The Propagation Behavior of Collinear Cracks under Compression

Peng YING, Zhe-Ming ZHU*, Yu-Qing DONG, Meng WANG and Lei ZHOU

Key Laboratory of Energy Engineering Safety and Disaster Mechanics, Ministry of Education and College of Architecture and Environment, Sichuan University, Chengdu, 610065, China

Abstract: In this paper, brittle materials with three collinear discrete cracks under compression have been studied, and a corresponding fracture condition has been proposed. In order to exam the effectiveness of the fracture condition and investigate the factors that control crack initiation and propagation, experiment study by using cement mortar and sandstone specimens with three collinear cracks has been implemented, and the effects of crack orientations, the distance between two cracks and the confining stresses have been investigated. The test results agree well with the fracture condition proposed in this paper.

1 Introduction

Cracks or faults are frequently encountered in many engineering structures, and they usually play a dominative role in structure stability. In order to predict and prevent engineering disasters induced by such cracks or faults, it’s necessary to investigate the properties of crack initiation and propagation so as to obtain a better understanding of the dominant parameters that control crack propagations. Therefore, a great deal of efforts from both theoretical and experimental points of view has been devoted to the study of the physics of crack propagations, and accordingly many significant results have been presented in the literature [1-12].

Cracks may be discrete, and they could be lined up which is called collinear discrete cracks. There are many factors which could affect crack stability, e.g. the stress state, the crack orientation [13-15], the crack friction and the target rock strength. Crack friction is considered as one important factor in resisting crack slip [11,16-17], and it is related to many factors, such as for geotechnical fault, it is related to frictional heat, thermal pressurization of pore fluids, and mechanical lubrication [18-21].

For an inclined crack under compression, whether axial splitting and shear faulting is a point of contention for the mechanism of wing-cracks [22-24]. It should be noted that, for an inclined crack under compression, the stress state can be converted into two kinds of stress states, one is biaxial compression, and the other is pure shear. Under pure shear state, the crack belongs to mode II crack, and the shear stresses will induce wing-cracks at the crack tips. For the wing-cracks developed under compression, the shear stresses actually have the function of tensile stresses, thus for inclined cracks under compression, tensile failure characteristic still can be observed.

In this study, the SIF of mode II crack will be focused. Under compression, if the material is fully homogeneous, there is no stress concentration at the crack tips, and therefore, the mode I crack under compression is not focused in this paper. Actually, the compressive stress may force the crack to close, and if the crack slides between the crack surfaces, there exists friction, and therefore, from this point of view, the compressive stresses do have effect on crack propagation. The friction between crack surfaces is considered in this study.

In this paper, three collinear discrete cracks inside an infinite plane under compression are investigated. The fracture condition for three collinear discrete cracks under compression is established based on the mode II crack propagation criterion, and experimental study is implemented by using sandstone and cement mortar specimens with three collinear cracks by using a true triaxial compression device.

2. Theoretical Results for an Infinite Plane with Collinear Cracks

In this study, an infinite plane containing a three-collinear-crack subjected to two far-field principal stresses $\sigma_1$ and $\sigma_3$ as shown in Fig. 1 is considered. Suppose the crack orientation angle with $\sigma_3$-axis is $\alpha$; the cracks are symmetrical about the origin; the crack lengths are equal, and the effective crack friction coefficient is $f$.

According to the result of Zheng et al. [9], the dimensionless SIF $Y_{II}$ at tips A, B and C can be expressed as
\[ Y_{II} = \frac{1}{2\sigma_1} \sqrt{\frac{1}{2(a^2-b^2)(a^2-c^2)}} [\lambda(c^2-b^2)-c^2+a^2] \]
\[ \{ (\sigma_1 - \sigma_3) \sin 2\alpha - [\sigma_1 + \sigma_3 + (\sigma_1 - \sigma_3) \cos 2\alpha] f \} \]
\[ Y_{II} = \frac{1}{2\sigma_1} \sqrt{\frac{b(c+b)}{b^2-a^2}} [\lambda - 1] \]
\[ \{ (\sigma_1 - \sigma_3) \sin 2\alpha - [\sigma_1 + \sigma_3 + (\sigma_1 - \sigma_3) \cos 2\alpha] f \} \]
\[ Y_{II} = \frac{1}{2\sigma_1} \sqrt{\frac{c(c+b)}{c^2-a^2}} [\lambda - 1] \]
\[ \{ (\sigma_1 - \sigma_3) \sin 2\alpha - [\sigma_1 + \sigma_3 + (\sigma_1 - \sigma_3) \cos 2\alpha] f \} \]

