Propagation Behavior Studies on Elliptically Polarized Wave within Chiral Negative Refraction Mediums

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Abstract. According to elementary transmission theories of electromagnetic wave and constitutive relation of mediums, the propagation behaviors are analyzed theoretically and numerically in detail when the elliptically polarized wave is incident upon the surface of a chiral negative refraction medium. The change rules of flux density of reflection and refraction with incident angle are given, which have distinct difference with conventional medium. It is also found that the left and right circularly polarized electromagnetic wave components have different propagation behaviors with the chiral negative refraction medium.

Introduction

Negative refraction materials have many peculiar characteristics[1,2], such as negative refraction effect, abnormal Doppler effect, abnormal Cherenkov radiation and perfect lens effect, which have received extensive attention in the fields of material science, optics, applied electromagnetics and solid physics in recent years. At present, scientists use artificial medium to achieve negative refraction dominantly with three methods: double negative-material, transmission line and photonic crystal. The propagation behaviors of electromagnetic wave within the chiral negative refraction medium have distinct differences from that within the conventional medium, which cannot be analyzed by classical electromagnetic theories and formulas. In this paper, the relevant formulas of electromagnetic wave within chiral negative refraction medium are given in section 2 firstly; then in section 3, based on propagation characteristics of monochromatic electromagnetic wave, constitutive relations and boundary conditions, we study the rules of reflection and refraction of elliptically polarized wave upon the chiral negative refraction medium’s surface in detail; and explore the differences with conventional medium; furthermore, according to the above theoretical results, the numerical calculation on reflectance and transmittance for three different chiral parameters are given, which are analyzed and discussed in section 4. Finally, section 5 is a brief summary.

Basic Theories of Monochromatic Wave in the Chiral Negative Refraction Medium

The chirality refers that a structure cannot be identical to its corresponding mirror by any spatial symmetrical operations like translation and rotation, which dominantly manifests as optical rotation and circular dichroism. Through the constitutive relations, the effects of mediums on the electromagnetic wave are described. The constitutive relations of monochromatic wave within isotropic chiral negative reflection mediums[3] are: \( \mathbf{D} = \varepsilon \mathbf{E} + i \chi \mu \varepsilon \mathbf{H} \), \( \mathbf{B} = \mu \mathbf{H} - i \chi \mu \varepsilon \mathbf{E} \), where \( \chi \) is the chiral parameter, \( \varepsilon ( \mu ) \) are the permittivity(permeability) in chiral medium.

For chiral medium, there exist two intrinsic waves, which are right-handed and left-handed circularly polarized waves. Intrinsic waves satisfy the Helmholtz equations: \( \nabla^2 \mathbf{E}_z + k^2 \varepsilon \mathbf{E}_z = 0 \), \( \nabla^2 \mathbf{B}_z + k^2 \mu \mathbf{B}_z = 0 \). Note that \( \mathbf{E}_z \) ( \( \mathbf{B}_z \) ) represents the electric (magnetic) field components of right-handed or left-handed circularly polarized wave respectively. Here the subscript + (-) denotes right-handed (left-handed) and the following are the same.
The wavenumber of the right-handed (left-handed) circularly polarized wave is: \( k_+ = n_+ k_0 \). Here is the effective refractive index of right-handed (left-handed) circularly polarized wave. \( n_+ = \sqrt{\mu_0/\mu_r \epsilon_0} \) is the refractive index in chiral medium, and \( k_0 = \omega/\sqrt{\mu_0 \epsilon_0} \) is the wavenumber of electromagnetic wave in vacuum.

When \( \chi > n_+ \), so \( n_- < 0 \), the negative refraction of the left-handed circularly polarized wave in chiral medium is achieved. For the left-handed circularly polarized wave, \( \vec{E}, \vec{H}, \vec{k} \) constitute a left-handed spiral relationship, that is, energy flux density vector \( \vec{S} \) is opposite to the wave-vector \( \vec{k} \). So the chiral medium which satisfies the condition \( \chi > n_+ \) is the so-called chiral negative refraction medium.

**Reflection and Refraction of Elliptically Polarized Wave at the Surface of Chiral Negative Refraction Medium**

The elliptically polarized wave can be written as a superposition of left-handed and right-handed circularly polarized waves. Therefore, in the following the case of circularly polarized wave should be discussed firstly [4].

