Fast SLAM Algorithm Based on Distributed Target Bayesian Detection in Compound Gaussian Noise

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Keywords: Simultaneous localization and mapping, Adaptive detection, Compound Gaussian noise, Bayesian detection.

Abstract. Fast detecting the distributed targets is appreciated in the SLAM algorithm. In this paper, Bayesian Rao detection and Wald detection methods are proposed for the distributed targets detecting in compound Gaussian noise, which can be applied to the rapid generation and iteration of SLAM (simultaneous localization and mapping) algorithm. Firstly, it is assumed that the covariance matrix of compound Gaussian noise obeys the inverse Wishart prior distribution, on this basis, the maximum posterior estimate of the covariance matrix is used to propose Bayesian Rao detection and Wald detection, then the affects by the prior distribution degree of freedom and the number of distance units spanned on the performance of the proposed detector are analyzed, Finally, Simulation and experiment results verify the effectiveness of the proposed method. Applying the detection method to the SLAM algorithm, there will be a significant improvement in detecting the distributed target.

Introduction

In recent years, with the rapid development of automatic driving and intelligent robot applications, the SLAM algorithm has become a research hotspot. Benefit from the high-distance resolution radar, adaptive detection of distributed targets has received much attention in SLAM algorithm. [1-6]

The main principle of SLAM algorithm based on Google Cartographer is to eliminate the cumulative error generated by the composition process through closed-loop detection. The basic unit for closed-loop detection is the sub-map. A sub-map is made up of a certain number of laser scans. When a laser scan is inserted into its corresponding sub-map, its optimal position in the sub-map is estimated based on the existing laser scan and other sensor data of the sub-map. However, as more and more submaps are created, the error accumulation between submaps will increase. It is necessary to properly optimize the pose of these sub-maps through closed-loop detection to eliminate these cumulative errors, which translates the problem into a pose optimization problem. The main point of the SLAM algorithm is to intergrate with multi-sensor data of local sub-map and the scan match strategy for closed-loop detection. However, when a new laser scan is a temporary obstacle, then the closed-loop search strategy will fail. In this case, a fast distributed target detection method is needed to avoid such problems.

Bayesian Rao detection and Wald detection methods for detecting the distributed targets are proposed in the paper, the Maximum A- Posterior (MAP) [7,8]estimate of the covariance matrix is used to derive Bayesian Rao detection and Wald detector of the distributed target in the compound Gaussian noise, which can be well applied to the SLAM algorithm.

Detection Model

Assuming that the receiving antenna array consists of a matrix of uniform elements, the target detection problem in compound Gaussian noise can be expressed as the following binary detection:

\[
\begin{align*}
H_0 : z_h &= a_h \cdot v + n_h; \quad h = 1, \cdots, H \\
H_1 : z_h &= n_h; \quad h = 1, \cdots, H
\end{align*}
\]  
(1)
$Z_h$ is the $N$ dimensional distance unit data to be measured, $\alpha_h$ is the target range, $v$ is the target-oriented vector of $N \times 1; R_b$ is the compound Gaussian noise component, $n_h = \sqrt{\tau_h} g_n$, $h = 1, \cdots, H$, $g_n$ is a complex-valued Gaussian vector with mean zero covariance matrix $R$, $\tau_h$ is the texture component obeying the Gamma distribution; $H$ is the number of distance units to be measured.

As the same, the training samples also obey the compound Gaussian distribution of the Gamma distribution. The training sample is $Z = [z_1, \cdots, z_K]$, While $K$ is the number of training samples.

Assuming $Z_k$ be a complex-valued Gaussian distribution with a mean covariance of $\tau_k R$ under $\tau_k$ and $R$, denoted as $Z_k | \tau_k, R \sim CN(0, \tau_k R)$, then the conditional probability density of $Z_k$ is:

$$f(z_k | \tau_k, R) = \frac{1}{\pi^{N/2} \tau_k^{N/2} |R|} \exp \left\{ - \frac{z_k^H R^{-1} z_k}{\tau_k} \right\}$$

(2)

While $| \cdot |$ represents the determinant of the matrix.

