The Auxiliary Function of Matlab Drawing in Mathematical Analysis Teaching

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Abstract. Aiming at the difficulty in mathematics analysis teaching, such as the convergence speed, uniform convergence and the concept of non-uniform convergence, matlab drawing auxiliary understanding of teaching content was introduced. Designed experiments are different from other mathematical experiments for the purpose of computing, it focuses on the understanding of the concept of mathematical analysis or provides a way of solving the problem of mathematical analysis. The purpose is to understand the difficult mathematical concepts, and to provide help for solving mathematical problems by analytical method.

Introduction

For a long time, mathematics has been considered to be a highly abstract subject, mathematics learning has been followed the theoretical framework of definition–proposition-theorem -proof. Mathematics teaching in higher education has always been emphasized highly abstract and rigorous logic, but not found the path of thinking problems to solve the problem. However, in the social background of the urgent need of engineering practice and innovation ability of outstanding talent under the expectation of outstanding talent can be closely combined with the mathematical and engineering practice, many schools carry out the "mathematics experiment" teaching.

The earliest mathematical experiment teaching in Jilin University 20 years ago[1], they set up basic courses in calculus, linear algebra and stochastic mathematics with curriculum experiment for over 2000 undergraduates, and laid solid foundation of mathematics experiment for students in mathematical modeling contest. After that, many mathematics experiment press were published [2][3][4]. At present, the domestic "mathematic experiment" teaching materials mainly focus on the calculation of some mathematical knowledge, such as the calculation of a limit, draw a picture, or a solution of linear equations, but not the understanding of mathematical concepts by the construction of experiment.

This paper focuses on building mathematical experiment about mathematical analysis. We hope students can under difficult definitions and concepts clearly through the mathematics experiment, thus help them learn mathematics, besides use mathematics. This is significantly different of our research from other.

Comparison of Convergence

An example of the speed of convergence is introduced by using the unfolding of different orders.

In the lecture about series, we know that the harmonic series \( \sum_{n=1}^{\infty} \frac{1}{n} \) is an important series of divergent series. It’s partial sums can be noted as \( \sum_{k=1}^{n} \frac{1}{k} = \ln n + \gamma + O \left( \frac{1}{n} \right) \) when n tends to infinity, where the limit of \( \sum_{k=1}^{n} \frac{1}{k} - \ln n \) called Euler constant, is approximately equal to 0.577.
Next, by using the expansion of higher order $\sum_{k=1}^{n} \frac{1}{k} = \ln n + \gamma + \frac{1}{2n} + o\left(\frac{1}{n}\right)$, we can show the significant difference in rate of convergence between two different approximate accuracy.

Calculate limit $\gamma$, and take the first three places.

```plaintext
syms k n; lim=limit(symsum(1./k,k,1,n)-log(n),n,inf);lim
lim = eulergamma
vpa(lim,3)
ans =0.577
```

Then in a graph, for $n$ from 1 to 10 different natural value, tracing points corresponding to $\sum_{k=1}^{n} \frac{1}{k} - \ln n$ and $\sum_{k=1}^{n} \frac{1}{k} - \ln n - \frac{1}{2n}$, and for comparison, see figure 1.

```plaintext
symsi;
f(i)=symsum(1./k,k,1,i)-log(i);
g(i)=symsum(1./k,k,1,i)-log(i)-1./i*1./2;h(i)=0.577;
end
figure,plot(1:10,f(1:10),'--k',1:10,g(1:10),'-.rd',1:10,h(1:10),':');
```

![Figure 1](image.jpg)

Figure 1. The Comparison between $\sum_{k=1}^{n} \frac{1}{k} - \ln n$ and $\sum_{k=1}^{n} \frac{1}{k} - \ln n - \frac{1}{2n}$.

Easy to find, $\left(\sum_{k=1}^{n} \frac{1}{k} - \ln n - \frac{1}{2n}\right)_{n\in\mathbb{N}^*}$ convergence to $\gamma$ faster than $\left(\sum_{k=1}^{n} \frac{1}{k} - \ln n\right)_{n\in\mathbb{N}^*}$.

Students can easily understand the different concepts of convergence speed.

**Uniform Convergence**

The uniform convergence concept of function sequences and function term series is difficult for beginners to learn. Two examples below can help to understand uniform convergence.

Define the function sequence $(f_n)$ as follows:

$$ f_n(x) = \frac{x}{1+n^2x^2} $$

We know $\forall x \in [0,1], f_n(x) = 0$, and can prove this function sequence uniform convergence to 0 in $[0,1]$.

Take $n=20, 50, 100, 200, 500, 1000, 2000, 5000, 10000$ respectively. We find each peak of $f_n(x) - f(x)$, $n \in \mathbb{N}$ appears in $x = \frac{1}{n}$, thus can make the function image close to 0 through increasing $n$.

Here is an example which is not uniform convergence. set $\forall n \in \mathbb{N}$, $f_n: \mathbb{R} \rightarrow \mathbb{R}, x \mapsto \frac{2nx}{1+n^2x^2}$. 

