An Interval-based Method for Calculating Performance Variation of Reflector Antenna

Bo ZHAO, Hong-bo MA*, Wei GAO, Xiao-huan WANG and Li-wei SONG

Research Center of Intelligent Manufacturing & Industrial Big Data, School of Mechano-Electronic Engineering, Xidian University, Xi’an 710071, P.R. China

*Corresponding author

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Abstract. This letter presents a non-probabilistic interval-based calculation method for antenna performance evaluation. After the interval function of antenna far field pattern is given, the interval calculation process for complex exponential is deduced. Then the power pattern function affected by distortion errors is derived in accordance with interval operation rules. The method is applied to an 8-meter antenna as a numerical example.

Introduction

Due to the inevitable errors of antennas structural parameters in manufacturing and assembly, an antenna always works in a distorted condition deviated from its nominal condition. These errors will deteriorate antennas’ performance. During the design phase, the traditional methods employ the worst condition (e.g. the largest structural errors) to validate the antenna designs. However, these methods may not obtain the worst performance of an antenna because the largest structural errors do not always correspond to the worst performance. There are also some works predicting antenna’s performance by treating structural errors as uncertain quantities that follow a few certain probabilistic distributions. For example, Refs. [1, 2] studied the effect of surface uniform and heterogeneous errors with the Normal distribution on power pattern function, and gave the relation between heterogeneous random errors and axial gain loss. Ref. [3] analyzed the effect of surface errors following the Gaussian distribution on average power pattern with numerical algorithm. Refs. [4,5,6] proposed methods to analyze the effect of system errors with the Gaussian distribution on reflector antenna performance. Ref. [7,8,9] studied the effects of surface errors following the chi-square-Rayleigh distribution on degradation of antenna side lobes.

Considering that the probabilistic distributions of the uncertain quantities are unavailable in many cases, this letter proposes an interval-based calculation method to determine the variation range of antenna performance by treating antenna structural parameters as interval quantities.

Interval-based Far Field Pattern Involving the Phase Error Intervals

For a reflector antenna, the far field pattern function is given as follows [5]:

$$ E(\theta, \phi) = \int \int_{\Omega} E_0(\rho', \phi') \cdot e^{j kp' \sin \theta \cos(\phi - \phi')} \rho' d\rho' d\phi' $$

(1)

where $E_0(\rho', \phi')$ is the field distribution function of the aperture plane $\Omega$. If the influence of the errors of the aperture plane on the phase distribution is considered, Eq. 1 can be rewritten as follows:

$$ E(\theta, \phi) = \int \int_{\Omega} E_0(\rho', \phi') \cdot e^{j kp' \sin \theta \cos(\phi - \phi')} \rho' d\rho' d\phi'. $$

(2)

where $\Delta\phi$ is the phase errors caused by antenna surface errors, which can be obtained as follows:
\[ \Delta \phi = \frac{4\pi}{\lambda} \Delta z \cos^2 \left( \frac{\xi}{2} \right). \]  

(3)

where \( \lambda \) is the working wavelength of the antenna, \( \xi \) is the radiation angle, and \( \Delta z \) is the antenna surface axial errors that can be obtained by finite element method (FEM). If the structural parameters are considered as interval quantities, \( \Delta z \) and \( \Delta \phi \) will be transformed into the interval quantities as \( \Delta z' \) and \( \Delta \phi' \), respectively. (The superscript ‘I’ indicates the parameter is an interval quantity). Then Eq. 2 can be written as follows:

\[ E^I(\theta, \phi) = \left[ \int_{\Omega} E_0 \left( \rho', \phi' \right) \cdot e^{j \Delta \phi'} e^{j \rho' \sin \theta \cos(\phi - \phi')} \rho' d\rho' d\phi' \right]. \]  

(4)

Figure 1. Mesh generation of reflector aperture \( \Omega \).

According to the numerical integration method [6], we divide the aperture plane into \( N \) annular regions, each of which is further divided into \( M \) zones (as shown in Fig. 1), to numerically solve the integral in Eq. 4. Then the Eq. 4 can be expressed as follows:

\[ E^I(\theta, \phi) = \sum_{n=1}^{N} \sum_{i=1}^{M} E_{r(n,i)}^I \cdot E_{r(n,i)}^I. \]  

(5)

where \( \Delta \phi'_{(n,i)} \) is the interval phase error of the \( i \) th zone in the \( n \) th region, \( E_{r(n,i)}^I = e^{\frac{j 4\pi}{\lambda} \Delta z \cos^2(\xi/2)} \) and \( E_{r(n,i)} = \int_{\Omega(n,i)} E_0 \left( \rho', \phi' \right) \cdot e^{j \rho' \sin \theta \cos(\phi - \phi')} \rho' d\rho' d\phi' \).

