Weak Signal Detection Based on Cascade Adaptive Stochastic Resonance

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Keywords: Cascade stochastic resonance, Weak signal detection, Particle swarm optimization.

Abstract. The gain of the output signal-to-noise ratio (SNR) is insufficient in traditional stochastic resonance (SR) method to detect weak signals. In order to obtaining a higher SNR gain, a kind of signal detection method of adaptive stochastic resonance that use cascade system is proposed. To achieve the optimal output, stochastic resonance parameters adjustment is transformed into the multi-parameter optimization of particle swarm optimization. The optimal result in weak signal detection can be reached by adjusting the parameters of two layers subsystem. The weak signal submerged in strong noise background can be extracted through this simple method that has a fast convergence speed. The threshold of the weak signal detection is lowered by this method, and the applicable scope of stochastic resonance can be enlarged.

Introduction

The earliest stochastic resonance phenomenon was proposed by Italian scholars Benzi and others to explain the cyclical alternation of the ancient glacial period and warming climate period in meteorology in 1981[1], which has been widely concerned in recent years. The phenomenon of stochastic resonance has been found in a variety of systems in many fields. Weak signal detection using the principle of stochastic resonance is a practical new technology [2]. It has a certain application in a lot of fields such as the biological signal, visual images, mechanical fault diagnosis and optical signal processing [3,4].

The stochastic resonance was used in weak signal detection from the 1990s. At first people study how to adjust the noise intensity of the system to produce stochastic resonance [5]. With the further research it was found that the effect of the stochastic resonance can also be produced by adjusting the parameters of the system when noise intensity is fixed [6]. What's more, the method of adaptive adjustment parameter provides the principle of stochastic resonance a better practicability [7].

The Principle of Stochastic Resonance

Stochastic resonance detection is a nonlinear method to enhance a weak signal with energy of the noise through a nonlinear bistable system. Part of the noise energy has been transformed into the energy of the weak signal. The characteristic of weak periodic signal is enhanced by the noise. A typical model of weak signal detection in stochastic resonance is bistable system. The bistable system influenced by the noise \( \epsilon(t) \) and periodic signal \( A \cos \omega_0 t \) can be described by the Langevin equation as follow.

\[
\dot{x} = ax - bx^3 + A \cos \omega_0 t + \epsilon(t)
\]

\( \epsilon(t) \) is a white noise with zero mean and unit variance. The potential function can be described as bistable system.

\[
U(x) = -\frac{1}{2}ax^2 + \frac{1}{4}bx^4
\]
As shown in Figure 1, the function $U(x)$ reaches the minimum value $U(x)_{\text{min}} = -\frac{a^2}{4b}$ at $x = \pm \sqrt{\frac{a}{b}}$, reaching the maximum value $U(x)_{\text{max}} = 0$ at $x=0$. The two minimum points of the function are separated by a barrier of a height of $\mathcal{V}U = \frac{a^2}{4b}$. Two minimum value is the double potential well for this function. When signal and noise achieve stochastic resonance in the system, the particle have an over damped motion in the double well.

When there is no input or noise, the particle is stable in a potential well of the double well. When there is no noise and the input signal $A$ is weak enough, the system makes periodic motion in a well. When the noise excitation is increasing, the particles will have the potential to jump from one well to the other. The transition frequency is $r_k = \frac{a}{\sqrt{2\pi}} \exp \left(-\frac{\mathcal{V}V}{D}\right)$. When the noise intensity is appropriate, input signal, noise and nonlinear bistable system achieve some matching, particle will switch back and forth in two wells. The switching frequency is identical with the input signal in the phenomenon of stochastic resonance.

**Cascade Stochastic Resonance Method**

A large number of research data show that the SNR can be enhanced by using the method of stochastic resonance. But if the input SNR is too low, the output signal is still submerged in noise. Therefore, the method of cascade stochastic resonance system can achieve a better signal detection effect. The output of the first layer is used as the input of the second layer, and the SNR gain can be further improved. Two layer cascade stochastic resonance system method is shown in Figure 2.

![Figure 1. Model of nonlinear bistable system.](image1)

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![Figure 2. Schematic diagram of two layer stochastic resonance system.](image2)

Stochastic resonance detection method is actually the realization the optimal matching of the input signal, noise interference and the system. The effect of stochastic resonance can be achieved by adding a signal or adjusting the system parameters. But noise intensity can only increase but not decrease in practical application. When the noise intensity has reached or exceeded the optimal matching need, the adding signal method has no effect. This makes the method of adjusting the parameters of the system has a better practical significance.

Because the frequency of weak signal to be measured is often unknown, how to adjust the parameters of the system is the key to solve the problem in the field of engineering. In the two layer stochastic resonance system, there are four unknown parameters $a_1, b_1, a_2, b_2$. Since the input of the second layer is the output of the first layer, the signal frequency of the two layer system is equal. In the case of no frequency offset. The first step, make $a_1 = a_2, b_1 = b_2$ to get the best $a_1, b_1$. The second step, make the value of the $a_1, b_1$ remains unchanged, search $a_2$ in the vicinity of the $a_1$.
search $b_2$ in the vicinity of the $b_1$. The optimization problem of the four parameters can be simplified to the optimization problem of the two parameters.

