2017 2nd International Conference on Architectural Engineering and New Materials (ICAENM 2017)
ISBN: 978-1-60595-436-3

Wind and Rain Vibration Excitation Model of Cables Considering the In-plane and Outer-plane Vibration

Guodong Zheng and Huali Cao

ABSTRACT

On the basis of summarizing the characteristics of rain-wind-induced vibration of stayed cables, the wind-induced vibration force of in-plane and outer-plane is studied, and the motion equation of cables when free vibrated and when wind-induced vibration and damper interacted. With the Galerkin multi-modal truncation method, the two expressions of the motion equations of the continuous system are studied, and their correlations are discussed. On the basis of discretization, the possibility of numerical calculation is considered. The solution strategy of the continuous system is presented when dampers are added.[1]

INTRODUCTION

As the main component of cable-stayed bridge, the cable has a greater flexibility, less damping, and lighter weight. In the wind or wind and rain weather conditions, cables are prone to large vibration, endangering the operational safety of the bridge. With the progress of science and technology and construction technology, the length of stay cable is also increasing with the span of bridge, and the vibration of cable is becoming more and more prominent.

In 2001, a strong rain-wind-induced vibration of the cables of the Nanjing Yangtze River Bridge happened before the opening, making part of the dampers installed in the root of the cables damaged. From January 2001 to April 2004, Z Q Chen et al [1] had a vibration observation study on the rain-wind-induced vibration of cables of Hunan Yueyang Dongting Lake Bridge. The field

[1] Guodong Zheng, Huali Cao. School of Civil Engineering and Architecture, Linyi University, Shandong, China

measurement data of wind and rain vibration on cables each year can be got 3 to 4 times.

A theoretical model which can accurately and comprehensively reflect the characteristics of wind and rain vibration of cable is established and analyzed. The mechanism of this phenomenon can be further studied and provided the basis for engineering design and wind and rain vibration control. T B PENG and M GU [2] established a three-freedom degrees model of cable when it is wind-rain-induced vibrated with the Lagrange equation, which includes the upper and lower water lines on the surface of the cable and the vertical vibration of the cable.

ESTABLISHMENT OF DYNAMIC EQUATION WHEN IN-PLANE AND OUTERSIDE-PLANE VIBRATED WHEN DAMPERS ARE ADDED

The vibration suppression measures commonly used include changing the cable cross-section itself and the usage of dampers. Such as axial slotting or pasting longitudinal ribs on the surface of the cable; playing a pit on the surface of the cable; winding a spiral on the surface of the cable; using a cable with a polygonal profile.

Auxiliary cable method can also effectively suppress cable wind and rain vibration. The principle of this method is to install the auxiliary cable between the cables, making the entire cable surface stiffness raised due to the free length of the cable is reduced, while the energy can be transmitted between the different cables by the auxiliary cable, so as to achieve the purpose of inhibition the wind and rain excitation of cables. The use of high damping material in the cable or the installation of dampers between the cable and the bridge deck can both improve the cable damping, but the former method is very difficult to make the cable damping raised due to the cable strain caused by vibration is very low. So the damper is the inevitable choice to improve the cable damping.

At present, according to its internal working medium the commonly used damper can be divided into the following four categories: rubber damper (HDR), oil damper, shear viscous damper (VSD) and magneto rheological damper (MRD).

In the process of wind-rain excitation, the in-plane and outer-plane vibrations occur synchronously. Most of the theoretical models only consider the in-plane vibration of the cable; or the outer-plane vibration is taken into account in modeling, and the influence of it is neglected in the calculation. In this paper, the vibration equation form in-plane and outer-plane when the damper is applied is given.

Free Vibration

First let's consider the free vibration of the cable. The vibration in the X-Y plane is defined as the in-plane vibration and the X-Z plane is the out-of-plane vibration, as is shown in Figure 1.

Considering the cable sag and bending stiffness and without the axial vibration, we can obtain the cable-damper system of in-plane coupling nonlinear vibration equation by S S Chen and D H Ren [3] with Newton's law:

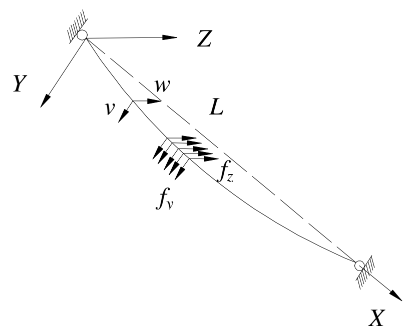

Figure 1. Forced Vibration Model of Stay Cables.

$$-EI\frac{\partial^4 v}{\partial x^4}+(H+h)\frac{\partial^2 v}{\partial x^2}+h\frac{d^2 y}{dx^2}=m\frac{\partial^2 v}{\partial t^2}+c_1\frac{\partial v}{\partial t}+C_{dy}\frac{\partial v}{\partial t}-f_y(x,t) \tag{1}$$

$$-EI\frac{\partial^4 w}{\partial x^4}+(H+h)\frac{\partial^2 w}{\partial x^2}=m\frac{\partial^2 w}{\partial t^2}+c_2\frac{\partial w}{\partial t}+C_{dz}\frac{\partial w}{\partial t}-f_z(x,t) \tag{2}$$

Where m is the unit length mass of the cable, c_1, c_2 is the in-plane and out-of-plane damping coefficient of the cable, y is the sag curve in the Y direction of the cable's weight, f_y, f_z are respectively the external load acting in Y and Z direction, EI is the bending stiffness of cable direction, H and h are respectively static and dynamic tensile force along X direction, which is approximately constant in the whole L range, C_{dy}, C_{dz} are the passive equivalent damping coefficient in the Y, Z direction produced by dampers.

