**Sliding Mode Control Based on Inversion Design and Bounded Input Constraint for Path Tracking of Mobile Robot**

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**Abstract.** In order to solve the problem of path tracking of mobile robot, a sliding mode control strategy is proposed. Firstly, a sliding mode controller based on the inversion algorithm is designed, and the low-pass filter is used to reduce the interference in the control process. On this basis, for solving the problem that the dynamic performance of the previous controller is not ideal, the bounded input constraint is used to improve the control law. The simulation results show that the improved controller not only has good tracking effect and fast dynamic response, but also ensures the global asymptotically stability.

**Introduction**

Mobile robot has developed rapidly in the past decades, and it has been widely applied in a variety of industries. However, as a highly complex nonlinear system, it is difficult to obtain high-precision results of path tracking of mobile robot. Accuracy issues of path tracking are getting more and more attention. In 1991, a stable tracking control rule for non-bolonomic vehicles is proposed, and it implements local asymptotic tracking. After soon, Walsh G et al. use a local exponential control law to stabilize the trajectory of the system and complete the trajectory tracking. The result has been successfully applied in several aspects and people gradually see the huge prospects of the related technology. After that, Yang J M et al. use polar coordinates to represent the attitude of robot and the calculated torque method to feedback linearize dynamic equation of robot, which has good tracking performance.

With the advent of the new century, more and more scholars have invested in the study of path tracking. They use the amount of virtual intermediate, transverse function, indirect guide angle and Lyapunov stability criterion for the trajectory tracking control to simplify the design of the controller. On the basis of predecessors, a control law based on reverse thrust technology is proposed to solve the problem of speed jump in traditional sliding mode tracking controller. Hadian G et al. use the Kalman uniform filter and the decentralized Kalman filter to reason the information received by the mobile robot and successfully applied to the tracking of stochastic linear dynamic targets. And Kigezi T N et al. greatly simplify analysis and produce significant flutter reduction by linearly combining two indirect control states on a sliding surface into one global variable, avoid nonlinear state variable coupling. The researches of these scholars have rapidly promoted the development of robotics and improve the tracking accuracy of mobile robots.

The contribution of this paper is proposing a sliding mode controller for path tracking of mobile robot. Firstly, we apply the inversion idea to the sliding mode controller. Secondly, a low-pass filter is used to reduce the interference of the control process. Besides, the bounded input constraint is used to improve the control law. At last, simulation experiments verify the feasibility and advancement of the design.

This paper is organized as follows. The modelling of the concerned mobile robot is represented in Section II. Inversion design applying the sliding mode controller of path tracking and the specific
design approaches are described in Section III. The improved controller based on bounded input constraint and the detailed design approaches are described in Section IV. Simulation results are analyzed and statistical results are summarized in Section V. The conclusion is finally given in Section VI.

**Model of Mobile Robot in Research**

The research object of this paper is an independent double rear wheel differential drive mobile robot which control direction of the robot through the different speeds of the two rear wheels. The rectangular coordinates are established in the working plane of the mobile robot, the schematic diagram of the posture error is shown in Figure 1.

![Figure 1. Schematic diagram of posture error of mobile robot.](image)

According to the above figure, the kinematic model equation (1) and the target path equation (2) of the robot are obtained:

\[
\begin{align*}
\dot{x} &= v \cos \theta \\
\dot{y} &= v \sin \theta \\
\dot{\theta} &= \omega 
\end{align*}
\]  
\(1\)

\[
\begin{align*}
\dot{x}_d &= v \cos \theta_d \\
\dot{y}_d &= v \sin \theta_d \\
\dot{\theta}_d &= \omega_d 
\end{align*}
\]  
\(2\)

achieve \(x\) tracking \(x_d\), \(y\) tracking \(y_d\), \(\theta\) tracking \(\theta_d\) through designing control law, the posture error equation (3) of mobile robot can be obtained as follow:

\[
\dot{e} = \begin{bmatrix}
\dot{e}_1 \\
\dot{e}_2 \\
\dot{e}_3
\end{bmatrix} = \begin{bmatrix}
\dot{x} - x_d \\
\dot{y} - y_d \\
\dot{\theta} - \theta_d
\end{bmatrix} = \begin{bmatrix}
v \cos \beta - x_d \\
v \sin \beta - y_d \\
\omega - \omega_d
\end{bmatrix}
\]  
\(3\)

**Sliding Mode Control of Path Tracking Based on Inversion Design**

In this section, a new controller of path tracking will be designed by the inversion algorithm. The method of the inversion design is to decomposture the complex nonlinear system into subsystems that do not exceed the order of the system, then design the Lyapunov function and the intermediate virtual control quantity for each subsystem respectively, and always retreat to the whole system until the design of the whole control law is completed.