Here Eq. 5 can be considered as the product of \( E_{r(n,i)}^I \) and \( E_{r(n,i)}^I \), \( E_{r(n,i)}^I \) is an interval term and \( E_{r(n,i)} \) is a deterministic term. The computation method of Eq. 5 should be constructed.

**Interval-based Power Pattern**

As the interval part, \( E_{r(n,i)}^I \) in Eq. 5 can be expressed in the trigonometric form as

\[ E_{r(n,i)}^I = \cos \Delta \phi'_{(n,i)} + i \sin \Delta \phi'_{(n,i)} = A_{r(n,i)} + i B_{r(n,i)} = \left[ A_{r(n,i)}, \overline{A_{r(n,i)}} \right] + j \left[ B_{r(n,i)}, \overline{B_{r(n,i)}} \right]. \]  

(6)

where \( A_{r(n,i)} \) and \( B_{r(n,i)} \) can be considered as the results of \( \cos \Delta \phi'_{(n,i)} \) and \( \sin \Delta \phi'_{(n,i)} \). It can be expression as follows: [10, 11]

\[ A_{r(n,i)} = \min \left( \cos \Delta \phi'_{(n,i)} \right), \overline{A_{r(n,i)}} = \max \left( \cos \Delta \phi'_{(n,i)} \right). \]  

(7)

\[ B_{r(n,i)} = \min \left( \sin \Delta \phi'_{(n,i)} \right), \overline{B_{r(n,i)}} = \max \left( \sin \Delta \phi'_{(n,i)} \right). \]  

(8)

So, \( E_{r(n,i)}^I \) can be written in the form of an interval as follows:
As a deterministic part of Eq. 5, $E_{(n,j)}$ can be expressed as follows:

$$E_{(n,j)} = A_{(n,j)} + jB_{(n,j)},$$

where $A_{(n,j)}$ and $B_{(n,j)}$ are real quantities.

According to the interval operation rules [12,13,14], substituting Eq. 9 and Eq. 10 into Eq. 5 yields.

$$E^i(\theta,\phi) = \sum_{n=1}^{N} \sum_{i=1}^{M} (A_{(n,j)} + jB_{(n,j)}) \cdot \left[ A_{(n,j)} + jB_{(n,j)}, A_{(n,j)} + jB_{(n,j)} \right]$$

$$= \sum_{n=1}^{N} \sum_{i=1}^{M} \left[ (A_{(n,j)} + jB_{(n,j)}) \cdot (A_{(n,j)} + jB_{(n,j)}) \cdot (A_{(n,j)} + jB_{(n,j)}) \right]$$

$$= \left[ E, \bar{E} \right].$$

where $E = \sum_{n=1}^{N} \sum_{i=1}^{M} (A_{(n,j)} + jB_{(n,j)})$, $\bar{E} = \sum_{n=1}^{N} \sum_{i=1}^{M} (A_{(n,j)} + jB_{(n,j)})$. $E$ and $\bar{E}$ are the lower and upper bounds of $E(\theta,\phi)^i$ in complex exponential form, respectively.

In order to achieve the boundaries of interval based power pattern $p(\theta,\phi)^i$, we take the smaller modulus value of $|E|$ and $\bar{E}$ as the lower bound of $|E(\theta,\phi)^i|$ and the larger one as the upper bound. It is obvious that the range of the power pattern can be determined as follows:

$$p(\theta,\phi)^i = 10 \log p(\theta,\phi)^i = 10 \log \left[ \frac{|E(\theta,\phi)^i|^2}{|E_M|^2} \right] = \left[ 20 \log \frac{\min(\{E\}, \bar{E})}{E_M}, 20 \log \frac{\max(\{E\}, \bar{E})}{E_M} \right].$$

Now the upper and lower bounds of the power pattern can be obtained. The range could involve all the power patterns affected by different surface distortions.

**Numerical Example**

The method is applied to a reflector antenna with uncertain-but-bounded structural parameters to demonstrate its effectiveness.

Fig. 2 shows the finite element model of a reflector antenna structure with 8m aperture and 3.2m focal length. It works at 15GHz and its edge taper is -10dB. The amplitude aperture distribution can be given as

![Figure 2. The 8m-diameter reflector antenna.](image)
\[ E(\rho') = B + C(1 - \rho'^2 / a^2)^2. \]  \hspace{1cm} (13)

Where \( B \) and \( C \) are both constants. \( a \) is the radius of the aperture plane.

