses through planning. The basic model parameters for the following calculations are leadT=10, pop( population size)=500, num(number of iterations)=50, \(\mu_j^p = \mu_j^P, \sigma_j^P = \sigma_j^P, Cap_i = Cap_i, Order_{size}^P = 10000, p=2, i-j=17\).

In genetic algorithms, the proper number of iterations and population size play a very important role in the stability and efficiency of the operation results. Therefore, we repeated 5, 10 for population sizes of 500, 800, and 1000, respectively, 30 times, 50 times, 70 times. Perform 10 calculations in each case. The final result is the average of 10 operations. We finally use the parameters with a population size of 500 and iterations of 50 as the final parameters.

Result Analysis

We analyze the model from four aspects: market fluctuations, warehouse capacity constraints, lead time, and order constraints. The figures in the Table.1 represent the objective function 1, objective function 2, and the number of reserved warehouses under the conditions of maximum cost, minimum cost, and average cost in the results obtained in each case, such as (5076, 57, 11) indicates that the inventory constraint is CAPi, the maximum total cost of the resulting offspring is 50.75 million yuan, the delivery time is 57 minutes, and 11 warehouses are required.

Marke Fluctuations

<table>
<thead>
<tr>
<th>((\sigma_j^P)^2)</th>
<th>max(cost, delivery time, DC num)</th>
<th>Min(cost, delivery time, DC num)</th>
<th>Avg(cost, delivery time, DC num)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1^*(\sigma_j^P)^2)</td>
<td>(5076, 57, 11)</td>
<td>(4550, 66, 4)</td>
<td>(4803, 61, 8)</td>
</tr>
<tr>
<td>(4^*(\sigma_j^P)^2)</td>
<td>(4961, 59, 8)</td>
<td>(4656, 65, 5)</td>
<td>(4841, 62, 7)</td>
</tr>
<tr>
<td>(8^*(\sigma_j^P)^2)</td>
<td>(5152, 60, 10)</td>
<td>(4780, 65, 6)</td>
<td>(4934, 62, 8)</td>
</tr>
<tr>
<td>(10^*(\sigma_j^P)^2)</td>
<td>(5121, 61, 10)</td>
<td>(4768, 69, 6)</td>
<td>(4919, 65, 7)</td>
</tr>
</tbody>
</table>

In general, the increase in the number of warehouses will shorten the delivery time, but as can be seen from Table.1, with the increase in market volatility, the number of warehouses in the planning results increased, and the transport time did not decrease, indicating that if the market demand for products increased volatility Large, the company needs more warehouses to face the market risk, and at the same time as the product safety stock increases, and the warehouse has capacity limitations, the number of feasible solutions will be reduced, which is very unfavorable for warehouse planning. Therefore, when planning, companies should fully consider the product demand volatility.
Warehouse Capacity Constraints

Table 2. The influence of warehouse capacity constraints.

<table>
<thead>
<tr>
<th>Inventory constraint</th>
<th>max(cost, delivery time, DC num)</th>
<th>Min(cost, delivery time, DC num)</th>
<th>Avg(cost, delivery time, DC num)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cap_i</td>
<td>(5076, 57, 11)</td>
<td>(4550, 66, 4)</td>
<td>(4803, 61, 8)</td>
</tr>
<tr>
<td>1.2*Cap_i</td>
<td>(5153, 58, 10)</td>
<td>(4667, 65, 4)</td>
<td>(4904, 61, 7)</td>
</tr>
<tr>
<td>1.5*Cap_i</td>
<td>(5302, 59, 9)</td>
<td>(4661, 73, 3)</td>
<td>(4816, 66, 6)</td>
</tr>
<tr>
<td>1.8*Cap_i</td>
<td>(5563, 58, 9)</td>
<td>(4826, 72, 4)</td>
<td>(5185, 64, 6)</td>
</tr>
<tr>
<td>2*Cap_i</td>
<td>(5700, 59, 9)</td>
<td>(4868, 74, 5)</td>
<td>(5267, 62, 7)</td>
</tr>
<tr>
<td>2.5*Cap_i</td>
<td>(5971, 58, 10)</td>
<td>(5024, 70, 5)</td>
<td>(5538, 63, 7)</td>
</tr>
</tbody>
</table>

From the Table 2, an increase in the storage capacity can lead to an increase in the total cost, which is due to the increase in fixed costs. At the same time, as the increase in the inventory capacity of a single warehouse leads to an increase in transportation time, the number of warehouses required does not decrease. This is because with the increase of the area of a single warehouse, in order to reduce the cost, the system will favor the selection of a warehouse with a smaller unit area and a lower fixed cost as a distribution warehouse. These warehouses are not necessarily closest to the customer area, and thus lead to transportation. Increased time. Therefore, for enterprises, if the market demand is relatively stable, the effect of integrated planning is better than the expansion of warehouses.

Lead Time Constraint

Table 3. The influence of order lead time constraint.

