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Analytical Stiffness Matrix Method for Flexural Vibration of Compression Bar

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ABSTRACT

According to differential equation for transverse flexural vibration of compression bar considering second-order effect and inertia force, displacement vector expression was achieved. Based on the displacement boundary condition, displacement coefficient expressed by nodal displacement vector was obtained. Internal force equations of compression bar were established and then internal force at bar ends expressed by nodal displacement vector was provided. Finally, dynamic stiffness matrix colligating mass matrix and geometry matrix was given and can be applied to accurate analysis for dynamic performance and dynamic responses of bar.[1]

INTRODUCTION

Dynamics of structures have been widely used in many fields involving building engineering, machine manufacturing, aerospace engineering, submarine and ship, etc. It is very necessary in theory and practice to research the analyze method of dynamics correctly.

In analyzing structure dynamic problem, scholars have put up all kinds of method, such as immediate integration, energy method, numerical method including finite element method (FEM) and dynamic stiffness matrix method, etc[1-3]. Clough[4] derived motion differential equation of beam under axial force by using immediate integration. Cao[5] derived accurate analytical shape function and stiffness matrix of analytical dynamic element for Euler-Beam.

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But flexural vibration of axial force is not taken into account. Tang[6] developed analytical compression bar element by using energy variation method and showed that the accuracy of energy variation method is identical to that of direct stiffness method. The axial force on bending member can change both natural vibration frequency and vibration mode. Field problem becomes complicated when both axial force and inertia force considered. Zhang et al.[7] studied the transverse vibration of Bernoulli-Euler beam under axial force and solved element shape function and stiffness matrix by using energy variation method. But because the explicit formula of stiffness matrix wasn't achieved, it cannot be applied in actual calculation. Energy method and FEM use the shape function only satisfying field boundary condition. All results are approximate solution because displacement can't be expressed accurately[8-10]. Stiffness matrix method is matrix format of analytical method. It is easy to solve stiffness matrix and accurate solution can be achieved.

In this paper, the displacement vector expression was also achieved according to equilibrium equation for flexural vibration of compression bar. And then displacement coefficient and internal force at bar ends expressed by nodal displacement vector were derived. Matrix-vector format of stiffness matrix for transverse flexural vibration of member considering axial force was established.

COMPRESSION BAR AND VIBRATION PARAMETER

Compression Bar

In order to express two elements conveniently, the coordinate system of compression bar is defined as shown in Figure 1.

Axis x coincides with bar axis, the positive is to the right, and zero locates at the left end of bar. The positive of axis y is to the vertical upward. The positive of axis z is in line with right-hand rule.

The positive of all loads and displacement is in accordance with the coordinate axis. If cross-section outer normal direction is in accordance with axis x, the positive of bar internal force is to the coordinate axis. If cross-section outer normal direction is opposite to axis x, the positive of bar internal force is opposite to the coordinate axis.

The forces on micro segment of bar is shown in Figure 2. Compression bar bears combined action of bar end force, inertial force and external load at any time during its movement.

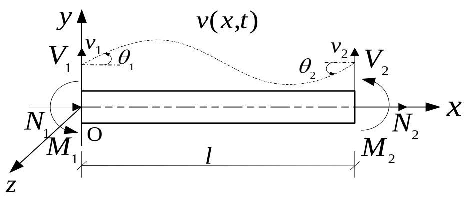

Figure 1. Coordinate definition of compression bar.

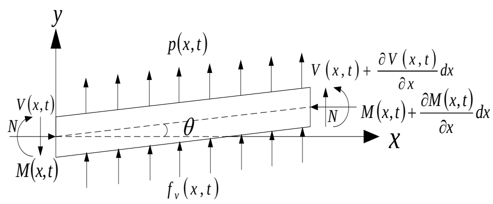

Figure 2. Forces on micro segment.

Vibration Parameter

In order to describe the equation for bar under compression-bending coupling, the following rules are defined: V_i, M_i, N, p are the external loads on bar, f_v is inertia force of bar. $V(x)$ and $M(x)$ are transverse internal force and bending moment at a time on the cross-section locating at x, respectively. Both are independent of time. v is deflection, and θ is rotation angle.

Nodal force vector of compression bar is $\boldsymbol{F}_v^e = \{V_1 \quad M_1 \quad V_2 \quad M_2\}^{\mathrm{T}}$.

Internal force vector at compression bar ends is $\boldsymbol{F}_{vi}{}^{\mathrm{e}} = \{V(\mathbf{0}) \quad M(\mathbf{0}) \quad V(l) \quad M(l)\}^{\mathrm{T}}$.

Nodal displacement vector of compression bar is $\boldsymbol{\delta}_v^e = \{v_1 \quad \theta_1 \quad v_2 \quad \theta_2\}^{\mathrm{T}}$.

CONTROL EQUATION

Suppose that the deformation of compression bar under all forces as shown in Figure 1 is so small that non-linear curvature can be ignored, and the second-order effect of compression bar needs to be considered due to large displacement. With equations including equilibrium equation, geometric equation, and physics equation, stiffness equation for flexural vibration of compression bar with uniformly distributed mass can be expressed as

$$EIv^{(4)} - Nv^{(2)} + m(x)\ddot{v} = p(x,t) \tag{1}$$

where, m is mass of bar per unit length, E is elastic modulus, I is inertia moment of bar cross-section and they are constants because the compression bar in this paper is uniform cross-section bar.

