Efficiency Studies of Fast Charging and Smooth Highway

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ABSTRACT

Imagine that you are about to end a long journey and take a hot bath at home while there are dozens or even hundreds of cars in front of you, you may lose hope all at once. Our goal is to establish a model of the shape, size and merging pattern of the highway toll plaza. We use the confluence model, queuing theory and other mathematical knowledge to analyze the performance of the different shapes of the toll plaza and the proportion of the tollbooths. Then we are expected to propose a practical strategy for New Jersey Turnpike Authority. We build two models: linear toll plaza (LM model) and fan-shaped (arc) toll plaza (AM model). By using the knowledge of fluid mechanics and queuing theory, we divide the whole process into three stages -- the stage of entry, waiting and departure. On the basis of reasonable assumptions and simplifications, we calculate the time of each model and then judge the operating efficiency of the toll station by time.

KEYWORDS

Fluid mechanics, queuing theory.

ASSUMPTIONS AND SYMBOLS

Because of the large number of vehicles and their high speed, we borrow the fluid mechanics theory concept in physics, regarding the traffic flow as the continuous fluid.

To simplify the area calculating, we regard the toll plaza as a regular geometry.

The construction cost is positive correlation of the land area. Because the area is the most important factor affecting the cost.

The coefficient of safety is proportional to the vehicle density and is inversely proportional to the land area. Because when the plaza is large enough, there is more space for vehicles to travel; when there are more vehicles, the collision probability will increase.

In the non-overlapping time, the number of vehicles to reach the toll station is independent of each other.

In a sufficiently small time $\Delta t$, the probability of arrival of the vehicle is independent of $t$ (the whole procedure) and is proportional to $\Delta t$. 
<table>
<thead>
<tr>
<th>Symbols</th>
<th>Descriptions</th>
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<tbody>
<tr>
<td>$W_w, W_c, W_a$</td>
<td>Width of the toll plaza</td>
</tr>
<tr>
<td>$L_u$</td>
<td>Length of the merging zone</td>
</tr>
<tr>
<td>$T_o, T_i, T_u$</td>
<td>Time for fanning out, waiting and fanning in</td>
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<tr>
<td>$d_1, d_2$</td>
<td>Width of the traffic lane and the distance between every two charging windows</td>
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<td>$v_0, v_2$</td>
<td>Velocity of fanning out and in</td>
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<tr>
<td>$q$</td>
<td>The traffic flow</td>
</tr>
<tr>
<td>$B, L$</td>
<td>The number of the charging windows and the lanes</td>
</tr>
<tr>
<td>$s$</td>
<td>The area of the toll plaza</td>
</tr>
<tr>
<td>$M$</td>
<td>The cost of construction</td>
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**THE LINEAR MODEL (LM MODEL)**

In the LM model [1], we assume the sides are linear, as they are regular and measurable. Considering the high traffic density in large-scale toll plazas, we research the model treating the traffic flow as continuous flow.
The model of entrance

According to the traffic flow theory [3], we have

\[ q_0 = v_0 \cdot k_0 \]  

(1)

When the traffic flow is at a point in the transition section, we have the following connection among traffic flow, traffic flow rate and traffic density:

\[ q_0 = v(x) \cdot k(x) \]  

(2)

Where:

- \( v(x) \) represents the traffic flow (km/h) and \( k(x) \) represents the traffic density.

Because the traffic flow \( q \) is constant, the traffic density and the width of the plaza have the following connection:

\[ k_0 \cdot w_n = k(x) \cdot w(x) \]  

(3)

According to the geometric knowledge, we can list the equation:

\[ \frac{y}{x} = \frac{w_f - w_n}{l_n} \]  

(4)

\[ W(x) = w_n + y \]  

(5)

\[ W_f = n \cdot d_2 \]  

(6)

\[ ln = n \cdot a_1 \]  

(7)

Where:

- \( w_f \) represents the total width of the toll station; \( n \) represents the number of the windows; \( a_1 \) is the coefficient of \( ln \).

So we can get \( v(x) \), then get \( T_n \)

\[ dT = \frac{dx}{v(x)} \]  

(8)

\[ T_n = \int_{0}^{l_s} dT \]  

(9)

We use Matlab to solve the equations above and get:

\[ T_n = \frac{nw_0 a_1}{v_0 (nd_z - w_n)} \ln \left( \frac{nd_z}{w_n} \right) \]  

(10)
The model for waiting

The meanings of the variables in the figure 3 are the same as figure 3. Because the traffic flow $q_0$ is fixed, we have

$$q_1 = q_0$$

According to the hydromechanics theory, $q_1$ is evenly distributed, so the traffic flow passing by every toll window is $q_1/n$. And based on Queuing Theory, every toll window obeys M/M/1 system. Thus, the waiting procedure is multiple M/M/1 system.

The average waiting time is:

$$T_w = \frac{1}{\mu - \lambda}$$

(11)

Where:
- $\lambda$ is the average arrival rate;
- $\mu$ is the average service rate.

Because $\lambda = q_1/n$, we can get:

$$T_w = \frac{n}{n\mu - q_1}$$

(12)

Considering the model needs to be fixed, we assume that $\lambda$ is always smaller than $\mu$ and $\lambda = q_1/n$. The constraint condition of $n$ is:

$$n > \frac{q_0}{\mu}$$

(13)
THE ARC MODEL (AM MODEL)

The model of entrance

According to the traffic flow theory, we have

\[ q_0 = v_0 \cdot k_0 \] (14)

When the traffic flow is at a point in the transition section, we have the following connection among traffic flow, traffic flow rate and traffic density:

\[ q_0 = v(x) \cdot k(x) \] (15)

Where:
\( V(x) \) represents the traffic flow (km/h) and \( k(x) \) represents the traffic density.

Because the traffic flow \( q \) is constant, the traffic density and the width of the plaza have the following connection:

\[ k_0 \cdot w_n = k(x) \cdot w(x) \] (16)

According to the geometric knowledge, we can list the equation:

\[
\begin{align*}
  y &= w_f \sin \theta - w_n \\
  \sin \theta &= \frac{w_n + y}{w_f} \\
  l_n - x &= w_f \cos \theta \\
  \cos \theta &= \frac{l_n - x}{w_f} \\
  w_f^2 &= w_n^2 + l_n^2 \\
  w(x) &= w_n + y
\end{align*}
\]

According to those equations, we can get \( T_n \)

\[
\begin{align*}
  dT &= \frac{dx}{v(x)} \\
  T_n &= \int_{0}^{l_n} dT
\end{align*}
\]

So we can get the final expression:

\[
T_n = \frac{nd_i \arctan\left( \frac{l_n}{\sqrt{(nd_i)^2 - l_n^2}} \right)}{v_0}
\]

CONCLUSION

According to the two models, we design the first, many performance curve model was superior to linear model, high safety, high transport efficiency, but the cost is slightly higher, but the current status of the improvement of the toll plaza better, hope that the relevant construction of the model to provide some help

We then designed a toll station model, how to control the various types of charging stations in the proportion of vehicles in different proportions, especially the
unmanned vehicle ETC only through the toll station, we finally through solving the optimization model, find the best proportion for toll station, how to set up all kinds of toll stations to provide recommendations

REFERENCES