Vehicle Routing Problem with Time Windows Study

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Abstract. Customer demands of logistics timeliness is getting higher and higher. The research on vehicle routing problem with time windows, can improve the level of logistics services. This paper studies the optimization of vehicle routing problem with time window based on the above background. Thesis assumes that customers tend to be in a specific time period to accept delivery services, beyond the delivery service time will give punishment cost of time, in order to reflect a decrease in the level of service. The paper takes the minimum of total cost as the target, establishes the nonlinear distribution path model, and in the model, corrects the errors of the time constraint in the literature. Finally, the practical case is introduced, the model is established, the genetic algorithm is used to solve the problem, and the optimal distribution path is obtained.

Introduction

In most of the previous vehicle routing problem studies, people often pay attention to the total mileage of the least, so that the total cost of the corresponding minimum. But with the customer's personalized demand is getting higher and higher, we need to consider the customer to receive goods time. The customer does not meet the delivery, but wants to deliver the goods on time. Different customer time requirements are different, personalized service more and more competitive advantage. Especially with the online shopping, the rapid development of e-commerce, almost everyone online shopping, so the number of express parcels increasing dramatically. At the same time, the demand for delivery has gone up, but the customer demand for distribution is also getting higher and higher. Therefore, the solution of vehicle routing problem with time window [1] (VRPTW) is proposed. Through the solution of adaptive genetic algorithm, the minimum distribution cost and optimal distribution route can be obtained, which can meet the realistic requirements of logistics company and customer time, improve vehicle utilization, reduce delivery costs. Therefore, for logistics and related enterprises, the study of this problem can improve the economic efficiency of enterprises, reduce energy consumption, improve customer service satisfaction, thereby enhancing the core competitiveness of enterprises, but also for energy conservation and emission reduction provides an effective reference program.

The Research Status at Home and Abroad

In the case of vehicle routing problems with time windows, domestic and foreign scholars have done a lot of research work. Liu Cheng and others in 2005 for the genetic algorithm in the solution VRPTW initial population of the unity, proposed a parallel genetic algorithm, the algorithm for different populations with different initialization methods, the effect is ideal. Yang Yudong [2] and so on in 2006 to introduce the direct arrangement of the solution, improve the simulated annealing algorithm, improve the efficiency of solving VRPTW. Zhang Liyan [3] is equal to 2006 using the
Zhang Haigang is equivalent to 2007 to study the improvement of vehicle scheduling problem with hard time window based on improved immune genetic algorithm, and improve the information entropy calculation of the algorithm, which greatly improves the global convergence reliability and convergence speed of the algorithm. Foreign scholars have done a lot of research work: Ombuki and others in 2002 proposed a genetic algorithm and tabu search based on hybrid genetic algorithm, genetic algorithm to optimize the use of the number of vehicles, using tabu search algorithm to optimize vehicle travel distance, simulation results are better than the solution of each single algorithm. Brware [4] proposed a four-stage heuristic algorithm based on variable neighborhood search in 2003 to solve VRPTW. Russell [5] and others used the scatter search algorithm to solve the VRPTW problem in 2006; Calvete[6] et al. Proposed a goal planning method for solving the VRPTW problem with soft time windows in 2007. Azi [7] and others in 2007 based on the shortest path method to propose a two-stage algorithm to solve the VRPTW problem.

Based on the above literature review, the current domestic and foreign scholars in taking into account vehicle routing problem with time window research has made many achievements, but not perfect. Different researchers think about the different angles of the problem, the decision to establish the model when the factors are different, how to build a more realistic reality of the theoretical model has become the focus of attention of researchers. Distribution not only to meet the physical needs of customers but also to meet customer demand for time, especially in the customer requirements to minimize inventory under the premise of timely distribution becomes more and more important, so consider to meet the customer's time window constraints of the vehicle path problem needs cause enough attention.

**Vehicle Routing Problem with Time Windows**

**Problem description**

The time window is a time period \([e_i, l_i]\), which is a service interval determined by the earliest service time \(e_i\) and the latest service time \(l_i\) that the \(i\) would like to receive the goods. The vehicle routing problem with time window (VRPTW) is an extension of general vehicle routing problem (VRP). The VRPTW problem can be simply described as a reasonable way to meet the requirements of vehicles at different geographical locations for goods and services at a minimum cost, without prejudice to constraints such as vehicle capacity limits and time constraints. If the distribution center violates the customer's time window constraints, it is bound to have some loss. In order to pursue cost minimization, it is necessary to take the time effect cost into consideration.

