Dynamic Characteristic Analysis of a Power Turbine Rotor System
Based on Finite Element Method

Yi DING, Gui-huo LUO* and Peng ZHANG
Nanjing University of Aeronautics and Astronautics, Jiangsu Key Laboratory of Aero Power System,
College of Energy and Power Engineering, Nanjing 210016, China
*Corresponding author

Keywords: Rotor system, Dynamic characteristics, Power turbine, Finite element method.

Abstract. This paper describes a series of work done to study the dynamic characteristics of a power turbine elastic-supported rotor system. Based on finite element method, the stiffness and damping of the rotor system were calculated, the critical speed and the unbalance response were analyzed and assessed. Through the numerical simulation, the design was quantitatively verified. This paper provides an effective methodology and process as theoretical reference for the verification of the aero-engine vibration safety.

Introduction

The vibration of the rotor system in an aero-engine has essential influence on the entire engine, therefore it is necessary and vital to study the dynamic characteristics of the rotor system’s vibration. Thanks to the development and the maturity of the finite element method, this kind of numerical computation and simulation has been widely applied, especially in the rotor system vibration domain. There is massive work done by other researchers based on the finite element method. The dynamic characteristics of a centrifugal impeller gas generator was calculated and analyzed in terms of critical speed and unbalance response based on finite element computation, and it provides a preliminary process to the verification of rotor system vibration safety [1]. During the study of the dynamic compartment of a two-spool rotor system, the impact of the rotational speed, number of intermediate bearing rollers and bearing parameters based on finite element method were discussed and compared with the experimental results in order to validate the method [2].

In this paper, the study object is a power turbine rotor system. The support stiffness and damping were analyzed. The critical speeds and the unbalance response of the rotor system were calculated and discussed. Finally, the design was assessed compared to the requirement, and the vibration safety of the rotor system was verified.

Modelling of the Rotor System

As illustrated in Figure 1, the elastic-supported rotor system consists of a power turbine fixed with the shaft, support I and support II. Support I comprises an angular contact ball bearing, a squirrel cage and a squeeze film damper with a bearing pedestal. Support II also contains an angular contact ball bearing and with the same bearing pedestal.

![Figure 1. Simplified diagram of the turbine rotor system.](image-url)
Bearing

In rotor dynamics and vibration, external radial load has essential influence on the stiffness of the bearing. As a result, one should always analyze the bearing loads of the system quantitatively at the first place before launching the stiffness computation.

The Bearing load distribution of the rotor system in this study is shown in Figure 1, where $A_0$ is the imbalanced force due to the upper compressor, $A_1$ is the weight of the turbine disk and $A_2$ is that of the turbine blades. Quantitatively, $A_1=32.40N$, $A_2=10.03N$, $A_0=\omega^2me$, where $\omega$ is the rotational speed of the compressor and $me=3\times10^{-3}Kg\cdot m$, the imbalanced value. The bearing damping was considered as consistent value, so the radial load was acquired by the rotational speed of 4400rad/s, middle point of the working speed range.

Afterwards, the stiffness of angular contact ball bearing was calculated and obtained by applying the radial load and bearings’ geometric parameters to Eq. 1.

$$K_b = 0.126096 \times 10^4(F_\| d^2 \omega^2 \sin^2 \beta)$$  \hspace{1cm} (1)

Where $d$ is the diameter of the roller [mm], $n$ is the number of roller, $\beta$ is the contact angle and $F_\|$ represents the radial load [N].

The stiffness of bearings in support I and support II are shown respectively in Table 1.

<table>
<thead>
<tr>
<th>Bearing location</th>
<th>Bearing type</th>
<th>Bearing stiffness [N/m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Support I</td>
<td>Angular contact ball bearing</td>
<td>9.9x10^6</td>
</tr>
<tr>
<td>Support II</td>
<td>Angular contact ball bearing</td>
<td>5.8x10^8</td>
</tr>
</tbody>
</table>

Squirrel Cage in Support I

The squirrel cage located in the support I dominates the support’s stiffness. By applying the geometric parameters of the squirrel cage to Eq. 2, the stiffness of the squirrel cage was obtained.

$$K_{sc} = nEb^2h^2L^{-3}$$ \hspace{1cm} (2)

Where $n$ is the number of ribs, $E$ is elastic modulus, $b$ is the width of the rib cross section, $h$ is the height of the rib cross section. $L$ is the length of the ribs.