Where
\[ \lambda = \frac{1}{k^2} \left[ 1 - \frac{1-\frac{1}{2}\frac{1}{2}\frac{1}{2}\frac{1}{2}}{1+\frac{1}{2}\frac{1}{2}\frac{1}{2}\frac{1}{2}} \right] \]
\[ k^2 = \frac{c^2 - b^2}{c^2 - a^2} \]

\[ \sigma_1 \]
\[ \sigma_2 \]
\[ \sigma_3 \]

\[ \sigma_1 \]
\[ \sigma_2 \]
\[ \sigma_3 \]

\[ \sigma \]

\[ (a) \text{ uniaxial compression} \quad (b) \text{ biaxial compression} \]

**Figure 1.** Sketch of an infinite plane containing a three-collinear-crack under compression.

The calculation results show that for three-collinear-crack as shown in Fig. 1, the SIF values at tip A are always larger than those at tips B and C, whereas at tip C it is always the smallest one, but the difference between those at tips A and C is slight.

**3. Fracture Condition for Three-Collinear-Crack under Compression**

As is well known, mode II crack propagation criterion can be written as

\[ K_{II} = Y_{II} \sigma_1 \sqrt{\pi l} \leq K_{IIC} \]  

where \( l \) is crack length, and in Fig. 1, \( l = c - b = 2a \), \( K_{IIC} \) is material fracture toughness, and \( Y_{II} \) can be obtained from Eq. (1). Because the value of \( K_{II} \) at tip A is larger than those at tips B and C, the \( Y_{II} \) at tip A is applied in Eq. (2).

Considering an infinite plane with three collinear cracks under uniaxial compression as shown in Fig. 2(a), the corresponding \( K_{II} \) (absolute value) can be obtained from Eq. (1) by taking \( \lambda = 0 \)

\[ K_{II} = Y_{II} \sigma_1 \sqrt{\pi l} = \frac{\sigma_1}{2} \sqrt{\frac{\pi a}{(a^2-b^2)(a^2-c^2)}} \]

\[ \{ \lambda(c^2-b^2)-c^2+a^2 \} [\sin 2\alpha - f(1+\cos 2\alpha)] \]

Under the critical condition, its critical uniaxial compressive stress is \( \sigma_{U} \), i.e. \( \sigma_1 = \sigma_{U} \), and \( K_{II} = K_{IIC} \), then from Eq. (3), one can have

\[ \frac{\sigma_u}{2} \sqrt{\frac{\pi a}{(a^2-b^2)(a^2-c^2)}} \{ \lambda(c^2-b^2)-c^2+a^2 \} [\sin 2\alpha - f(1+\cos 2\alpha)] = K_{IIC} \]

**Figure 2.** A specimen containing three inclined cracks under different loads.

When the horizontal stress is applied as shown in Fig. 2(b), its critical stress will be \( \sigma_{V} \), and the corresponding \( K_{II} \) can be obtained from Eq. (1) as

\[ K_{II} = Y_{II} \sigma_1 \sqrt{\pi l} \]

\[ \frac{1}{2} \left[ \lambda(c^2-b^2)-c^2+a^2 \right] \]

\[ \{ (\sigma_1 - \sigma_3) \sin 2\alpha - [\sigma_1 + \sigma_3 + (\sigma_1 - \sigma_3) \cos 2\alpha] f \}

Substituting Eqs. (4) and (5) into Eq. (2), the fracture condition can be rewritten as

\[ \{ (\sigma_1 - \sigma_3) \sin 2\alpha - [\sigma_1 + \sigma_3 + (\sigma_1 - \sigma_3) \cos 2\alpha] f \]

\[ \leq \sigma_1[\sin 2\alpha - f(1+\cos 2\alpha)] \]

From Eq. (6), the fracture condition for three collinear cracks under compression can expressed as

\[ \{ (\sigma_1 - \sigma_3) \sin 2\alpha - [\sigma_1 + \sigma_3 + (\sigma_1 - \sigma_3) \cos 2\alpha] f \]

\[ \leq \sigma_1[\sin 2\alpha - f(1+\cos 2\alpha)] \]
\[ \sigma_1 \leq \sigma_U + \frac{1 + f \cdot \tan \alpha}{1 - f \cdot \cot \alpha} \sigma_3 \] (7)

From Eq. (7), it can be seen that the critical stress \( \sigma_1 \) is related to four parameters, crack surface friction coefficient \( f \), crack inclination angle \( \alpha \) with \( \sigma_3 \)-axis, confining stress \( \sigma_3 \), and uniaxial compressive strength \( \sigma_U \). It should be noted that here \( \sigma_U \) is different from the general material uniaxial compressive strength, i.e. UCS, because the specimens used in measuring \( \sigma_U \) should contain three collinear cracks, whereas for the general material UCS, the specimens don’t contain any macro-cracks. The crack number should be the same as that shown in the target rock investigated. One may deduce that Eq. (7) could be applied for any (finite) number of collinear cracks under compression, and the specimens used in measuring \( \sigma_U \) should contain the same collinear cracks. In some special cases, this crack propagation criterion can be simplified.