Rewriting of The Motion Equation When The Wind-induced Vibration and The Damper Act Together

The motion equation can be expressed as

$$m\frac{\partial^2 v}{\partial t^2}+c_y\frac{\partial v}{\partial t}+F_{Dy}\delta(x-x_d)=-EI\frac{\partial^4 v}{\partial x^4}+H\frac{\partial^2 v}{\partial x^2}+h(\frac{\mathrm{d}^2 y}{\mathrm{d}x^2}+\frac{\partial^2 v}{\partial x^2})+F_y \tag{3}$$

$$m\frac{\partial^2 w}{\partial t^2}+c_z\frac{\partial w}{\partial t}+F_{Dz}\delta(x-x_d)=-EI\frac{\partial^4 w}{\partial x^4}+(H+h)\frac{\partial^2 w}{\partial x^2}+F_z \tag{4}$$

Calculation Model of Continuous System

In general, the vibration curve of the cable is a catenary. When the deflection-span ratio is very small (less than 1/6), it can be replaced by a parabola.

$$y=\frac{mg\cos\alpha}{2H}(xl-x^2)=\frac{4d}{l}(x-\frac{x^2}{l})\,,d=\frac{mgl^2\cos\alpha}{8H}\,,L_e=l\left[1+8\left(\frac{d}{l}\right)^2\right] \quad (5)$$

With Galerkin multi-modal truncation, we have

$$v(x,t)=\sum_{i=1}^{\infty}V_i(t)\cdot\sin\frac{i\pi x}{l}\,,w(x,t)=\sum_{j=1}^{\infty}W_j(t)\cdot\sin\frac{j\pi x}{l} \quad (6)$$

Substituting them into the differential equations of motion, multiplied by $\sin n\pi x/l$ at the both sides of the equations and integrated from 0 to l, we can get

$$\ddot{V}_n(t)+2\xi_{vn}\omega_{vn}\dot{V}_n(t)+\omega_{vn}{}^2V_n(t)+\chi_{vn}V_n{}^2(t)+\vartheta_{vn}V_n{}^3(t)+P_{vn}=0 \quad (7)$$

$$\ddot{W}_n(t)+2\xi_{wn}\omega_{wn}\dot{W}_n(t)+\omega_{wn}{}^2W_n(t)+\vartheta_{wn}W_n{}^3(t)=0 \quad (8)$$

and

$$\omega_{vn}{}^2=\frac{n^4\pi^4EI}{l^4m}+\frac{n^2\pi^2H}{ml^2}+\frac{128d^2EA}{mn^2\pi^2l^3L_e}(1-\cos n\pi)^2 +\frac{8n^2\pi dEA}{ml^3L_e}\cdot\left(\sum_{i=1}^{\infty}V_i(t)\cdot\frac{1-\cos i\pi}{i}\right)+\frac{n^2\pi^4EA}{4ml^3L_e}\left(\sum_{\substack{i=1\\i\neq n}}^{\infty}i^2V_i^2(t)+\sum_{j=1}^{n}j^2W_j{}^2(t)\right)$$

$$P_{vn}=\frac{128d^2EA}{mn\pi^2l^3L_e}\cdot(1-\cos n\pi)\sum_{\substack{i=1\\i\neq n}}^{\infty}V_i(t)\cdot\frac{1-\cos i\pi}{i}+\frac{4d\pi^2EA}{mn\pi l^3L_e}\cdot(1-\cos n\pi)\cdot\left(\sum_{\substack{i=1\\i\neq n}}^{\infty}i^2V_i^2(t)+\sum_{j=1}^{\infty}j^2W_j{}^2(t)\right)$$

$$c_y=2\xi_{vn}\omega_{vn}m\,,\chi_{vn}=\frac{4dEAn\pi(1-\cos n\pi)}{ml^3\cdot L_e}\,,\vartheta_{vn}=\frac{\pi^4n^4EA}{4ml^3L_e}\,,c_z=2\xi_{wn}\omega_{wn}m\,,$$

$$\vartheta_{wn}=\frac{\pi^4n^4EA}{4ml^3L_e}$$

$$\omega_{wn}{}^2=\frac{n^4\pi^4EI}{l^4m}+\frac{2Hn^2\pi^2}{l^3m}+\frac{8dEAn^2\pi}{m\pi l^3L_e}\sum_{i=1}^{\infty}V_i(t)\cdot\frac{1-\cos i\pi}{i} +\frac{\pi^4n^2EA}{4ml^3L_e}\sum_{i=1}^{\infty}i^2V_i^2(t)+\frac{\pi^4n^2EA}{4ml^3L_e}\sum_{\substack{j=1\\j\neq n}}^{\infty}j^2W_j{}^2(t)$$