In this study, the elastic modulus of the material is 210Gpa. Finally, the squirrel cage’s stiffness was calculated as $K_{sc}=2.35\times10^6N/m$.

Bearing Pedestal

Figure 2 shows the finite element model of the two bearing pedestals. During the numerical computation, the install side of the bearing pedestal was constrained as boundary condition. In addition, a central node and a rigid region were created where the bearing was located.

(a) In support I (b) In support II

Figure 2. Finite element models of bearing pedestal.

By imposing a unit force on the central node, its displacement was obtained. In our case, the displacement of the bearing pedestal in support I is $X_{p,1}=0.29113\times10^{-4}m$ and that of the support II is $X_{p,11}=0.75478\times10^{-4}m$. Afterwards the stiffness was calculated by Hooke law shown in Eq. 3 as $K_{p,1}=3.4\times10^8N/m$ and $K_{p,11}=1.32\times10^8N/m$. 

46
\[ K_p = f \cdot X_p^{-2} \]  \hspace{1cm} (3)

**Entire Support**

The entire support stiffness of the rotor system is constructed by all of the components, as the bearing, the squirrel cage and the bearing pedestal. In this paper the stiffness of the support were considered as the combination of two springs in series. Thus the support stiffness can be obtained by Eq. 4.

\[ K_s = \frac{K_b \cdot K_p}{K_b + K_p} \]  \hspace{1cm} (4)

Where \( K_s \) is the support stiffness, \( K_b \) is the bearing stiffness and \( K_p \) is the squirrel cage stiffness for support I and the bearing pedestal stiffness for support II.

According to the results of section 2.1 to 2.3, the entire support stiffness of the supports were respectively calculated and shown in Table 2.

<table>
<thead>
<tr>
<th>Location</th>
<th>Bearing stiffness ([\text{N/m}])</th>
<th>Bearing pedestal/Squirrel cage stiffness ([\text{N/m}])</th>
<th>Support stiffness ([\text{N/m}])</th>
</tr>
</thead>
<tbody>
<tr>
<td>Support I</td>
<td>9.9x10^6</td>
<td>2.4x10^6</td>
<td>1.9x10^6</td>
</tr>
<tr>
<td>Support II</td>
<td>5.8x10^8</td>
<td>1.3x10^8</td>
<td>1.1x10^8</td>
</tr>
</tbody>
</table>

**Squeeze Film Damper (SFD)**

The SFD brings additional stiffness and damping to the rotor system. In this rotor-support system, only the film damping was taken into account, since that it is much stronger than the other damping. The short bearing assumption and the \( \pi \)-film boundary condition were applied, and the equivalent damping can be obtained by Eq. 5:

\[ C_0 = \mu R L^2 e^{-3} \cdot \frac{\pi}{2} \left(1 - \varepsilon^2\right)^{\frac{3}{2}} \]  \hspace{1cm} (5)

Where \( \mu \) is the oil film dynamic viscosity, \( L \) is the film width, \( R \) represents oil film ring radius, \( e \) represents the oil film clearance radius, and \( \varepsilon \) represents the eccentricity epsilon.

By applying the SFD’s parameters to Eq. 5, the damping of the SFD in this rotor system is \( C_0 = 304.3 \text{N} \cdot \text{s/m} \).

**Dynamic Characteristics of the Rotor System**

Based on the acquired results, the finite element model of the rotor system was established in software ANSYS. From the actual rotor shown in Figure 3, the model was equivalently simplified shown in Figure 4, in order to facilitate the simulation.

![Figure 3. Actual rotor model.](image)

![Figure 4. Simplified rotor model.](image)

During the modeling, the turbine disk was shaped as cylindrical with a diameter limited to the mortise bottom of the disk, and the blades with the left part of the disk were re-added as equivalent
mass and rotational inertia. A rigid surface was created to couple the degree of freedom of central
element and disk edge nodes in order to reduce the computation complexity [3].