The Bayesian detector generally selects the complex value inverse Wishart distribution as the prior distribution of the covariance, because a large number of simulation results show that the prior distribution can reflect the environmental information, and it is the conjugate prior distribution of the covariance of the multidimensional normal distribution. Assuming $R$ obeys the complex-valued inverse Wishart distribution $R \sim CW((N), v)$ with degree of freedom $v$, and then the probability density function is:

$$f(R) = \frac{\Gamma(v-N)}{\Gamma_N(v) |R|^{v-N} \exp \left\{ - (v-N)R^{-1} \bar{R} \right\} }$$

(3)

where $\exp(\cdot)$ represents the exponential power of the trace of the matrix and $\Gamma_N(v)$ is the product of the N Eulerian Gamma functions:

$$\Gamma_N(v) = \pi^{N(N-1)/2} \prod_{n=1}^{N} \Gamma(v-n+1)$$

(4)

**Bayesian Detection of Distributed Targets**

Assuming $\tau_k$ be independent of the Gamma distribution with parameters $\alpha_k$ and $\beta_k$. The probability density function can be expressed as:

$$f(\tau_k) = \frac{\beta_k^{\alpha_k}}{\Gamma(q_k) \tau_k^{q_k+1}} \exp(-\frac{\beta_k}{\tau_k}), \quad \tau_k \geq 0, \alpha_k \geq 0, \beta_k \geq 0$$

(5)

$$\tau = [\tau_1, \cdots, \tau_K]$$, there is:

$$f(\tau) = \prod_{k=1}^{K} \frac{\beta_k^{\alpha_k}}{\Gamma(q_k) \tau_k^{q_k+1}} \exp(-\frac{\beta_k}{\tau_k})$$

(6)

In order to obtain the posterior distribution $f(\tau|R|Z)$, the joint posterior distribution of $\tau$ and $R$ must be obtained first. From (2), (3) and (6), ignoring constants, there is:

$$f(\tau, R|Z) \propto f(Z|\tau, R) f(\tau) f(R)$$

$$\propto \frac{1}{|R|^{v-N} \exp \left\{ - (v-N)R^{-1} \bar{R} \right\} } \prod_{k=1}^{K} \frac{1}{\tau_k^{q_k+1}} \exp\left(-\frac{z_k^H R^{-1} z_k}{\tau_k} + \frac{\beta_k}{\tau_k}\right)$$

(7)
While \( \propto \) represents a positive proportional relationship. Calculation \( f(\tau, R|Z) \) for the integral of \( \tau \), there is:

\[
 f(R|Z) \propto \int f(\tau, R|Z) d\tau
\]

\[
 f(R|Z) \propto \frac{\text{etr}\left\{-(v-N)R^{-1}R\right\}}{\prod_{k=1}^{K} \tau_{k}^{N+N_k}} \exp\left\{-z_{k}^{H}R^{-1}z_{k} + \beta_{k}\right\} d\tau_k
\]

\[
 f(R|Z) \propto \frac{\text{etr}\left\{-(v-N)R^{-1}R\right\}}{\prod_{k=1}^{K} \tau_{k}^{N+N_k}} \left(z_{k}^{H}R^{-1}z_{k} + \beta_{k}\right)^{(N+N_k)}
\]

(8)

\[ \hat{R}_{\text{MAP}} = \arg \max_{R} f(R|Z) \]

(9)

The following matrix equation can be obtained by finding the derivative of \( R \) on the logarithm of \( f(R|Z) \) and deriving the derivative to zero to obtain the solution of equation (8):

\[
 (v+N+K)R = (v-N)\hat{R} + \sum_{i=1}^{K} (N+q_i)z_i^{H}z_i^{-1}R^{-1}z_i + \beta_i
\]

(10)

Get \( \hat{R}_{\text{MAP}} \) from (10).

After obtaining the MAP estimate of the covariance matrix in the compound Gaussian noise, we derive the Rao detector and the Wald detector of the distributed target proposed in the following.