Considering the wind and rain excitation, the vibration equations are

$$\ddot{V}_n(t)+2\xi_{vn}\omega_{vn}\dot{V}_n(t)+\omega_{vn}{}^2V_n(t)+\chi_{vn}V_n{}^2(t)+\vartheta_{vn}V_n{}^3(t)+P_{vn}=\frac{2}{ml}\int_0^l F_y(x,t)\sin\frac{n\pi x}{l}\mathrm{d}x \tag{9}$$

$$\ddot{W}_n(t)+2\xi_{wn}\omega_{wn}\dot{W}_n(t)+\omega_{wn}{}^2W_n(t)+\vartheta_{wn}W_n{}^3(t)=\frac{2}{ml}\int_0^l F_z(x,t)\sin\frac{n\pi x}{l}\mathrm{d}x \tag{10}$$

The damping term becomes to the left of the equation when the damper is added

$$\frac{2}{ml}\int_0^l F_{Dy}(x,t)\delta(x-x_d)\sin\frac{n\pi x}{l}\mathrm{d}x\,,\frac{2}{ml}\int_0^l F_{Dz}(x,t)\delta(x-x_d)\sin\frac{n\pi x}{l}\mathrm{d}x \tag{11}$$

DISCUSSION OF THE BASIC WIND INDUCED VIBRATION EQUATION

The first model, equations (9) and (10) is proposed by D LI [4], Z J LIU [5], and the second one is proposed by B WANG [6] and so on. But the result is derived only by substituting the Galerkin first-order truncation equation into the basic differential equation with no derivation process.

$$\ddot{p}+2\xi_1\dot{p}+c_1p+c_2p^2+c_3q^2+c_4p^3+c_5pq^2=0 \tag{12}$$

$$\ddot{q}+2\xi_2\omega\dot{q}+c_6q+c_7pq+c_8p^2q+c_9q^3=c_{10}\sin\theta t \tag{13}$$

Li Weiyi [7] and so on think that in the Lagrangian equation, if we neglect the impact of extraneous items, that is $\partial w/\partial x=0$, the second equation can be transformed into the first, the decoupling equations.

The simultaneous equations are coupled second order nonlinear differential equations. It is difficult to obtain the definite solution with analytic method. By programming the calculation procedures in MATLAB, we can obtain the vibration response of the cable and the waterline with the numerical method (Runge-Kuta method) to solve the simultaneous equations. Runge-Kutta method is a widely used high-precision single-step algorithm in engineering. The method is improved on the basis of the principle of Taylor series expansion. Some measures are taken to suppress the error, and the calculation precision is improved, so the realization principle is also complicated.

In addition, B WANG [6] used the equivalent stochastic linearization method to solve the vibration equation and analyzed the stability of the coupling vibration, but did not add the dampers to control it. This is the next step of research.

CONCLUSIONS

(1)In this paper, a theoretical model of in-plane and outer-plane vibration of cable-stayed bridge is proposed by applying the theoretical model of wind-rain-induced vibration of cable.

(2)The theoretical models of wind-rain-induced vibration induced by in-plane and out-plane vibrations, which is established in this paper, can be used to analyze the vibration characteristics of cable and various influencing factors.

(3)The equations established in this paper can be calculated by the general difference method through numerical calculation, and the bifurcation behavior can also be studied with nonlinear dynamics theory.

(4)The wind load and damping force of the equation can be either a sinusoidal function of vortex-induced vibration or a random function of wind-rain excitation. Therefore, equivalent stochastic linearization method is an effective way to study the vibration solution.

ACKNOWLEDGEMENT

This research was financially supported by the National Training Programs of Innovation and Entrepreneurship for Undergraduates.

REFERENCES

1. Z Q Chen. 2005. “On-site observation of wind-rain induced vibration of stay cables and its control”. *Journal of Architecture and Civil Engineering*. 22(4):5-10.
2. T B Peng, M Gu. 2001. “Mechanism of Rain-wind-induced Vibration of Cables”. *Journal of Tongji University*. (1):35-39.
3. S S Chen, D H Ren. 2005. “MR Fuzzy Semi-Active Control of Cables in Cable-Stayed Bridges Using Magnetorheological Dampers”. *Journal of East China Jiaotong University*.05:7-10.
4. D Li. “Study on the Theoretical Models of Rain-wind-induced Vibration of Cable and Analysis on Vibration Characteristics”. *Hunan University*, 2013.
5. Z J Liu, G P Chen, J W Zhong. 2007. “Application of Finite Difference Method in Measurement of Cable Tension by Vibration Method”. *Journal of Basic Science and Engineering*. 01:104-110.
6. B Wang. “Characteristic and Influence of Local Cable Vibration on Long-span Cable-Stayed Bridges”. *Huazhong University of Science & Technology*. 2008.
7. Li Weiyi. “Nonlinear dynamics research of rain-wind-induced vibration of stayed cable”. *Tianjin University*, 2011.