**Model setting and hypothesis**

**Model settings.** 1. The goods flow to one-way, i.e. pure delivery, not receiving reverse logistics; 2. There is only one distribution center, and each line starts and ends at the distribution center. All vehicles must return to the distribution center within the prescribed time; 3. Location coordinates of distribution centers and customers' points; 4. The vehicle is the same type and the maximum capacity is known; 5. The vehicle shall not exceed its rated capacity during delivery; 6. The requirements of each customer node are known; 7. Each customer must and can only be accessed once; 8. Each customer has a designated service time window \([e_i, l_i]\), and it is best to arrive at the client time. 9. Each car serves only one route

**Model hypothesis.** 1. The cost of punishment is represented by function; 2. When the vehicle is served within the time window of the customer, the distribution center shall not pay any penalty costs; 3. The narrower the time window widths, the higher the marginal effect of its penalty costs; 4. The cost of punishment increases linearly with increasing violations, whether early or late.

**Parameter meaning**

\(N\): the total number of customers to be served by the distribution center; \(i,j\): single customer, \(i,j=0,1,\ldots,N\); \(i,j=0\) is the distribution center; \(k\): each vehicle number, \(k=0,1,\ldots,K\); \(c_{ij}\): transportation
The objective function Eq.1 means to minimize the total cost of delivery. It is made up of three vehicle driving distance costs, the cost of vehicle commissioning and time delay or early penalty cost; Constraint Eq.2 indicates that the number of vehicles starting from the distribution center does not exceed \(K\); Constraints Eq.3 indicate that each vehicle starts from the distribution center and eventually returns to the distribution center; Constraints Eq.4, Eq.5 indicate that each customer point is accessed by a single vehicle; Constraints Eq.6 indicate that the total number of tasks per vehicle is not exceeding the load weight limit of the vehicle; The constraint Eq.7 indicates that the transport vehicle \(k\) which is responsible for the distribution of the distributor \(j\) is driven from the distributor \(i\), namely the uniqueness of the preorder node; The constraint Eq.8 indicates that the departure time and return time of the vehicle are within the specified time window; Constraint Eq.9 means the time constraint of the vehicle from customer \(i\) to customer \(j\); Constraint Eq.10 represents the integer constraint, which limits \(x_{ij}\) to 0 or 1; The constraint Eq.11 indicates that if the cargo
transport task of the distributor \( j \) is completed by vehicle \( k \), then \( y_{jk} = 1 \), otherwise equal to 0; Constraint Eq. 12 is the expression of the penalty cost function.

**Case Analysis**

Study in this paper, the time window of vehicle routing problem, assumes that the case for a Courier company in a certain area is a distribution center, to 50 clients within the region (numbered 1, 2... 50) delivery of goods, distribution center no. 0 and all customers directly transport goods by vehicles. And customers have different requirements for delivery time. The vehicles in the distribution center are of the same quality, with the same load and weight of 200 units. The distance of the distribution is calculated from the formula of eucrid distance, namely, the time and distance are converted to the same unit. The service time of the vehicle is 15 units at all customers. The time window of the distribution center is defined as [0, 350]. The details of the 50 nodes in the case are shown below.

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The first data in Table 1. represents the number of the distribution center and the customer node, and 0 represents the distribution center. Respectively the second and third column data distribution.
center or client coordinates x and y coordinates, this paper assumes that the customer is the Euclidean distance, the distance between the vehicle from the customer i drive to customer j drive travel is equal to the time, and \( d_{ij} = d_{ji}, i \neq j \). Distance satisfy the triangle inequality, the fourth column represents the customer demand for the goods; The fifth and sixth columns respectively represent the earliest acceptable service hours and the latest available time of service; The seventh column is the customer's service time.

**Test parameter setting**

In the above case, the path optimization problem of how to arrange the vehicle and the driving path to minimize the total cost of the final distribution. In the process of solving, the relevant test parameters are set as below.

**Model parameter setting.** Unit distance cost alpha = 8; The cost of starting unit car is equal to 80; Waiting unit time cost \( p = 0.8 \); Delay unit time cost \( q = 1.5 \);

**Genetic algorithm runs parameter setting.** The operation parameters of genetic algorithm have a great influence on the performance of the algorithm. The operating parameters that the algorithm needs to set in this paper mainly have chromosome length \( L = 50 \). Population size \( S = 200 \); Cross probability \( P_c = 0.85 \); Variation probability \( P_m = 0.15 \); Termination of evolutionary algebra \( Gen = 200 \).

**Case result**

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The optimal solution of the above case test results is 22560, which consists of the optimal distribution path composed of 7 paths. The optimal distribution path scheme is shown in Fig. 1.
Conclusion

With the rapid development of the economic Internet, people's living standards are getting higher and higher, and the market is specialized, subdivided and individualized. People's logistics delivery time requirements continue to increase. Therefore, VRPTW is the point that logistics distribution center should pay attention to the direction of development and improvement. More and more customers want the distribution center to deliver the goods at the customer's desired time. In this paper, the vehicle routing optimization problem with time Windows, has carried on the exploration and research on vehicle routing problem under the time Windows optimization, in analyze the model of the relevant literature, found that are more or less defects, deficiencies. Therefore, with the help of my tutor, professor wu, I improved it and finally formed the model of this paper.

Reference