Analytical Formulae and Algorithm for Substation Optimization Planning with Interruption Cost Being Considered

Yu-sheng HU, Man ZHANG, Jin-qian ZHAI, Zhu-ding WANG, Xiao TAN

ABSTRACT

Substation optimization planning is the key to avoid wasteful investment. In the absence of detailed data, new analytical formulae are presented for the substation optimization planning with the customer interruption cost being considered, based on the assumption of uniform load distribution. In the simplified models of analytical formulae, the total cost is represented by a function of substation number, and the line cost is calculated according to the estimated number and power supply radius of feeders. Based on the extreme value method, an algorithm is proposed for the single-stage substation optimization planning. Through the application of proposed models and algorithm to a real distribution network, it is shown that the larger the proportion of cable circuits is or the higher the customer interruption cost is, the greater the optimal number of substations is and the smaller the substation capacity is, and larger load density corresponds to bigger substation capacity and number.

INTRODUCTION

Substation optimization planning is to obtain the planning scheme of correctness and scientificalness, and much work of data preparation and input is required for complicated models and algorithms.\textsuperscript{1-3} In the absence of data (such as those of detailed load distribution), substation optimization planning may be performed under some simplified conditions. Therefore, it is important to develop simplified models and algorithms which can satisfy a certain calculation precision based on small amount of related data.

Currently, there are some related models and algorithms for the approximate estimation of substation number, capacity and power supply radius,\textsuperscript{4-10} in which the substation capacity is regarded as a decision variable. In reference 6, the interruption cost is taken into account in the objective function of annual total cost with a separated reliability evaluation model being employed.

In this paper, based on the assumption of uniform load distribution, new simplified models are presented for substation optimization planning, resulting in the new analytical formulae of optimal substation number and capacity with the interruption cost being considered. Also, a simple algorithm is proposed for single stage substation optimization planning.

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planning. The validity and effectiveness of the proposed models and algorithm is demonstrated by applying them to a real distribution network.

OPTIMIZATION MODELS

Objective Function

In this paper, the objective function is the annual total cost $C_N$ of substations and feeders. With the substation capacity-load ratio (i.e., the ratio of total transformer capacity to total active power demand) and the maximum allowable supply radius ($R_{X_{MAX}}$) being satisfied, the objective function can be represented by

$$\min \; C_N = C_B + C_X = (C_{BT} + C_{BK} + C_{BF}) + (C_{XT} + C_{XS} + C_{XK})$$

Where, $C_B$ is the annual cost of substations, which includes the corresponding investment cost $C_{BT}$, transformer no-loading loss cost $C_{BK}$ and loading loss cost $C_{BF}$; $C_X$ is the annual cost of feeders, which includes the corresponding investment $C_{XT}$, line energy loss cost $C_{XS}$ and line customer interruption cost $C_{XK}$.

Annual Cost of Substations

Based on the capacity-related investment statistics of substations, a linear fitting curve is used to represent the investment cost of substation $i$ as follows.

$$C_{BT,i} = a_B + b_B S_{B,i}$$

Where, $C_{BT,i}$ and $S_{B,i}$ are respectively the investment cost and capacity of substation $i$, $a_B$ and $b_B$ are the coefficients of formula (2).

Considering the annual operation and maintenance rate $\alpha$, annual asset profit rate $\beta$ and equipment depreciation rate $\gamma$, the annual investment cost $C_{BT}$ of all substations can be calculated by

$$C_{BT} = \varepsilon \sum_{i=1}^{N_B} C_{BT,i} = B_{BT} N_B + B'_{BT}$$

\[\varepsilon = \alpha + \beta + \gamma\]

$$B_{BT} = \varepsilon a_B$$

$$B'_{BT} = \varepsilon b_B \left\{ \sum_{i=1}^{N_B} S_{B,i} \right\} = \varepsilon b_B \left( K_S P_Z \right)$$

Where, $N_B$ is the number of substations, $P_Z$ is the total system power demand, $K_S$ is the substation capacity-load ratio.