The support was represented by COMBI214 elastic element. Element SOLID95 was applied to
mesh the model and divided it into 26005 elements and 29813nodes, as shown in Figure 5.

![Finite element model of the rotor.](image)

**Critical Speed**

The critical speed was calculated and analyzed based on the finite element model presented above.
The actual working rotational speed range was within the range applied to the finite element simulation in order to take into consideration all of the provided working conditions. The Campbell
diagram of the rotor system was obtained and illustrated in Figure 6, and the critical speeds of the first
two orders are shown in Table 3.

![Campbell diagram of the rotor system.](image)

It is strictly required, in aero-engine design criterion, that the critical speed must be higher than the
highest working speed with at least 20% safety margin, and lower than the engine’s idling speed with
at least 20% safety margin [4].

<table>
<thead>
<tr>
<th>Order</th>
<th>Critical speed frequency [Hz]</th>
<th>Critical speed [r/min]</th>
</tr>
</thead>
<tbody>
<tr>
<td>First</td>
<td>64</td>
<td>3840</td>
</tr>
<tr>
<td>Second</td>
<td>1150</td>
<td>69000</td>
</tr>
</tbody>
</table>

According to Table 3, the critical speed of the first order 3840 r/min is largely lower than the
minimum stable rotational speed 35381r/min, with a safety margin of 89.1%. For the second order,
the obtained critical speed 69000r/min is higher than the maximal stable rotational speed 55051r/min
with a safety margin of 25.3%. As conclusion, the safety margins of the numerically simulated critical
speeds of this rotor system theoretically satisfy the aero-engine design criteria, with no harmful
resonance. And it may serve as a theoretical reference for the verification of aero-engine’s vibration
airworthiness.
Unbalance response

In order to study the unbalance response of the rotor system, an unbalanced force of 3gcm was imposed at the center of the turbine disk finite element model. As the critical speed of the rotor system is out of the working rotational speed range, from 35381r/min to 47305r/min (590Hz to 788Hz), only at the position of the two extremities can the peak amplitude occur. Regarding the finite element simulation, it turned out that the relative peak amplitudes laid on the outer edge of the turbine disk and the back edge of the shaft.

The displacement evolution by the rotational speed of these two interesting positions was done and illustrated in the Figure 7, and Tables 4 shows more details under several main working conditions.

![Figure 7. Radial displacement evolution by the rotational speed.](image)

Table 4. Response amplitude under main working conditions.

<table>
<thead>
<tr>
<th>Main working condition</th>
<th>Rotational speed [r/min]</th>
<th>Frequency [Hz]</th>
<th>Displacement amplitude [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Disk outer edge</td>
</tr>
<tr>
<td>Lowest stable speed</td>
<td>35381</td>
<td>589.7</td>
<td>1.06E-05</td>
</tr>
<tr>
<td>Rated speed</td>
<td>39095</td>
<td>651.6</td>
<td>1.00E-05</td>
</tr>
<tr>
<td>Highest stable speed</td>
<td>42418</td>
<td>706.8</td>
<td>9.19E-06</td>
</tr>
</tbody>
</table>

In Figure 7, the two radial displacements have opposite tendencies. The maximum amplitude of the turbine disk’s outer edge is 1.06×10⁻⁵m and the peak displacement of the shaft’s back edge is 1.02×10⁻⁵m, and both of them are under the clearance between the turbine rotor and the casing which is 9×10⁻⁴m. As conclusion, the imbalance would not generate excessive vibration in the working speed range, and the unbalance response of the rotor system is effectively safe.

Conclusions

By numerical computation based on finite element method, the rotor system would not generate excessive vibration in the working speed range, and the imbalance would not occur a rub neither. Therefore, the critical speed and unbalance response effectively satisfied the design requirement in the aspect of vibration safety. The whole work and methodology may serve as theoretical reference for vibration safety evaluation and aero-engine rotor system vibration airworthiness assessment.

Reference

