A Quotient Function Method for Selecting Adaptive Dynamic Load Identification Optimal Regularization Parameter

Yu-ping FENG and Wei GAO*
China, Northeast Petroleum University, School of Mathematics and Statistics, 163318
*Corresponding author

Keywords: Dynamic load, Morbidity, Adaptive, Quotient function method, Regularization method.

Abstract. This paper proposes an adaptive innovation for selecting regularization parameters with optimal features. Firstly, the least squares solution of the optimization problem is investigated by the Tikhonov regularization method to construct a quotient function with regularization parameters as independent variables. Secondly, for the quadratic programming theory, an adaptive innovation for selecting the optimal regularization parameters put forward by regularizing the relationship between the regularization and the function values. Finally, the proposed method is compared with traditional methods which are widely used through numerical simulation. The results show that the quotient function method can effectively overcome the ill-posed problem caused by the ill-posedness of the system matrix since it has certain anti-noise performance, and can theoretically obtains an approximate stable solution with better accuracy.

Introduction

Problems in vibration engineering are generally expressed in three parts: input, vibration system, and output. Dynamic load identification is to find the external excitation of the system through measured response data [1]. Therefore, it is first necessary to determine the relation between the observed data and the dynamic external excitation. This paper conducts a basic theoretical application research on the limitations of the application of dynamic load identification methods in practical engineering. Firstly, the dynamic load identification process on account of regularization method and the analysis of the ill-posedness of dynamic load identification problem are introduced [2]. Secondly, compared with the traditional method of regularization, a quotient function method on the theoretical basis of quadratic programming is presented [3]. Finally, through numerical simulation, the rationality and effectiveness of the quotient function method are illustrated.

Regularization Method

The most effective way to solve the ill-posed problem caused by the ill-posedness of the system matrix is the regularization method. Its fundamental idea is to use the bounded operator that approximates the original ill-posed problem to approximate the operator of the original problem, and the ill-posed problem is converted into a well-posed problem, so that a better solution can be obtained.

Establishment of Dynamic Load Identification System Model

Based on the first type of Fredholm integral equation in structural dynamics systems [4]

\[ z(x,t) = \int_{0}^{\infty} g(x,t,\sigma)y(x,\sigma)d\sigma. \] (1)

Where \( y(x,\sigma) \) denotes any load action function for \( x \); \( g(x,t) \) denotes a structural system operator function; \( z(x,t) \) denotes the structural response of the system structure at time \( t \). In practice, the integral equation can degenerate into...
\[ z(x,t) = \int_0^t g(t,\sigma)y(\sigma)d\sigma. \]  
For the convenience of calculation, the integral equation is the Duhamel integral

\[ z(t) = \int_0^t g(t,\sigma)y(\sigma)d\sigma. \]

When the Eq (3) is discretized in the time domain, the resulting dynamic load identification system is based on the model

\[ z = Gy. \]

**Regularization Parameter Selection Strategy**

The dynamic load method based on the Tikhonov regularization method is mainly divided into three steps: the establishment of the model, the regularization of the system model, and the selection of the optimal regularization parameters. The selection of the optimal regularization parameters is the core of the load identification regularization method [5]. The traditional regularization parameters are selected by Engl criterion [6], Morozov deviation principle [7], L-curve [8] criterion and generalized cross-checking criterion (GCV) [9]. Since the traditional method still has some drawbacks, we propose quotient function method. Different regularization methods define these folds. The criteria for the method of determining the compromise are different [10].

When the matrix \( G^T G + \gamma^2 E \) is non-pathological, that is, the minimum singular value is strictly greater than 0 in the SVD decomposition [11]. The formula is

\[
\begin{align*}
\min_{f \in \mathbb{R}} & \left\| G \beta - f \right\|_2^2 + \gamma^2 \left\| \beta \right\|_2^2. \\
\text{Where } & \gamma > 0 \text{ is the regularization parameter and } \left\| \beta \right\|_2 \text{ is the Euclidean norm. As the parameter } \gamma \text{ gradually decreases, there must be a certain parameter value that makes the best compromise between the "norm" between the residual norm } \left\| G \beta - f \right\|_2 \text{ and the regularized norm } \left\| \beta \right\|_2. \\
\text{High-precision least squares solution } & [2] \text{ satisfies}
\end{align*}
\]

\[ G^T G \hat{\beta}_\gamma = \gamma^2 \hat{\beta}_\gamma. \]

quadratic programming theory to solve the optimization problem. The quadratic programming solution for the optimization problem in equation (5). The Euclidean norm on both sides of the above formula has

\[
\left\| G^T f - G^T G \hat{\beta}_\gamma \right\|_2 = \gamma^2 \left\| \hat{\beta}_\gamma \right\|_2.
\]

Finished up

\[
\gamma^2 \left\| \hat{\beta}_\gamma \right\|_2 = 1.
\]

As the value of parameter \( \gamma \) decreases gradually, the ill-conditionedness of matrix \( G^T G + \gamma^2 E \) decreases. There is
Matrix morbidity is enhanced while the regularization parameter $\gamma$ decreases continuous, there is

$$\left\| G^T f - G^T G \hat{\beta} \right\|_2 \approx 1.$$  \hfill (9)

The quotient function is defined as

$$\Phi(\chi) = \left\| G^T f - G^T G \hat{\beta} \right\|_2.$$  \hfill (11)

Then the optimum regularization parameter $\gamma_o > 0$ satisfies the following condition

(i) When $\gamma \geq \gamma_o$, there is $\varphi(\gamma) = 1$; (ii) When $\gamma < \gamma_o$, there is $\varphi(\gamma) \neq 1$.