**Cascade Adaptive Stochastic Resonance Method Based on PSO Algorithm**

Particle swarm optimization (PSO) is an iterative algorithm that takes each individual in the target population as a particle in the search space. Each particle represents a possible solution to the optimization problem. First, the position and velocity of a group of particles are initialized. Second, each particle swarm updates its velocity and position according to the current optimal particle. Searching in the multi-dimensional space, the optimal solution could be found out through several iterations. In each iteration, the particle updates its velocity and position by tracking its own current optimal solution and the current optimal solution of the whole particle swarm, until the maximum number of iterations or the optimal solution is reached. According to the current individual extreme value and the global extremum, the formula of the velocity and the position of the $i$ particle at $t$ time is as follows.

$$v_i(t + 1) = wv_i(t) + c_1r_1(t)[P_{best_i}(t) - x_i(t)] + c_2r_2(t)[N_{best}(t) - x_i(t)]$$  (3)

$$x_i(t + 1) = x_i(t) + v_i(t + 1)$$  (4)

$W$ is the inertia weight, and its formula is as follows.

$$w(t) = w_{max} - \frac{(w_{max}-w_{min}) \times t}{T_{max}}$$  (5)

The SNR of the system output is used as the fitness function, and the optimal solution of the adaptive stochastic resonance method is obtained. The formula of objective optimization function can be obtained according to the definition of SNR.

$$F(a_1, b_1, a_2, b_2) = \text{SNR}_{out} = 10 \log_{10} \frac{S(f_0)}{N(f_0)}$$  (6)

The specific algorithm steps are as follows.

Step 1, PSO algorithm initialization. Set the population size, the maximum number of iterations, the dimensions and the search scope of each dimension.

Step 2, the evaluation of each particle. $a_1 = a_2, b_1 = b_2$. Calculation of the corresponding output SNR. The SNR of the first generation is used as a single particle of the local optimal value. The maximum value is used as the global optimal value.

Step 3, update the particle position and velocity according to the global optimal value. If the local optimal solution or the global optimal solution of a single particle is better than the last one, the velocity and position of the individual particles are updated, and the local and global optimal solutions are updated.

Step 4, get the optimal parameters. After the maximum number of iterations, according to the position of the final particle to get the best parameters. make the value of the $a_1, b_1$ remains unchanged, search $a_2$ in the vicinity of the $a_1$, search $b_2$ in the vicinity of the $b_1$.

Step 5, Output final signal, according to the optimal solution $a_1, b_1$ of the first iteration and the optimal solution $a_2, b_2$ of the second iteration.

**The Experiment Results and Discussion**

Adaptive cascade stochastic resonance based on PSO get the optimal output of cascaded bistable system by automatic combined adjustment parameters. Compared with the traditional method, it has a greater SNR gain. The simulation data is used to verify the effectiveness of the proposed method.

Input signal is a sine signal.

$$U(t) = A_0 \sin(2\pi f_0) + n(t)$$  (7)
\[ A_0 = 0.1, f_0 = 0.01, n(t) \] is the Gauss white noise with mean value of 0 and variance \( D = 2.5 \). The time domain waveform and frequency spectrum of the signal are shown in Figure 3. As can be seen in Figure 3, the intensity of the noise intensity is much greater than the periodic signal. Input signal is completely overwhelmed by the noise, it is difficult to distinguish the signal of the periodic component.

Figure 3. Time domain waveform and frequency spectrum of input signal.

The input SNR of the original signal is lower than -20dB. The target of signal detection can not be achieved through the first level stochastic resonance system. The signal is submerged by the noise. Figure 4 is the waveform of the input signal in the first layer stochastic resonance system.

Figure 4. Output signal of the first layer stochastic resonance system.

The input signal is processed by the cascade adaptive stochastic resonance method. Using PSO algorithm to find the optimal system parameters. Set the number of population is 30, the maximum number of iterations is 50, and the search range of the parameters of the SR system.

First, \( a_1 = [0.1:0.01:2], b_1 = [0.1:0.01:2] \), \( a_1 = a_2, b_1 = b_2 \). The optimal \( a_1, b_1 \) is obtained by searching. The optimal output parameters are \( a_1 = 1.15, b_1 = 1.22 \). Second, the optimal \( a_2, b_2 \) is searched according to the value of \( a_1, b_1 \). \( a_2 = [0.9 \times a_1:0.01:1.1 \times a_1], b_2 = [0.8 \times b_1:0.01:1.2 \times b_1] \). The optimal output parameters are \( a_2 = 1.13, b_2 = 1.28 \).
The time domain and frequency domain waveform of output signal of the second layer stochastic resonance system are shown in figure 5. The comparison shows that, after two layers of stochastic resonance subsystem, the SNR can be improved.

![Output signal of the second layer stochastic resonance system](image)

Figure 5. Output signal of the second layer stochastic resonance system.

Conclusion

A method for detecting weak signals of cascaded stochastic resonance is proposed in this paper. The output SNR of the cascaded adaptive stochastic resonance system is used as the fitness function. Two times of global search are used to adjust the parameters of the first and second layer stochastic resonance subsystems, and the weak signal is detected in the strong noise background. Compared with a layer of stochastic resonance in traditional system, the output SNR is significantly increased in series of two or more layers of stochastic resonance system. This method can reduce the threshold of weak signal detection and improve the range of application of stochastic resonance method.

Acknowledgement

This Project is from National Natural Science Foundation of China.(61263004)

References