![Figure 1. Electromagnetic wave incident on the surface between conventional and chiral negative refraction mediums.](image)

As shown in Fig. 1, when circularly polarized wave is incident onto the surface of negative refraction medium, the left-handed circularly polarized component is negatively refracted. According to electromagnetic parameters given in Fig. 1, the amplitudes of incident circularly polarized wave are expressed as:

\[
\begin{align*}
\vec{E}_{i\pm} &= (\mp i \hat{y} + \cos \theta \hat{x} - \sin \theta \hat{z}) \exp \left[ i(k_x \hat{x} + k_z \hat{z}) \right] \\
\vec{H}_{i\pm} &= \frac{k_c}{\omega \mu_1} \times \vec{E}_{i\pm} = \pm i \sqrt{\frac{\epsilon_0}{\mu_1}} \vec{E}_{i\pm} = \pm i \frac{1}{\eta_i} \vec{E}_{i\pm} 
\end{align*}
\]

Propagation behaviors of reflected wave in conventional medium can be expressed as a superposition of TE and TM waves. According to its propagation in the medium 1, supposing \( a_m \) \((b_m)\) as the amplitude of TE (TM) wave, the complex expressions for the electromagnetic field of reflected wave and refracted wave are written as Eq. 2 and Eq. 3 separately:

\[
\begin{align*}
\vec{E}_{r\pm} &= (a_m \hat{y} - b_m \cos \theta \hat{x} - b_m \sin \theta \hat{z}) \exp \left[ i(k_x \hat{x} - k_z \hat{z}) \right] \\
\vec{H}_{r\pm} &= \frac{k_c}{\omega \mu_1} \times \vec{E}_{r\pm} = -\frac{1}{\eta_i} \left( b_m \hat{y} + a_m \cos \theta \hat{x} + a_m \sin \theta \hat{z} \right) \exp \left[ i(k_x \hat{x} - k_z \hat{z}) \right] 
\end{align*}
\]
\[
\begin{aligned}
\begin{cases}
\tilde{E}_{\pm} = A_{m\pm} \left( \mp i \hat{y} + \cos \theta_{\pm} \hat{x} - \sin \theta_{\pm} \hat{z} \right) \exp \left[ i \left( k_{m\pm} \hat{x} + k_{m\pm} \hat{z} \right) \right] \\
\tilde{H}_{i\pm} = \mp i \frac{1}{\eta_{\pm}} \tilde{E}_{i\pm}
\end{cases}
\]
\end{aligned}
\]  

(3)

Here \( m=1,2 \) refer to the incidence of right-handed and left-handed circularly polarized wave respectively, \( \theta_{\pm} \) denotes the refraction angle of the right-handed (left-handed) circularly polarized component. By the energy flux density formula \( \mathcal{S} = \mathbf{E} \times \mathbf{H}^*/2 \), the expressions for energy flux of reflectance \( (R_m) \) and transmittance \( (T_m) \) are

\[
R_m = \frac{|A_m|^2 + |B_m|^2}{2}, T_m = \frac{\eta_1\left[A_m \Re(\cos \theta_m) + |A_m|^2 \Re(\cos \theta_m) \right]}{\eta_1 \cos \theta}.
\]

(4)

Arbitrary elliptically polarized wave can be transformed into a normal elliptically polarized wave by rotation operation of coordinate system, so for simplification, the case of the incidence of normal elliptically polarized wave is discussed. If the electric field component along y-axis is \( \hat{y} \) and the electric field component parallel to \( y=0 \) plane is \( E_z \), by unit amplitude of circularly polarized wave [6], the complex expressions for electric field of incident normal right-handed (left-handed) elliptically polarized wave can be written as

\[
\begin{aligned}
\tilde{E}_{x\pm} &= \frac{1}{2} \left( E_{1\pm} \mp E_{2\pm} \right) \tilde{E}_{1}\mp \frac{1}{2} \left( E_{2\pm} \mp E_{1\pm} \right) \tilde{E}_{2}, \\
\tilde{H}_{y\pm} &= i \frac{1}{\eta_1} \left[ \left( E_{2\pm} \mp E_{1\pm} \right) \tilde{E}_{1\pm} - \left( E_{1\pm} \mp E_{2\pm} \right) \tilde{E}_{2\pm} \right].
\end{aligned}
\]

(5)

Similarly, the expressions for reflected and transmitted components are given as Eq. 6:

\[
\begin{aligned}
\tilde{E}_{x\pm} &= \frac{1}{2} \left( E_{1\pm} \mp E_{2\pm} \right) \tilde{E}_{1\pm} + \frac{1}{2} \left( E_{2\pm} \mp E_{1\pm} \right) \tilde{E}_{2\pm}, \\
\tilde{H}_{y\pm} &= i \frac{1}{\eta_1} \left[ \left( E_{2\pm} \mp E_{1\pm} \right) \tilde{E}_{1\pm} - \left( E_{1\pm} \mp E_{2\pm} \right) \tilde{E}_{2\pm} \right].
\end{aligned}
\]

(6)