DIFFERENTIAL EQUATION SOLUTION AND DISPLACEMENT

When only nodal load and distributed inertia force are considered to analyze free vibration of compression bar, nothing but general solution of differential equation should be solved without regard to particular solution. If external load is zero, partial differential equation for flexural vibration according to Eq.(1) can be obtained as following:

$$EIv^{(4)} - Nv^{(2)} + m\ddot{v} = p(x,t) \tag{2}$$

By using the method of variable separation, $v(x,t) = \phi_v(x)Y_v(t)$, the transformation can be obtained:

$$\phi^{(4)}(x) + n^2\phi''(x) - \omega^2 m\phi(x)/EI = \mathbf{0} \tag{3}$$

where, $n^2 = |N|/EI$, $\omega = \xi^2\sqrt{EI/m}$

The solution of Eq.(3), which is the displacement general format for transverse flexural vibration of compression bar, can be expressed as

$$\phi(x) = \sum_{i=1}^{4} f_i(x)b_i = \boldsymbol{fb} \tag{4}$$

where, primary function is $\boldsymbol{f}(x) = \{\sin\alpha x \quad \cos\alpha x \quad e^{\beta x} \quad e^{-\beta x}\}$, $\alpha = \sqrt{\left(\omega^2 m/EI + n^4/\mathbf{4}\right)^{1/2} + n^2/\mathbf{2}}$, $\beta = \sqrt{\left(\omega^2 m/EI + n^4/\mathbf{4}\right)^{1/2} - n^2/\mathbf{2}}$, displacement coefficient of displacement equation is $\boldsymbol{b} = \{b_1 \quad b_2 \quad b_3 \quad b_4\}^T$, and it can decide the shape of bar.

For simplified calculation, displacement primary function can be transformed as following:

$$\boldsymbol{f}' = \boldsymbol{fZ} \tag{5}$$

where, $\boldsymbol{Z}$ is differential transformation matrix.

DEFINITE SOLUTION OF DISPLACEMENT COEFFICIENT

According to the definition, rotation angle of bar at x is $\theta(x) = \phi' = \boldsymbol{fZb}$. With the displacement boundary condition, Eq.(6) can be obtained

$$\boldsymbol{\delta}_v^e = [\phi(x=\mathbf{0}) \quad \theta(x=\mathbf{0}) \quad \phi(x=l) \quad \theta(x=l)] = \boldsymbol{Ab} \tag{6}$$

where $\boldsymbol{A} = [\boldsymbol{f}(\mathbf{0}) \quad \boldsymbol{f}'(\mathbf{0})\boldsymbol{Z} \quad \boldsymbol{f}(l) \quad \boldsymbol{f}'(l)\boldsymbol{Z}]^{\mathrm{T}}$

Then, displacement coefficient can be obtained

$$\boldsymbol{b} = \boldsymbol{A}^{-1}\boldsymbol{\delta}_v^e \tag{7}$$

STIFFNESS MATRIX

From physics equation, moment equilibrium equation of bar, and Eq.(4)-Eq.(7), internal forces of compression bar can be expressed as Eq.(8) and Eq.(9).

$$M(x) = EI\boldsymbol{fZZb} = EI\boldsymbol{fZZA}^{-1}\boldsymbol{\delta}_v^e \tag{8}$$

$$V(x) = -EI\boldsymbol{fZZZb} + N\boldsymbol{fZb} = -EI\boldsymbol{fZZZA}^{-1}\boldsymbol{\delta}_v^e + N\boldsymbol{fZA}^{-1}\boldsymbol{\delta}_v^e \tag{9}$$

According to the definition, the relationship of force and internal force at bar end can be expressed as

$$\boldsymbol{F}_v^e = \{V_1 \quad M_1 \quad V_2 \quad M_2\}^{\mathrm{T}} = \{-V(\mathbf{0}) \quad -M(\mathbf{0}) \quad V(l) \quad M(l)\}^{\mathrm{T}} \tag{10}$$

Due to $n^2 = |N|/EI$ and axial force is negative, $N = -n^2 EI = -(\alpha^2 - \beta^2)EI$. With Eq.(8) and Eq.(9), Eq.(10) becomes

$$\boldsymbol{F}_v^e = EI\boldsymbol{DA}^{-1}\boldsymbol{\delta}_v^e \tag{11}$$

where,

$$\boldsymbol{D} = [\boldsymbol{f}(\mathbf{0})\boldsymbol{ZZZ} + (\alpha^2 - \beta^2)\boldsymbol{f}(\mathbf{0})\boldsymbol{Z} \quad -\boldsymbol{f}(\mathbf{0})\boldsymbol{ZZ} \quad -\boldsymbol{f}(l)\boldsymbol{ZZZ} - (\alpha^2 - \beta^2)\boldsymbol{f}(l)\boldsymbol{Z} \quad \boldsymbol{f}(l)\boldsymbol{ZZ}]^{\mathrm{T}}$$

Then, $\boldsymbol{F}_v^e = \boldsymbol{K}^e\boldsymbol{\delta}_v^e$, $\boldsymbol{K}^e = EI\boldsymbol{DA}^{-1}$. $\boldsymbol{K}^e$ is the stiffness matrix for flexural vibration of compression.

CONCLUSIONS

(1) Accurate displacement formula for flexural vibration of compression bar expressed by primary function vector and displacement constant vector is achieved.

(2) Internal force equation of bar end expressed by displacement vector is obtained.

(3) Accurate dynamic stiffness matrix colligating mass matrix and geometry matrix is provided.

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