The uncertain parameters of antenna include the elastic modulus of steel used for the backup frame \( E_1 = [1.785, 2.415] \times 10^{11} \text{ N/m}^2 \). The elastic modulus of aluminum for the surface panels \( E_2 = [0.595, 0.805] \times 10^{11} \text{ N/m}^2 \). The thickness of the surface panels \( T = [1.7, 2.3] \times 10^{-3} \text{ m} \). The cross-sectional areas \( A_i \sim A_2 \) of the backup frame are shown in Fig. 3 and listed in Table 1. The uncertain parameters would lead to the variation range of surface distortions.

![Figure 3. Backup frame of the 8m-diameter reflector antenna.](image)

<table>
<thead>
<tr>
<th>( A_1 )</th>
<th>( A_2 )</th>
<th>( A_3 )</th>
<th>( A_4 )</th>
<th>( A_5 )</th>
<th>( A_6 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>[2552.5,3453.4]</td>
<td>[1700,2300]</td>
<td>[1275,1725]</td>
<td>[2550,3450]</td>
<td>[1700,2300]</td>
<td>[1275,1725]</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( A_7 )</th>
<th>( A_8 )</th>
<th>( A_9 )</th>
<th>( A_{10} )</th>
<th>( A_{11} )</th>
<th>( A_{12} )</th>
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</thead>
<tbody>
<tr>
<td>[850,1150]</td>
<td>[850,1150]</td>
<td>[1275,1725]</td>
<td>[1275,1725]</td>
<td>[2550,3450]</td>
<td>[850,1150]</td>
</tr>
</tbody>
</table>

The interval-based power pattern curves under the impact of interval surface errors are shown in Fig. 4. Fig. 5 shows the enlarged view of the central part of Fig. 4. As shown in the figures, due to the consideration of surface errors with interval forms, the power pattern has its lower and upper bounds. The main beam of the distorted power pattern became wider and the lower and upper bounds of the gain were lower than the undistorted power pattern curve. The lower and upper bounds of sidelobe levels lied under the nominal one, and the fluctuation amplitude and variation ranges of sidelobe levels at the far zones became greater. From Table 2, the interval values of gain and sidelobe levels in Table 2 could indicate the same conclusions. The gain loss was from 0.36dB to 0.87dB, which was a rather large loss for the antenna performance. The influences of interval errors on the first left sidelobe were about 0.3dB greater than those on the first right sidelobe.

![Figure 4. The power pattern for undistortion and distorted reflector.](image)

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Table 2. The effect of surface errors on gain and sidelobe.

<table>
<thead>
<tr>
<th></th>
<th>Nominal [dB]</th>
<th>Distorted [dB]</th>
<th>Sidelobe Increase [dB]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gain Loss</td>
<td>0</td>
<td>[-0.87,-0.36]</td>
<td></td>
</tr>
<tr>
<td>Left Sidelobe</td>
<td>-27.10</td>
<td>[-26.91,-26.10]</td>
<td>[0.19,1.00]</td>
</tr>
<tr>
<td>Right Sidelobe</td>
<td>-27.10</td>
<td>[-27.03,-25.99]</td>
<td>[0.07,1.11]</td>
</tr>
</tbody>
</table>

To validate the effectiveness of this method, the Monte Carlo method was introduced. The elastic modulus $E_1$, $E_2$, the thickness of the surface panels $T$ and the cross-sectional areas $A_1$ $A_2$ were considered as the initial uncertain parameters and had the same intervals as shown in the previous part. Thus 1000 examples were generated by uniformed random numbers. According to the examples, 1000 power pattern curves were obtained by conventional GO method. Among all the pattern curves, 100 curves were selected randomly and shown in Fig. 6. The figure shows that all pattern curves are located in the expected pattern interval.

The numerical example suggests that when certain sets of structural tolerance given by antenna designers are considered as interval-based structural errors, the interval-based antenna performance can be achieved. If the performance doesn’t satisfy the requirement, designers can modify the design according to the proposed method.

Conclusions

In this letter, an interval-based computation method for rotationally symmetric reflector antenna performance evaluation was proposed. The interval-based relationship between the surface errors and the far field pattern was given. The lower and upper bounds of power pattern can be obtained by
applying interval operation rules. The performance variation range of an 8m antenna was calculated to demonstrate the effectiveness of the proposed method. The proposed method can be used as an alternative of probabilistic methods when the distributions of the structural parameters cannot be achieved. Future work will be focused on investigating the different effects on antenna performance caused by different interval-based structural parameters. The method can provide a reference for reflector antenna designers to achieve the optimized design.

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Reference