Bayesian Rao Detection of Distributed Targets

For convenience of description, the parameters in the Rao detector are expressed as follows:

\[
 \alpha = \alpha_{h} + j \alpha_{h}, \ h=1,\ldots,H \quad \theta_{i} = \left[\alpha_{R,h}, \alpha_{I,h}, \alpha_{R,h}, \alpha_{I,h}\right]^{T}, \quad \theta = \left[\tau_{1},\ldots,\tau_{H}\right]^{T}, \quad \theta' = \left[\theta^{T}, \theta'^{T}\right]^{T}.
\]

Thus, the Bayesian Rao detection problem can be expressed as follows:

\[
 \frac{\partial \ln f(z_{1},\ldots,z_{H}|\alpha, \tau, R)}{\partial \theta_{i}} \left|^{\theta_{i} = \hat{\theta}_{i}, \theta = \hat{\theta}}\right. > \xi
\]

(11)

While \( \partial / \partial \theta_{i} \) represents the gradient operator of \( \theta_{i} \); \( \xi \) is the detection threshold; \( f(z_{1},\ldots,z_{H}|\theta, R) \) is the conditional probability density of \( z_{1},\ldots,z_{H} \) under the assumption of \( H_{1} \):

\[
 f(z_{1},\ldots,z_{H}|\theta, R) = \prod_{i=1}^{H} \frac{1}{\sqrt{\tau_{i}}} \exp\left\{-\frac{(z_{i}-\alpha_{i,v})^{H}R^{-1}(z_{i}-\alpha_{i,v})}{\tau_{i}}\right\}
\]

(12)

\[
 J(\theta) = J(\theta', \theta') \]

is the Fisher information matrix, and :

\[
 J^{-1}(\theta) = J_{a,a}'(\theta) - J_{a,a}(\theta)J_{a,a}'(\theta)J_{a,a}(\theta)\]

(13)

\[
 J_{a,a}(\theta) = 2\text{diag} \left[ \frac{v_{H}^{H}R^{-1}v}{\tau_{i}}, \frac{v_{H}^{H}R^{-1}v}{\tau_{i}}, \ldots, \frac{v_{H}^{H}R^{-1}v}{\tau_{i}}, \frac{v_{H}^{H}R^{-1}v}{\tau_{i}} \right], \text{diag}(\cdot) \]

While \( J_{a,a}(\theta) = 0 \) and \( 0 \) denote zero matrix.

From (12), we can get:

\[
 \left[ J^{-1}(\theta) \right]_{a,a} = J_{a,a}^{-1}(\theta)
\]

(14)

The maximum likelihood estimation problem for \( \tau \) under \( H_{1} \) can be expressed as:
\( \hat{\theta}_{o,0} = \arg \max f(z_1, \ldots, z_H | \tau, \hat{R}_{MAP}, H_0) \) \hfill (15)

\[
f(z_1, \ldots, z_H | \tau, \hat{R}_{MAP}, H_0) = \prod_{h=1}^{H} \frac{1}{\pi^{N_h} \tau_h^{N_h}} \exp \left( - \frac{z_h^T \hat{R}_{MAP}^{-1} z_h}{\tau_h} \right)
\] \hfill (16)

Take the logarithm of (14):

\[
F(\hat{\theta}) = \ln(f(z_1, \ldots, z_H | \tau, \hat{R}_{MAP}, H_0)) = \text{Re} \left[ \sum_{h=1}^{H} \frac{v^T \hat{R}_{MAP}^{-1} z_h}{\tau_h} \right] + \text{Im} \left[ \sum_{h=1}^{H} \frac{v^T \hat{R}_{MAP}^{-1} \alpha_H z_h}{\tau_h} \right]
\] \hfill (17)

For (15) deriving about \( \tau_h \) and zeroing the derivative, there is:

\[
\frac{\partial F(\hat{\theta})}{\partial \tau_h} = - \frac{N}{\tau_h} + \frac{z_h^T \hat{R}_{MAP}^{-1} z_h}{\tau_h^2} = 0
\] \hfill (18)