Therefore, the corresponding energy flux reflectance and transmittance are

\[
\begin{aligned}
R_{\pm} &= \frac{\left( E_{1\pm} \mp E_{2\pm} \right)^2 R + \left( E_{2\pm} \mp E_{1\pm} \right)^2}{2 \left( E_{1\pm}^2 + E_{2\pm}^2 \right)}, T_{\pm} &= \frac{\left( E_{1\pm} \mp E_{2\pm} \right)^2 T + \left( E_{2\pm} \mp E_{1\pm} \right)^2}{2 \left( E_{1\pm}^2 + E_{2\pm}^2 \right)}.
\end{aligned}
\]

(7)

**Numerical Analysis and Discussion**

Here we only analyze the propagation behaviors of normal elliptically polarized wave with \( E_1 = 1, E_2 = 2 \), such as the rules of average energy flux reflectance, transmittance and total reflection in detail. Supposing conventional medium 1 is the vacuum with \( \mu_1 = \mu_0, e_1 = e_0 \), and the chiral parameter \( \chi > n \quad (\chi = 0.4) \) is required for negative refraction medium 2 with \( \mu_2 \approx \mu_0, e_2 = 0.16 \).

1. When \( \chi = 0.5 \), that is \( n_2 < n_1 < n_3 \). In Fig. 2 there are two critical angles for both the right-handed and left-handed circularly polarized components, and it can be seen obviously that \( \theta_{1\pm} < \theta_{2\pm} \). The sum of reflectance and transmittance always keeps constant as 1, which satisfies the law of conservation of energy.
Figure 2. Energy flux reflectance and transmittance in the case of $|n| < n_+ < n_1$. (a) and (b) are for the incidence of normal right-handed and left-handed elliptically polarized wave respectively. The horizontal coordinate denotes incident angle $\theta$, and the vertical coordinate denotes reflectance or transmittance, similarly hereinafter.

For the incidence of normal right-handed elliptically polarized wave, from Fig. 2(a) it can be found that when incident angle $\theta$ increases to $\theta_{c1}$, the left-handed circularly polarized component in refracted wave becomes a surface wave, which means that it cannot propagate in the medium. In the process of incident angle changing, the transmittance keeps decreasing while reflectance keeps increasing. When $\theta$ increases to $\theta_{c2}$, the right-handed circularly polarized component also becomes a surface wave. So reflectance turns into 1 while transmittance turns into 0, that is, the total reflection occurs. For the incidence of normal left-handed elliptically polarized wave, transmittance decreases significantly with $\theta$ increasing around $\theta_{c1}$. When $\theta = \theta_{c2}$, the total reflection also occurs.

(2) For $\chi = 0.9$, that is $n_+ < n_1 < n_-$. In this case, from Fig. 3 we can see only the left-handed circularly polarized component has a critical angle $\theta_{c1}$.

Figure 3. Energy flux reflectance and transmittance in the case of $|n| < n_1 < n_+$. In the incidence of normal right-handed elliptically polarized wave, two curves in Fig. 3 are relatively smooth. When incident angle increases to $\theta_{c1}$, both reflectance and transmittance have slight steps, and the left-handed circularly polarized component becomes a surface wave. After that, in this case the total reflection never appears. Like the incidence of normal right-handed elliptically polarized wave, similar rules of normal left-handed elliptically polarized wave can be obtained, besides a significant step at $\theta = \theta_{c2}$ is compared to the normal right-handed one.

(3) Supposing $\chi = 1.5$, that is $n_1 < n_+ < n_-$. The average energy flux reflectance and transmittance varying with incident angles are shown in Fig. 4. It indicates that the curves are smooth all the time, and no total reflection occurs for incidence of both normal right-handed and left-handed elliptically polarized waves.
Figure 4. Energy flux reflectance and transmittance in the case of $n_1 < k_1 < n_\ast$.

For the cases of arbitrary elliptically polarized waves, the results can be obtained by the rotation operation of a normal elliptically polarized wave, and here we shall not mention it too much.

Summary

According to constitutive relations in chiral negative refraction medium, the reflection and refraction characteristics of elliptically polarized wave in chiral negative refraction medium are discussed, and the relevant results show that there are two intrinsic waves in the chiral medium, which have different propagation characteristics: (1) The left-handed circularly polarized component of intrinsic waves will achieve negative refraction in the case that the chiral parameter is greater than refractive index of the medium, which is also called chiral negative refraction medium. (2) When chiral parameters are taken as different values, there may be situations of two angles, one angle or even no critical angle for two intrinsic waves. (3) In terms of the elliptically polarized wave in different polarization states, various reflective and refractive phenomena for incidence upon the surface of chiral negative refraction medium can be observed.

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