Based on related loss statistics of transformers, a linear fitting curve is used to formulate the no-loading loss of transformer $j$ as follows.

$$P_{K,j} = a_K + b_K S_{Z,j}$$

Where, $P_{K,j}$ and $S_{Z,j}$ are respectively the no-loading loss and capacity of transformer $j$, $a_K$ and $b_K$ are the coefficients of formula (4).

The annual no-loading loss cost $C_{BK}$ of all transformers is calculated by
\[
C_{\text{BF}} = C_e \sum_{i=1}^{N_z} \left( a_{k,i} + b_{s,i} S_{Z,i} \right) T_B = C_e \sum_{i=1}^{N_z} \left( n_i a_{k,i} + b_{s,i} S_{B,i} \right) T_B = B_{\text{BF}} N_B + B_{\text{BF}}',
\]
\[
B_{\text{BF}} = C_e N_z a_{k,i} T_B, \quad B_{\text{BF}}' = C_e b_{s,i} \left( P_z K_s \right) T_B
\]
Where, \( n_z \) is the number of transformers for a substation; \( T_B \) is the annual service hours of a transformer; \( C_e \) is the cost per unit energy loss.

Also, a linear fitting curve is used to represent the loading loss of transformer \( j \) as follows.
\[
P_{F,j} = a_F + b_F S_{Z,j}
\]
Where, \( P_{F,j} \) is the loading loss of transformer \( j \), \( a_F \) and \( b_F \) are the coefficients of formula (6).

The annual loading loss cost \( C_{\text{BF}} \) of all transformers is calculated by
\[
C_{\text{BF}} = C_e \sum_{i=1}^{N_z} \left( \frac{S_{F,i}}{S_{B,i}} \right)^2 \left( n_z a_{F,i} + b_{F,i} S_{B,i} \right) \tau_{\text{max}} = B_{\text{BF}} N_B + B_{\text{BF}}'
\]
\[
B_{\text{BF}} = \frac{C_e n_z a_{F,i}}{\left( K_s \cos \theta \right)^2} \tau_{\text{max}}, \quad B_{\text{BF}}' = \frac{C_e b_{F,i} P_z}{K_s \cos^2 \theta} \tau_{\text{max}}
\]
Where, \( S_{F,i} \) is the maximum power demand of substation \( i \); \( \tau_{\text{max}} \) is the maximum power loss hours; \( \cos \theta \) is the power factor.

With formulae (3), (5) and (7) being combined, the annual cost of substations is given by
\[
C_B = B_{\text{BT}} N_B + B_{\text{BF}}'
\]
Where, \( B_B = B_{\text{BT}} + B_{\text{BK}} + B_{\text{BF}}, \quad B_B' = B_{\text{BT}}' + B_{\text{BK}}' + B_{\text{BF}}'\).

**Annual Cost of Feeders**

The annual investment cost of feeders can be represented by
\[
C_{\text{XT}} = \varepsilon n_i L_x \left( k_{A} C_D + k_{J} C_J \right)
\]
Where, \( k_{J} \) and \( k_{D} \) are respectively the proportions of aerial lines and cable circuits; \( C_{J} \) and \( C_{D} \) are respectively the investment costs per unit length of aerial lines and cable circuits; \( L_x \) is the average length of a feeder; \( n_i \) is the total number of feeders, which is computed by
\[
n_i = \text{int} \left\{ \frac{P_{D}}{P_{D}} \right\} + 1
\]
Where, \( P_{D} \) is the preset power demand (such as the economic power demand) for a feeder.

The power supply radius \( R_x \) and average length \( L_x \) of a feeder can be calculated by
\[
R_x = \frac{A_z}{N_{a}} K_Z, \quad L_x = \frac{A_z}{N_{a} \pi} K_Z
\]
Where, \( A_z \) is the total area of a power supply region; \( K_Z \) is the correction factor of line length, which generally is 2.0.

With formulae (9), (10) and (11) being combined, the annual investment cost of feeders is represented by
\[ C_{XT} = \frac{B_{XT}}{\sqrt{N_B}} \]  

Where, \( B_{XT} = \varepsilon n_k k_z \sqrt{\frac{A_T}{\pi}} (k_D C_D + k_J C_J) \).