Calculate the maximum likelihood estimate of \( \tau \) under \( H_0 \) from (16):

\[
\hat{\tau}_h = \frac{z_h^T \hat{R}_{MAP}^{-1} z_h}{N}, \quad \text{there is:}
\]

\[
\hat{\theta}_{o,\tau} = \frac{1}{N} \left[ z_1^T \hat{R}_{MAP}^{-1} z_1, \ldots, z_H^T \hat{R}_{MAP}^{-1} z_H \right]^T
\] \hfill (19)

\[
\frac{\partial \ln f(z_1, \ldots, z_H | \theta, \tau, R)}{\partial \alpha_{x,n}} = 2 \text{Re} \left[ \frac{v^T \hat{R}_{MAP}^{-1} (z_n - \alpha_n v)}{\tau_h} \right]
\] \hfill (20)

\[
\frac{\partial \ln f(z_1, \ldots, z_H | \theta, \tau, R)}{\partial \alpha_{x,n}} = 2 \text{Im} \left[ \frac{v^T \hat{R}_{MAP}^{-1} (z_n - \alpha_n v)}{\tau_h} \right]
\] \hfill (21)

Thus there is

\[
\frac{\partial \ln f(z_1, \ldots, z_H | \theta, \tau, R)}{\partial \hat{\theta}} = \frac{\partial \theta}{\partial \hat{\theta}} = 2 \left[ \text{Re} \left[ \frac{v^T \hat{R}_{MAP}^{-1} (z_n - \alpha_n v)}{\tau_h} \right], \text{Im} \left[ \frac{v^T \hat{R}_{MAP}^{-1} (z_n - \alpha_n v)}{\tau_h} \right], \ldots, \text{Re} \left[ \frac{v^T \hat{R}_{MAP}^{-1} (z_h - \alpha_H v)}{\tau_h} \right], \text{Im} \left[ \frac{v^T \hat{R}_{MAP}^{-1} (z_h - \alpha_H v)}{\tau_h} \right] \right]^T
\] \hfill (22)

Substituting (14), (22), \( \hat{R}_{MAP} \), and \( \hat{\theta}_o \) into (11), after a simple mathematical calculation, there is:

\[
\sum_{h=1}^{H} \left( \frac{v^T \hat{R}_{MAP}^{-1} z_h}{\tau_h} \right)^2 \xrightarrow{H_1} \mathcal{J}_i
\] \hfill (23)

**Bayesian Wald Detection of Distributed Targets**

The Wald detection criteria can be expressed as:

\[
\hat{\theta}_i, \left[ J^{-1}(\hat{\theta}_i) \right]^{-1}_{\theta, \phi} \xrightarrow{H_1} \mathcal{J}_i > \xi_w
\] \hfill (24)

While, \( \hat{\theta}_i = \left[ \hat{\theta}_i, \hat{\theta}_i \right]^T \) represents the estimated value of \( H_1 \) at \( \theta \):

\[
\hat{\theta}_i = \arg \max_{\phi} f(z_1, \ldots, z_H | \theta, \hat{R}_{MAP}, H_1)
\] \hfill (25)

While:
\[ f(z_1, \ldots, z_H|\theta, \hat{R}_{MAP}, H_1) = \prod_{h=1}^{H} \frac{1}{\pi^H \Sigma_h} \exp \left( - \frac{(z_h - \alpha \nu)^T \hat{R}_{MAP}^{-1} (z_h - \alpha \nu)}{\tau_h} \right) \]  

(26)

In the same way, for (26) to maximize \( \theta \), the maximum likelihood estimate \( \hat{\alpha}_h \) is obtained, so that:

\[ \hat{\alpha}_h = \frac{\nu^T \hat{R}_{MAP}^{-1} z_h}{\nu^T \hat{R}_{MAP} v} \]

(27)