It can be considered that the parameter achieves the optimal compromise between the “size” of the regularization norm and the residual norm.

**Numerical Simulation**

Based on the finite element model of the cantilever beam structure [2], the system model in Eq (4) is established, and the external dynamic load is divided into sinusoidal load $f(t) = 40\sin(40\pi t)$, wherein the load acts on the 8th node with a duration of 0.3s. The sampling frequency is 1000 Hz. The half-sine wave of load $f(t) = 40\sin(200\pi t)$ in the time interval of 0~0.005s is used to simulate the impact load acting on the 8th node. The whole time history is 0.03s, and the sampling frequency is 5000Hz during the load identification process. For the impact load and sinusoidal load, the Tikhonov regularization method is used to solve the dynamic load identification system model in equation (4) by different methods at different noise levels, at the same time, the results are compared and analysed.

**Impact Load Identification**

The impact load is identified at the 1%, 3%, and 5% noise levels by the acceleration response at the 6th node. The threshold is taken as $e = 10^{-4}$. It can be seen from the graph that the impact load identified by the quotient function method can effectively reflect the time history of the real load. The impact load identified by the L-curve method has the same good fitting property as the real load. The relative error of the GCV method load recognition result decreases as the noise level increases. In addition, the relative error of the identification load obtained by the L-curve method increases with the increase of the noise level.
Figure 1. SISO system shock load identification quotient function curve.

Figure 2. SISO system impact load identification GCV function curve.

Figure 3. SISO system shock load identification L-curve.

Figure 4. SISO system impact load identification result.

Table 1. SISO system impact load identification result.

<table>
<thead>
<tr>
<th>Real amplitude /N</th>
<th>Noise level %</th>
<th>Parameter selection method</th>
<th>Identification/N</th>
<th>Identification/N</th>
<th>Identification/N</th>
<th>Identification/N</th>
<th>RE/%</th>
<th>RE/%</th>
<th>RE/%</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>QFM</td>
<td>39.52</td>
<td>40.72</td>
<td>41.87</td>
<td>41.87</td>
<td>2.00</td>
<td>2.00</td>
<td>4.89</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>QFM</td>
<td>39.52</td>
<td>40.72</td>
<td>41.87</td>
<td>41.87</td>
<td>2.00</td>
<td>2.00</td>
<td>4.89</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>QFM</td>
<td>39.52</td>
<td>40.72</td>
<td>41.87</td>
<td>41.87</td>
<td>2.00</td>
<td>2.00</td>
<td>4.89</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>GCV</td>
<td>38.67</td>
<td>39.39</td>
<td>39.61</td>
<td>39.61</td>
<td>1.33</td>
<td>1.33</td>
<td>0.76</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>GCV</td>
<td>38.67</td>
<td>39.39</td>
<td>39.61</td>
<td>39.61</td>
<td>1.33</td>
<td>1.33</td>
<td>0.76</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>L-curve</td>
<td>40.19</td>
<td>41.62</td>
<td>43.04</td>
<td>43.04</td>
<td>4.25</td>
<td>4.25</td>
<td>7.81</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>L-curve</td>
<td>40.19</td>
<td>41.62</td>
<td>43.04</td>
<td>43.04</td>
<td>4.25</td>
<td>4.25</td>
<td>7.81</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>L-curve</td>
<td>40.19</td>
<td>41.62</td>
<td>43.04</td>
<td>43.04</td>
<td>4.25</td>
<td>4.25</td>
<td>7.81</td>
</tr>
</tbody>
</table>
Impact Load Identification

The acceleration response of the 7th node is shown in the 1%, 3%, and 5% noise level as shown in the quotient function curve, the GCV function curve and the L curve. At 1% noise level, QFM, GCV function curve and L-curve are shown in Figure 5-7. The three regularized parameter selection methods correspond to the identification load shown in Figure 8. The result in Table 2 is the load identification results at different noise levels. It can be seen that the three selection of regularization parameter can effectively obtain the optimal regularization parameter at 1% noise level. Moreover, a load recognition result similar to that at 1% noise level can be obtained when the noise level is 3%, and the load recognition result is relatively poor when the noise level is at a higher level of 5%.

![Figure 5. SISO system shock load identification quotient function curve.](image1)

![Figure 6. SISO system impact load identification GCV function curve.](image2)

![Figure 7. SISO system shock load identification L-curve.](image3)

![Figure 8. SISO system impact load identification result.](image4)

<table>
<thead>
<tr>
<th>Noise level%</th>
<th>α_{opt}</th>
<th>RE/%</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>QFM</td>
<td>GCV</td>
</tr>
<tr>
<td>1</td>
<td>0.0009</td>
<td>0.2110</td>
</tr>
<tr>
<td>3</td>
<td>0.1032</td>
<td>0.2110</td>
</tr>
<tr>
<td>5</td>
<td>0.0650</td>
<td>0.2110</td>
</tr>
</tbody>
</table>

In summary, the QFM can effectively determine the optimal regularization parameter value for different types of dynamic loads under different noise levels, and it has good applicability for different forms of load and the lower level of measurement noise.

Summary

In this paper, in view of the dynamic load identification problem, the dynamic load identification method based on regularization method is studied in the time domain. A quotient function method for optimal regularization parameter selection is proposed on account of the quadratic programming theory. Compared with the commonly common GCV method, it is easier and more efficient to obtain the optimal regularization parameters. In other words, it has better stability. Therefore, the QFM can
effectively overcome the limitations of the current common methods. The adaptive QFM is a preferable choice when using the Tikhonov regularization method for load recognition. This method provides a basis for engineering safety and reliability.

References