The annual energy loss cost of feeders can be calculated by

\[ C_{XS} = n_j G_p \Delta P_{\text{max}} \tau_{\text{max}} C_e \]  

(13)

Where, \( G_p \) is the power loss factor which is related to a load distribution mode, \( \Delta P_{\text{max}} \) is the maximum power loss of a feeder on the assumption that the feeder’s power demand is regarded as a single concentrated load at its end.

\[ \Delta P_{\text{max}} = \frac{(P_z / (n_z \cos \theta))^2}{U_N^2} (k_D r_D + k_J r_J) L_x \]  

(14)

Where, \( r_j \) and \( r_D \) are the resistances per unit length for aerial lines and cable circuits respectively, \( U_N \) is the rated voltage of feeders.

With formulae (13) and (14) being combined, \( C_{XS} \) can be represented by

\[ C_{XS} = \frac{B_{XS}}{\sqrt{N_B}} \]  

(15)

Where, \( B_{XS} = G_p \frac{(P_z / (n_z \cos \theta))^2}{U_N^2} (k_D r_D + k_J r_J) K_Z \sqrt{\frac{A_T}{\pi}} \tau_{\text{max}} C_e \).

In this paper, the annual customer interruption cost \( C_{XX} \) can be expressed by

\[ C_{XX} = P_z \xi (k_D \text{SAIDI}_D + k_J \text{SAIDI}_J) C_E \]  

(16)

Where, \( \xi \) is the line load rate (i.e., the ratio of average power demand to maximum one), \( \text{SAIDI}_J \) and \( \text{SAIDI}_D \) are the System Average Interruption Duration Indexes of aerial lines and cable circuits respectively, \( C_E \) is the customer interruption cost per unit energy loss and can be estimated by the production electricity ratio (i.e., the ratio of GDP to energy consumption for a region).

According to reference 11, \( \text{SAIDI}_J \) and \( \text{SAIDI}_D \) can be represented through formula transformation as follows.

\[ \text{SAIDI}_J = D_J L_J + D_J', \quad \text{SAIDI}_D = D_D L_D + D_D' \]  

(17)

Where, \( D_J, D_D, D_J' \) and \( D_D' \) involve the reliability related parameters such as equipment failure rates and switching times, and change with tie-connection modes and switch types (load switches or circuit breakers) of feeders.

With formulae (11), (16) and (17) being combined, we have.

\[ C_{XX} = \frac{B_{XX}}{\sqrt{N_B}} + B_{XX}' \]  

(18)

Where, \( B_{XX} = P_z \xi K_Z \sqrt{\frac{A_T}{\pi}} C_E (k_D D_D + k_J D_J) \); \( B_{XX}' = P_z \xi C_E (k_D D_D' + k_J D_J') \).

With formulae (12), (15) and (21) being combined, the annual costs of feeders can be represented by

\[ C_X = \frac{B_{X}}{\sqrt{N_B}} + B_{XX} \]  

(19)

Where, \( B_X = B_{XT} + B_{XS} + B_{XX} \).
Annual Total Cost

With formulae (8) and (19) being combined, the annual total cost is given by

\[ C_N = C_B + C_X = B_B N_B + \frac{B_S^{0.5}}{N_B^{0.5}} + \left( B_B' + B_X' \right) \]

(20)

MODEL SOLUTION

Concept of Optimal Substation Number

According to formula (20), the annual total cost of substation planning is a function of substation number \( N_B \). The more \( N_B \) is, the larger \( C_B \) is and the smaller \( X_C \) is. As shown in figure 1, the total cost \( C_N \) has a minimum value and the corresponding \( N_{B,ZY} \) is the optimal substation number.

![Figure 1. Diagram of the optimal substation number.](image)

Single-Stage Optimization Algorithm

In this paper, based on the extreme value method, a heuristic single-stage optimization algorithm is proposed to solve the objective function of formula (20) with the substation capacity-load ratio and \( R_{X,MAX} \) being considered. The proposed algorithm consists of the following steps.