1. Crack surface friction free

If the friction between crack surfaces is zero, i.e. \( f = 0 \), then Eq. (7) becomes

\[ \sigma_1 \leq \sigma_U + \sigma_3 \quad \text{or} \quad \sigma_1 - \sigma_3 \leq \sigma_U \] (8)

Eq. (8) is similar to the traditional maximum shear stress failure criterion for materials without cracks.

2. For the most unfavourable cracks

Differentiating Eq. (7) with respect to \( \alpha \) and taking \( K_{II} = 0 \), the most unfavorable crack orientation \( \alpha_m \) can be obtained

\[ \alpha_m = \frac{\pi}{2} - \frac{1}{2} \tan^{-1} \frac{1}{f} \] (9)

It can be seen that the most unfavorable crack orientation is only related to crack friction coefficient \( f \). When \( f = 0 \), \( \alpha_m = 45^\circ \), and when \( f = 1.0 \), \( \alpha_m = 67.5^\circ \).

For the most unfavourable crack, substituting Eq. (9) into Eq. (7), the relation between \( \sigma_1 \) and \( \sigma_3 \) can be rewritten as

\[ \sigma_1 \leq \sigma_U + \left( \sqrt{1 + f^2} + f \right)^2 \sigma_3 \] (10)

It can be seen that for the most unfavourable crack, the critical stress \( \sigma_1 \) is only related to three parameters, confining stress \( \sigma_3 \), uniaxial compressive strength \( \sigma_U \), and crack surface friction coefficient \( f \).

3. Materials without pre-existing macrocracks

Brittle materials contain randomly distributed microcracks. If there is no pre-existing macrocracks, the fracture process will be governed by the most unfavourable microcracks, which will first start to propagate and cause the brittle materials to failure. Therefore, for materials without pre-existing macrocracks under compression, Eq. (10) can be adopted as the crack propagation condition.

For brittle materials without pre-existing macrocracks under compression, the sliding friction between the microcrack surfaces is the same as the internal friction defined by Coulomb [25-26], i.e. \( f = \mu = \tan \phi \), where \( \mu \) is the internal friction coefficient, and \( \phi \) is the angle of internal friction defined by Coulomb. The well-known Coulomb-Mohr failure criterion can be written as

\[ \sigma_i \leq \text{UCS} + \sigma_i \left( \sqrt{1 + \mu^2} + \mu \right)^2 \] (11)

or in another form, \( r \leq \text{UCS} + \sigma \tan \phi \) which is more familiar for readers. For materials without pre-existing macrocracks, evidently \( \sigma_U = \text{UCS} \), then comparing Eq. (11) with Eq. (10), it can be seen that the new fracture condition for materials without pre-existing macrocracks is just the well-known Coulomb-Mohr criterion.

4. Experimental Study

In order to examine the effectiveness of the fracture condition for collinear discrete cracks under compression and investigate the factors that control fracturing, an experimental study was implemented. The specimens were square plates, 180 mm x 180 mm x 30 mm, with two or three artificial and penetrated cracks, which measure 20 mm in length. Two kinds of materials, cement mortar and sandstone, were selected. The specimens were loaded by a triaxial loading device; the vertical loading is the major principal stress \( \sigma_1 \), and the two horizontal confining stresses are kept as constants during the process of vertical loading. One of the horizontal stress is \( \sigma_3 \), and the other one is \( \sigma_2 \) which is kept as 10% larger than \( \sigma_3 \). In order to avoid the effect of the friction between the specimen and the loading device, the specimen surfaces were smeared with oil before testing.

For sandstone specimens, we first drilled a very small hole in the center of the predesigned crack, and then using a very thin steel rope and a burnished thin saw to cut slowly along the crack predesigned. In order to simulate the friction between crack surfaces, we inserted a thin blade inside the cracks, thus under compression, the friction between crack surfaces will occur. The blade surfaces have different roughness in order to study the effect of friction.

For cement mortar specimens, the ratio of cement, sand and water is 1:1.0:0.35 by weight. The cracks were made by using a very thin (0.1 mm) film. The films were placed inside the specimens during the process of casting in molds until they were loaded. The curing period of the specimens is 28 days, and after the specimens have been stored in a heating apparatus with controlled temperature.
for several hours, the films can be pulled out from the cracks.