The design steps of the inversion controller are as follows:

Firstly, we define the Lyapunov function as

\[
V_i = \frac{1}{2} e_1^2 + \frac{1}{2} e_2^2
\]  
\(4\)

Available from the formula (1), (2) and (3)
\[ V_1 = \frac{1}{2} e_1^2 + \frac{1}{2} e_2^2 \]  
(5)

Then take the switching surface function as \( s_1 = e_1, s_2 = e_2 \). And design the virtual control quantity \( \beta \) to make

\[
\begin{align*}
(m_1 &= v \cos \beta = x_d - k_1 e_1 \\
(m_2 &= v \sin \beta = y_d - k_2 e_2 \\
\end{align*}
\]  
(6)

Substitute the formula (6) into the formula (5), get

\[ V_1 = -k_1 e_1^2 - k_2 e_2^2 \leq 0 \]  
(7)

According to formula (1), the virtual control law and linear velocity can be designed as

\[
\begin{align*}
\beta &= \arctan \frac{m_1}{m_2} \\
v &= u_2 / \sin \beta
\end{align*}
\]  
(8)

The next task is to achieve \( \theta \) tracking \( \theta_d \) by designing the angular velocity control quantity, while ensuring that \( \theta \) tracks \( \beta \). Lyapunov function is defined as

\[ V_2 = V_1 + \frac{1}{2} e_3^2 \]  
(9)

So we can get

\[ V_2 = -k_3 e_3^2 + e_3 (\omega - \omega_d) \]  
(10)

Take the switching surface function as \( s_3 = e_3 \), the angular velocity control law is designed as

\[ \omega = \omega_d - k_3 s_3 - q \text{sgn}(s_3) \]  
(11)

At this time, \( \dot{V}_2 \leq 0 \), so the system satisfies the theoretical condition of Lyapunov stability\(^{[10]} \).

The following low-pass filter is used in order to reduce the interference in the control process.

\[ Q(s) = \frac{\alpha_1}{s_i + \alpha_i}, \alpha_i > 0 \]  
(12)

**Improved Sliding Mode Control of Path Tracking Based on Bounded Input Constraint**

In this section, the bounded input constraint is used to improve the control law. It is required to set the path tracking \( \phi^*(\cdot) = [x^*(\cdot), y^*(\cdot), \theta^*(\cdot)] \). Traceable if the robot can converge and follow constraints.

\[
\begin{align*}
0 < V_{\text{min}} < v^*(t) < V_{\text{max}} \\
-W_{\text{max}} < \omega^*(t) < W_{\text{max}}
\end{align*}
\]  
(13)

Where, \( v \) and \( \omega \) are the control input linear speed and angular velocity.

According to the model of the mobile robot, the input of the mobile robot must also meet the constraints (13) if \( \phi^*(\cdot) = [x^*(\cdot), y^*(\cdot), \theta^*(\cdot)] \) is trackable. That is to say for \( \forall t \geq 0 \), there is

\[ V_{\text{min}} < \sqrt{x^*2(t) + y^*2(t)} < V_{\text{max}} \]  
(14)

\[ \frac{|y^*(t)x^*(t) - x^*(t)y^*(t)|}{x^*2(t) + y^*2(t)} < W_{\text{max}} \]  
(15)

Conditions (14) and (15) ensure that the robot tracks \( \phi^*(\cdot) \) under constraint input. So they are necessary for \( \phi^*(\cdot) \) to be a trackable trajectory. Next, define that

\[
\begin{align*}
\dot{e}_1 &= x_d - \text{atanh}(pe_1) \\
\dot{e}_2 &= y_d - \text{btanh}(qe_2)
\end{align*}
\]  
(16)

\( e_1, e_2 \rightarrow 0 \) can be realized. Design the virtual control quantity \( \beta \) to make
\begin{align*}
\{m_1 = v \cos \beta = x_d - \dot{e}_1 \\
m_2 = v \sin \beta = y_d - \dot{e}_2 \}
\end{align*}

Similarly, the virtual control law and linear velocity are designed as

\begin{align*}
\{ \beta = \arctan \frac{m_1}{m_2} \\
v = u_2 / \sin \beta \}
\end{align*}

The angular velocity control law (11) is unchanged and a low-pass filter (12) is used to eliminate interference.

After building the controller, we also need to verify that the controller meets the stability of the system which is very necessary. Because, if the design is unreasonable, the system will be unstable during the control process, and the system error will become larger and larger, which will lead to the test failure.