<table>
<thead>
<tr>
<th>Lead time</th>
<th>max(cost, delivery time, DC num)</th>
<th>Min(cost, delivery time, DC num)</th>
<th>Avg(cost, delivery time, DC num)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>(5073, 59, 10)</td>
<td>(4540, 66, 4)</td>
<td>(4752, 62, 7)</td>
</tr>
<tr>
<td>10</td>
<td>(5076, 57, 11)</td>
<td>(4550, 66, 4)</td>
<td>(4803, 61, 8)</td>
</tr>
<tr>
<td>15</td>
<td>(5004, 60, 9)</td>
<td>(4629, 65, 5)</td>
<td>(4816, 62, 7)</td>
</tr>
<tr>
<td>20</td>
<td>(4935, 59, 9)</td>
<td>(4636, 65, 5)</td>
<td>(4808, 62, 7)</td>
</tr>
<tr>
<td>25</td>
<td>(4933, 59, 9)</td>
<td>(4641, 65, 5)</td>
<td>(4832, 61, 7)</td>
</tr>
<tr>
<td>30</td>
<td>(5157, 58, 11)</td>
<td>(4684, 63, 8)</td>
<td>(4867, 60, 9)</td>
</tr>
<tr>
<td>45</td>
<td>(5256, 55, 13)</td>
<td>(4526, 69, 6)</td>
<td>(4857, 61, 10)</td>
</tr>
</tbody>
</table>

In fact, warehouses often set different lead times in advance to facilitate the operation of the company. So whether such a strategy has an impact on the inventory location is a very worthy issue. From the figure, we can see that with the extension of the lead time, the cost of the warehouse will first decline and then increase. The longer the lead time, the more warehouses will be needed, and the transportation time will be reduced. Therefore, companies should not be too long when designating lead time, nor should they be too short, and need to be planned rationally. At the same time, once the company has designated a good lead time and implemented a layout plan, it should minimize drastic changes because of the sharp lead time Changes will have a very big impact on the final outcome of the plan. This will result in the inability to achieve the desired results after implementing the layout plan.
Minimum Order Quantity Constraint

Table 4. The influence of Minimum order quantity constraint.

<table>
<thead>
<tr>
<th>num</th>
<th>$Order_{size}^1$</th>
<th>$Order_{size}^2$</th>
<th>max(cost,delivery time,DC num)</th>
<th>Min(cost,delivery time,DC num)</th>
<th>Avg(cost,delivery time,DC num)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>NO</td>
<td>NO</td>
<td>(5076, 57, 11)</td>
<td>(4550, 66, 4)</td>
<td>(4803, 61, 8)</td>
</tr>
<tr>
<td>2</td>
<td>20</td>
<td>40</td>
<td>(4900, 60, 8)</td>
<td>(4629, 66, 5)</td>
<td>(4751, 63, 6)</td>
</tr>
<tr>
<td>3</td>
<td>40</td>
<td>40</td>
<td>(5028, 59, 10)</td>
<td>(4550, 66, 4)</td>
<td>(4770, 62, 7)</td>
</tr>
<tr>
<td>4</td>
<td>40</td>
<td>80</td>
<td>(5073, 57, 11)</td>
<td>(4647, 66, 5)</td>
<td>(4865, 60, 8)</td>
</tr>
<tr>
<td>5</td>
<td>80</td>
<td>80</td>
<td>(5071, 57, 11)</td>
<td>(4550, 66, 4)</td>
<td>(4793, 61, 7)</td>
</tr>
<tr>
<td>6</td>
<td>80</td>
<td>160</td>
<td>(5169, 56, 12)</td>
<td>(4550, 66, 4)</td>
<td>(4848, 60, 8)</td>
</tr>
<tr>
<td>7</td>
<td>160</td>
<td>160</td>
<td>(5210, 54, 13)</td>
<td>(4513, 69, 6)</td>
<td>(4769, 62, 9)</td>
</tr>
<tr>
<td>8</td>
<td>160</td>
<td>320</td>
<td>(5324, 53, 14)</td>
<td>(4513, 69, 6)</td>
<td>(4801, 61, 9)</td>
</tr>
</tbody>
</table>

We have increased the minimum order quantity of the two kinds of products into multiples respectively. As a result, as shown in the Table 4, when the two products have the same constraint (num. 3, 5, 7), the cost function will be significantly reduced, and the transportation time will also be reduced. Obviously reduced. From the overall result of numbers 2, 4, 6, and 8, when the order constraint increases, the cost function will increase significantly and the transport time will be shortened. The reserve warehouse will increase, so for the enterprise, if There are two or more types of products, which should be as consistent as possible when formulating minimum order quantities. If the company pays more attention to the customer experience, it can increase the minimum order quantity, and if the enterprise has cost concerns, it can relax the restrictions on the order quantity. Of course, companies can temporarily ignore the constraints of order size when doing inventory planning because they have little impact on the final outcome of planning.

Summary

This paper integrates the logistics network planning on the basis of the existing warehouse to ensure the inventory location problem of transportation time, studies the problem of reducing the company's logistics cost, and analyzes the planning results from the three aspects of market changes, warehouse expansion and storage operational constraints. In the following research, we will consider the interaction of various parameters of model. The transport time of the model is not a general time between the two places. It is a variable dependent on the actual factors such as traffic congestion and so on.

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References