Substituting (14), (27), and (28) into (24), after a simple calculation, the Wald detector can be obtained

\[ \sum_{h=1}^{H} \nu^T \hat{R}_{MAP}^{-1} v \left( z_h^T \hat{R}_{MAP}^{-1} z_h - \frac{\nu^T \hat{R}_{MAP}^{-1} z_h}{\nu^T \hat{R}_{MAP} v} \right) \begin{cases} > \gamma & \text{for } H_1 \\ < \gamma & \text{for } H_0 \end{cases} \]

(29)

**Fast SLAM Algorithm Simulation Analysis**

In this section, simulation experiments and performance analysis of the modified Cartographer algorithm for SLAM will be carried out. The Monte Carlo method was used for simulation. Based on 100 independent experiments, the false alarm probability was taken as \( 10^{-3} \). Other simulation parameters are set to: \( N = 8 \), \( H = 2 \), \( q_1 = \cdots = q_K = 10 \), \( v = 10 \), \( \beta_1 = \cdots = \beta_K = q_K - 1 \), \( \hat{R}(i, j) = 0.9^{|i-j|} \), \( v = 1/\sqrt{N} [1, \ldots, 1]^T \), \( \alpha_1 = \cdots = \alpha_H \), \( \text{SNR} = H \| \theta \| E[\nu^T M^T \nu] \).

Fig. 1 shows the variation of the detection probability with the signal-to-noise ratio (SNR) of the traditional non-Bayesian detection method and the Bayesian detection method proposed in the condition of the training samples as \( K = 8 \). It can be seen from Fig. 1 that the detection probability of the traditional non-Bayesian Rao detection method is low, and the detection probability is only 0.38 when the SNR is 15 dB. In contrast, the detection probability of the Bayesian KA-Rao (KA means Knowledge-aided) method is improved, which can reach to 0.99 when the SNR is 15 dB. Similarly, the traditional non-Bayesian Wald detection method has a low detection probability, and the detection probability is only 0.53 when the SNR is 15 dB. In contrast, the detection probability of the Bayesian KA-Wald method is greatly improved, and the detection probability is 0.99 when SNR is 15 dB. It shows that in the case of limited training samples in compound Gaussian noise, the proposed Bayesian detection methods can make full use of the prior distribution, which can greatly improve the detection performance.

Fig. 2 shows the variation of the detection probability with the signal-to-noise ratio of the traditional non-Bayesian detection method and the Bayesian detection method when the training samples as \( K = 24 \). As can be seen from Fig. 1 and Fig. 2, as the number of training samples increase, the performance of the conventional detector gradually increases. When the number of samples is greater than 2 times the degree of system freedom, the performance of the conventional detector is close to that of the Bayesian detector.
Fig. 3 shows the effect of the prior distribution degree of freedom $v$ of the covariance matrix on the detection performance. As can be seen from Fig. 3, the detection performance increases as $v$ increases, because the degree of freedom is large, and the covariance matrix is closer to $R$. Fig. 4 is a graph showing the variation of the detection probability with the signal to interference and noise ratio under the number of different distance units to be measured, and Fig. 5 is a partial enlarged view of Fig. 4. As can be seen from Fig. 4 and Fig. 5, as the number of target spanning distance units increase, the detection performance of the Bayesian Rao detection method slightly decreases. Similarly, as the number of target spanning distance units increases, the detection performance of the Bayesian Wald detection method also decreases to some extent.
Summary
Fast SLAM algorithm based on distributed target Bayesian detection in compound Gaussian noise is proposed in this paper. Using the maximum posteriori estimate of the covariance matrix, the maximum likelihood estimation of $\tau$ under $H_0$ and the maximum likelihood estimation of $\theta$ under $H_1$ are derived respectively, in the condition of that, Bayesian Rao detection and Wald detection methods for distributed targets are proposed. Finally, the simulation experiment is carried out to verify the effectiveness of the proposed detector. Apply this algorithm to modify the Cartographer of SLAM algorithm can avoid the temporary obstacles in the SLAM process and has rapid discharge and convergence effect.

Acknowledgement
This research was financially supported by the Science and Technology Plan Project of Guangzhou of China (201705030010, 201707010416), Science and Technology Plan Project of Guangdong province of China (2017A010101022)

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