In this step, we first construct the Lyapunov function as

\[ V = \frac{1}{2} s_1^2 + \frac{1}{2} s_2^2 \]

and get

\[ \dot{V} = s_1 \dot{s}_1 + s_2 \dot{s}_2 \]

Design the switching function so that \( s_1, s_2 \) tend to 0, and select the constant velocity approaching law, let

\[ \begin{bmatrix} \dot{s}_1 \\ \dot{s}_2 \end{bmatrix} = \begin{bmatrix} -k_1 \text{sgn} s_1 \\ -k_2 \text{sgn} s_2 \end{bmatrix} \]

Substitute the formula (21) into the formula (20) and combine to get

\[ \dot{V} = -k_1 s_1 \text{sgn} s_1 - k_2 s_2 \text{sgn} s_2 \leq 0 \]

Because \( V \geq 0 \) is continuously differentiable, \( \dot{V} \leq 0 \), the global asymptotic stability of the system can be determined by Lyapunov stability theory\cite{lyapunov}.

**Simulation Analysis**

The Matlab simulation of path tracking of mobile robot is carried out by using the above two methods, and the initial posture is \( [0.4 \quad -0.2 \quad 0] \), the expected path is \( x_d = t, y_d = \sin(0.5x_d) \).

When the sliding mode controller based on inversion design is used to track the sinusoidal path, we take \( k_1 = k_2 = 0.3, k_3 = 0.5, q = 3 \). And when the improved sliding mode controller based on bounded input constraint is used, we take \( a = b = 1.0, p = q = 10 \). The simulation results are shown in Figures 2-4.

According to Figure 2, we know that the improved controller based on the bounded input constraint achieve accurate path tracking within 0.2s, but the sliding mode controller based on inversion design realizes accurate path tracking after 6.3s. Apparently, the improvement of bounded input constraint makes significant advance in dynamic performance.

Figure 3 shows that the tracking effects of the two methods are not much different in the x direction, but the effect of the improved controller based on bounded input constraint is still slightly better than that based on inversion design. Moreover, the former is accurately tracking the target path faster than the latter in the y direction.
Figure 2. The comparison of tracking effect about sine curve.

Figure 3. The comparisons of tracking effect on the x and y directions.

Figure 4. The control inputs of linear velocity and angular velocity.

Figure 4 shows the control input of linear and angular velocity. It can be seen from the figure that the systems of the two methods are both stable. But comparatively speaking, the controller based on the bounded input constraint is more ideal than the one based on inversion design and the former has better robustness.

Figure 5. The tracking absolute errors of the two methods.
The tracking absolute errors of the two methods are presented in Figure 5. As can be seen from the figure, the tracking absolute errors of the two methods can converge to 0 in the end. The absolute error of the controller based on the bounded input constraint converges to 0 at 1.6s, but the absolute error of the controller based on inversion design does not converge to 0 until 18s. Therefore, the rate of convergence of the controller based on the bounded input constraint is faster than the latter, the dynamic response has been significantly improved.

Table 1. Statistical comparisons of the two methods.

| The controllers                                      | $E[|e|]$ | $\sigma_\varepsilon$ | CC  |
|------------------------------------------------------|---------|----------------------|-----|
| The controller based on inversion design             | 0.0436  | 0.0615               | 0.9960 |
| The controller based on bounded input constraint     | 0.0010  | 0.0130               | 0.9998 |

To further clarify the tracking results, their statistical properties include the mean absolute deviation $E[|e|]$, the standard deviation of the tracking error $\sigma_\varepsilon$, and the CC between the actual and the tracking value are used for comparison. The statistical results are summarized in Table 1. Table 1 shows that all $E[|e|]$ and $\sigma_\varepsilon$ results of the controller based on bounded input constraint are smaller than that of the controller based on inversion design. These statistical properties show that the tracking of the improved controller results in a smaller error. Furthermore, the CC results of the improved controller are bigger than that of the controller based on inversion design. And the outstanding performance of the improved controller is validated by the result that the value of the CC is much closer to 1.0 than that of the controller based on inversion design.

Conclusion

In this paper, the path tracking problem of mobile robot based on sliding mode control is studied. Firstly, a sliding mode controller is designed by applying inversion design, and the low-pass filter is used to effectively reduce the interference to improve the robustness of the system. In addition, the bounded input constraint is used to improve the control law. Last but not least, matlab simulation results show that under the same parameter conditions, the sliding mode controller based on bounded input constraint have better performance in tracking effect, dynamic response and robustness.

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