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Pointwise convergence to 0 can be proved easily. Since $\forall x \in \mathbb{R}, f_n'(x) = \frac{2n(-3n^2 x^4 + 1)}{(1 + n^2 x^2)^2}$, extreme points $x = 1/\sqrt{3n^2}$ can be obtained, $\sup_{\mathbb{R}} |f_n| \geq 1$. Therefore, $(f_n)_{n \in \mathbb{N}}$ is not uniform convergence.

Figure 2. Take $n=20,50,100,200,500,1000,2000,5000,10000$ for $f_n(x) = \frac{x}{1 + n^2 x^2}$.

Figure 3. Take $n=20,50,100$ and 100,500,1000 Respectively for $f_n(x) = \frac{2nx}{1+n^2 x^2}$.

Take $n=20,50,100,100,500,1000$ respectively. From the image point of view, the different x values converge to 0 step inconsistencies. There will be such a problem as long as 0 points exist. Therefore, we consider the study of uniform convergence on one smaller interval where 0 is removed.

$$\forall n \in \mathbb{N}, \forall x \in \mathbb{R}^+, |f_n(x)| \leq \frac{2n|x|}{n^2 x^4} = \frac{2}{n|x|^3}$$

Set $a > 0$, then $\forall n \in \mathbb{N}^+, \forall x \in \mathbb{R}$ and $|x| \geq a, |f_n(x)| \leq \frac{2}{n|x|^3} \leq \frac{2}{na^3}$. Thus, $\forall n \in \mathbb{N}^+, \sup_{|x| \geq a} |f_n(x)| \leq \lim_{n \to \infty} \frac{2}{na^3} = 0$. We conclude that $\forall a > 0, (f_n)_{n \in \mathbb{N}}$ is uniform convergence to $f = 0$ in interval $]-\infty, -a| \cup [a, +\infty[$.

In the example above, we can choose smaller $N$ when $x$ is large, and choose greater $N$ when $x$ is small, to insure the peak of $f_n$ locates between 0 and $x$ when $n \geq N$. Through the image, it is easy for students to understand why the following convergence is studied in $]-\infty, -a| \cup [a, +\infty[$.

Analysis of Convergence and Divergence of Function Term Series with Parameters

The convergence and divergence judgment of the series of function terms with parameters is also a difficulty in learning.
Set \( \varphi : \mathbb{N} \rightarrow \mathbb{N}, \ n \mapsto (n + 1)^2 \), and \( v_n = (-1)^n \sum_{i=\varphi(n)}^{\varphi(n+1)-1} \frac{1}{i} \). We have noticed that this series is a staggered series, and the series convergence can be proved as long as it is proved that the absolute value of the sum term is monotonically decreasing.

When \( \alpha = 1 \), \( |v_{n+1}| \leq \int \frac{(n+3)^2-1}{x} \ dx \leq \int \frac{(n+2)^2-1}{x} \ dx \leq |v_n| \leq \int \frac{(n+2)^2-1}{x} \ dx \),

We conclude that \( (|v_n|)_{n \in \mathbb{N}} \) descent to limit 0, the staggered series is convergent, and thus the original series converges.

When \( \alpha = 3/4 > 1/2 \), we draw the picture as follows:

![Figure 4. Figure for \( (v_n)_{n \in \mathbb{N}} \) when \( \alpha = 3/4 \), Where \( n=2:20 \) and \( v_n = (-1)^n \sum_{i=\varphi(n)}^{\varphi(n+1)-1} \frac{1}{n^\alpha} \).](image1)

When \( \alpha = 1/3 < 1/2 \) and \( \alpha = 1/2 \), we draw the picture as follows:

![Figure 5. Figure for \( (v_n)_{n \in \mathbb{N}} \) when \( \alpha = 1/2 \) and 1/3, Where \( n=2:20 \) and \( v_n = (-1)^n \sum_{i=\varphi(n)}^{\varphi(n+1)-1} \frac{1}{n^\alpha} \).](image2)

We estimate the series converge when \( 1/2 < \alpha < 1 \) and diverge when \( 0 < \alpha \leq 1/2 \).

By drawing, prior to the detail demonstrate convergence of series is a rough estimate. Analytical proof process will be appropriate scaling direction, reduce the calculation process, or directly inspired that diverges conclusion from the image.

In mathematical analysis we can construct many similar experiments, which can be used to understand mathematical definitions, familiar with a complex calculation process, or just see the figure of a complex equation.

**Summary**

SIAE of Civil Aviation University of China is a college of engineers run by China and France together. In the process of mathematics teaching in preparatory stage, we complete with reference to the French engineer preparatory stage syllabus, and focus on understanding definition and proving theorem, which is similar to domestic mathematics department teaching requirements. In this
process, French mathematics reference books give me an inspiration to understand mathematical definitions and theorems by using drawing[5][6][7].

On the one hand, we designed mathematics experiment to help understanding definitions and theorems in mathematics inspired by French Maple course. On the other hand, we use Matlab platform which is more popular in domestic undergraduate students. In combination with the current problems appears in SIAE teaching, we demonstrated the auxiliary role of MATLAB in the teaching of mathematical analysis.

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References