4.1 Effect of Crack Orientation

In order to investigate the effect of crack orientations, the crack inclined angles with $\sigma_3$-axis were designed as $0^\circ$, $15^\circ$, $30^\circ$, $45^\circ$, $60^\circ$, $75^\circ$ and $90^\circ$. The two horizontal stresses are fixed as $\sigma_2 = 2.5$ MPa, $\sigma_3 = 2.0$ MPa, and we measure the critical vertical stress $\sigma_1$. The fracture patterns are shown in Fig. 3. It can be seen that the fracture patterns vary with crack inclined angles $\alpha$. As $\alpha$ is less than $30^\circ$, the fracturing occurs in almost all the crack tips; and as $\alpha$ is larger than $75^\circ$, several new cracks which are roughly parallel to the pre-existing crack can be observed.

![Figure 3](image1.png)

**Figure 3.** Fracture patterns for three collinear cracks with different inclined angles $\alpha$ under compression

![Figure 4](image2.png)

**Figure 4.** The relation of the fracture stresses versus crack inclined angles.

As stated in section 2, for three collinear cracks under compression, the middle crack tip A shown in Fig. 1 has the largest mode II SIF value, but the difference between the values at crack tip A and C is slight, and for a finite specimen, the boundary effect are also significant, and therefore, new cracks initiating from crack tip C and extending to the loading boundary of $\sigma_1$ can be observed frequently.

Fig. 4 shows the relationship between the fracture stress $\sigma_1$ and crack inclined angles from the test results and from the prediction results by Eq. (7). It can be seen that as the inclined angle is in the range between $40^\circ$ and $60^\circ$, the corresponding critical stress is low. This is to be expected as in this range, the shear stress along the crack surface is large. The test results generally agree with the prediction result from Eq. (7).

4.2 Effect of the Distance between Two Cracks

Cracks may affect each other, but as their distance is larger than a certain value, the effect could be ignored. In this study, only the case of two collinear cracks is investigated. Fig. 5 shows the relationship between the fracture stress $\sigma_1$ and crack tip distance $d$. It can be seen that the testing results by using sandstone specimens with two collinear cracks generally agree with the prediction results from Eq. (7). The fracture stress $\sigma_1$ increases as crack tip distances increase, which means that the larger the distance between two cracks, the less the effect between them. Fig. 6 shows the test results for two sandstone specimens with crack tip distances 1.0 cm and 5.0 cm, respectively. It can be seen that for the specimen with 1.0 cm crack tip distance, the two pre-existing cracks are coalesced, whereas for the specimen with 5.0 cm crack tip distance, the two cracks are not connected, and they propagate independently. This indicates that as the distance between two cracks is larger than a certain value, the effect between them could be ignored.

![Figure 5](image3.png)

**Figure 5.** The relationship between the fracture stress and the...
distance between two crack tips.

The calculation result from Eq. (1) shows that as the ratio of crack tip distance to crack length is 0.5, the SIF increases by 16.73% as compared with the corresponding single crack’s SIF value, and as the ratio is 3.0, it increases by 1.63%, and as the ratio is 5.0, it increases by 0.69%. In rock engineering, 2% error is acceptable, and therefore, as crack tip distance is larger than 3 times crack length, the effect from its adjacent cracks is ignorable.

Figure 6. The test results for sandstone specimens with 1 cm and 5 cm crack tip distances.

4.3 Effect of Confining Stresses

The effect of confining stress on crack stability are investigated by using three collinear crack specimens, and the test results as well as the prediction results from Eq. (7) are shown in Fig. 7. The crack inclined angle is 60°, and the crack length is 2.0 cm. One can find that the fracture stress $\sigma_1$ increases linearly with the confining stress $\sigma_3$, and the test results agree with the fracture criterion in Eq. (7) for collinear discrete cracks under compression. This indicates that the fracture condition for collinear discrete cracks under compression is reliable.

Figure 7. The relationship between the fracture stress $\sigma_1$ and the confining stress $\sigma_3$.

Conclusions

(1) The fracture condition of three collinear discrete cracks under compression, i.e. Eq. (7), has been developed in this paper, which could be applicable for any (finite) number of collinear cracks under compression. When apply this fracture criterion, the uniaxial compressive strength $\sigma_U$ should be measured by using the specimens containing collinear cracks.

(2) Crack orientation affect crack stability, as the angle with $\sigma_3$-axis is the most unfavorable, which should be between 45° and 67.5°, the crack stability is very low.

(3) As the distance between two collinear cracks decreases, the effect between them increases, and accordingly the crack tip stress intensity factor increases. As crack tip distance is larger than 3 times crack length, the effect from its adjacent cracks is negligible (increases by 1.63% for three collinear cracks).

(4) For the case of brittle materials without microcracks, the fracture condition becomes the well-known Coulomb-Mohr criterion.

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References