Step 1: Calculate the substation number

Firstly, take the derivative of formula (20) with respect to the substation number and make it be zero in order to get the substation number \( N_{B,ZY} \).

\[ \frac{d C_N}{d N_B} = B_B - \frac{B_S^{0.5}}{2 N_B^{0.5}} = 0 \]

(21)

\[ N_{B,ZY} = \text{int} \left\{ \sqrt[3]{\frac{B_X}{2 B_B}} \right\} + 1 \]

(22)

Then, the power supply radius \( R_X \) is calculated according to formula (11) with the formula’s \( N_B \) being replaced by \( N_{B,ZY} \). If the resulting \( R_X \) is greater than \( R_{X,MAX} \), let \( N_{B,ZY} = N_{B,ZY} + 1 \) and calculate \( R_X \) again until \( R_{X,MAX} \) is satisfied.

Step 2: Select the substation capacity

Firstly, the continuous value of \( S_{B,j} \) is calculated by

\[ S_{B,j} = \frac{S_J}{N_{B,ZY}} = K_5 P_z \]

(23)

Then, in reference to the continuous value of \( S_{B,j} \), the discrete value of \( S_{B,j} \) can be selected from the preset candidate substation capacity combinations (such as those of...
$n_z \times 31.5\text{MVA}$, $n_z \times 40\text{MVA}$, $n_z \times 50\text{MVA}$, $n_z \times 63\text{MVA}$, where $n_z$ is 1, 2, 3 or 4), with the minimum total cost and the allowable capacity-load ratio being considered.

Step 3: Calculate the annual total cost

Based on the substation capacity combination corresponding to the selected discrete value of $S_{b,i}$, the annual total cost is recalculated according to formula (1) or (20).

Step 4: Let $C_{N,1}$ be the annual total cost corresponding to $N_{b,z}$. Let $N_{b,z}=N_{b,z}-1$, repeat Steps 2 and 3, and then let $C_{N,0}$ be the new annual total cost.

Step 5: Select the substation number and capacity corresponding to a smaller value of $C_{N,0}$ and $C_{N,1}$ as the optimal or suboptimal substation combination.

APPLICATION AND ANALYSIS

The single-stage substation optimization planning of a real distribution network is performed by using the proposed models and algorithm. For the distribution system, the total area $A_z$ is 910km$^2$, the total power demand $P_z$ is 4551MW, and the calculation results are shown in Table 1.

<table>
<thead>
<tr>
<th>$C_E$ (yuan/kWh)</th>
<th>Load density (MW/km$^2$)</th>
<th>Aerial lines</th>
<th>50% aerial lines and 50% cable circuits</th>
<th>Cable circuits</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$N_{b,z}$</td>
<td>$C_N$ (billion yuan)</td>
<td>$S_b$ (MVA)</td>
<td>$R_x$ (km)</td>
</tr>
<tr>
<td>0</td>
<td>0.1</td>
<td>3</td>
<td>0.04</td>
<td>61</td>
</tr>
<tr>
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<td>15</td>
<td>0.20</td>
<td>121</td>
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<tr>
<td></td>
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<td>2.10</td>
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<td>3.76</td>
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</tr>
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<tr>
<td></td>
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<td>231</td>
<td>4.28</td>
<td>315</td>
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</tbody>
</table>

From Table 1, we have the following conclusions.

1) The larger the proportion of cable circuits is and/or the higher the customer interruption cost per unit energy loss is, the greater the optimal number of substations is and the smaller the substation capacity is.
2) With the increase of customer interruption cost per unit energy loss, the number of substations varies relatively greatly for the cases of aerial lines, by contrast with the cases of cable circuits.
3) Larger load density corresponds to more substations and bigger capacity.

CONCLUSIONS AND DISCUSSIONS

In this paper, based on the assumption of uniform load distribution, the analytical formulae are derived for the optimal substation number and capacity with the customer interruption cost being considered, and also a simple algorithm of single-stage substation optimization planning is proposed. With the total area and power demand being known, the simplified models and algorithm can conveniently generate a quantitative basis of scientific and reasonable decision making about the investment of a distribution network. Through the application of the proposed models and algorithm to a real distribution network, it is shown that larger feeder costs of investment and customer interruption correspond to bigger number and smaller capacity of substations, and the larger the load density is, the bigger both the number and the capacity of substations are.

REFERENCES