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Application of Nonlinear Adaptation Method for Discrete Economic Objects

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Abstract. An application of the method of analytical design of aggregated regulators for constructing systems of control over integrated economic entities with positive and negative feedback loops is discussed. The controlled objects are presented as a system of ordinary non-linear differential or difference equations with chaotic behavior in the case of certain combinations of parameters. A number of control problem formulations are given and the algorithms of control synthesis are reported and theoretically validated. Some illustrative examples of application of the proposed algorithms are provided along with their numerical simulation data. The results obtained would be useful in designing a smart control system and for real-time decision making in the financial policies of a variety of economic entities.

Introduction

Nonlinear models of economic entities of different scales [1] and business application areas are becoming increasingly more appealing to the researchers, as they approach real physical models and are described by multivariable nonlinear indices responsive to the positive and negative feedback loops. This paper deals with an application of the techniques of nonlinear adaptation on manifolds [2] for economic models; in particular, based on a unified methodology we formulate and solve a number of different control problems for the following types of objects with nonlinear models and unstable behavior: capital expansion of a corporate giant, changes in sales volumes of two competing companies producing two similar commodities, and financial operations of a small business enterprise.

In the present paper, the problem of the correct application of nonlinear adaptation methods on manifolds for economic models continues. In the early article [3], we considered the problem of controlling a continuous economic object developing on the basis of the Lorenz model. The purpose of this study is the development of software control systems for several discrete economic models of a mixed type, for example, public and private productions of the same type of products. The objects of the study are developed on the basis of the Feigenbaum model [4], which, in turn, takes into account a number of processes, including temporary changes in economic indicators [5] and environmental indicators.

All these objects are represented by systems of difference equations of 2-4 dimensions with chaotic behavior in cases of certain combinations of parameters (models of deterministic chaos).

In the open-loop states, the mathematical models of economic objects are thought to be unstable, which in essence means the following: there are such initial conditions and parameter values inherent in these objects that the property of economic self-regulation is no longer valid for them [1].

Setting Problems and Designing Scalar Control Systems

Discrete Scalar Control Problem

Let us look at a controlled object described by the following system of equations:

$$X_{i+1} = X_i(\alpha C_0 - \mu\beta_X X_i Y_i),$$
$$Y_{i+1} = u_i(\alpha C_0 - \mu\beta_Y X_i Y_i),\ i = \overline{1, N}, \tag{1}$$

where X_{i+1}, Y_{i+1} are expected volumes of regular sales of the same type of products during the *i*-th period, sold by private and state enterprises, respectively; X_i, Y_i are volumes of previous sales; C_0 is an average buyer's income in the region; β_X, β_Y are prices for goods from private and state enterprises, respectively; u_i is the variable of control and interprets quantitative characteristics of state needs in this type of product; α, μ are proportionality factors.

It is required to carry out control in the state space of the object transferring this object (1) from its given initial state (X_0, Y_0) into the neighborhood of the target manifold (target attractor) $\psi(X,Y) = 0$, where $\psi(X,Y)$ is a certain function meeting the desired physical properties of the target state, and minimizing the quality functional Φ_1, taking into account 1-dimensional control variable

$$\Phi_1 = \sum_{i=0}^{\infty}\left(\omega^2\psi_i^2 + (\Delta\psi_i)^2\right),\ \omega > 0\,. \tag{2}$$

Let's look at the target set, described by the following equation $\psi_i = 0,\ \psi_i = \beta_X X_i - k\beta_Y Y_i,\ i \ge 1$, k>0. The target manifold $\psi_i = 0,\ i \ge 1$ in essence represents the desired relationship between the object's variables.

Solution of the Discrete Scalar Control Problem

Statement 1. Let there exist a variational problem $(\Phi,\ \psi)$: $\Phi = \sum_{i=0}^{\infty}\left(\omega^2\psi_i^2 + (\Delta\psi_i)^2\right) \to \min$, $F(i, \psi_i, \dot\psi_i) = \psi_i^2 + \omega_i^2\dot\psi_i^2$ with a restriction, $\psi_i = 0$. The Euler–Lagrange equation for the problem $(\Phi,\ \psi)$ would be given by equation $\psi_{i+1} + \lambda\psi_i = 0,\ |\lambda| < 1,\ i \ge 1$ where $\lambda = 0.5\left(2 + \omega^2 - \sqrt{(2+\omega^2)^2 - 4}\right)$.

The proof of this statement relies directly on the results of theoretical mechanics []. This statement plays the main role in finding control with predetermined target properties of an object.

Remark. Weighting factor ω is the parameter of the regulator settings; it is proportional to the duration of motion of the image point to the target manifold (target attractor) $\psi_i = 0,\ i \ge 1$.

A solution of variational problem $(\Phi,\ \psi)$ yields an equation for control $u_i,\ i \ge 1$ (in accordance with statement): $\psi_{i+1} + \lambda\psi_i = \beta_X X_{i+1} - k\beta_Y Y_{i+1} + \lambda\psi_i = 0,\ |\lambda| < 1,\ i \ge 1.$

We solve this functional equation for variable u_i:

$$u_i = \left(\beta_X X_i(\alpha C_0 - \mu\beta_X X_i Y_i) + \lambda\psi_i\right)\left(k\beta_Y(\alpha C_0 - \mu\beta_Y X_i Y_i)\right)^{-1},\ i \ge 1.. \tag{3}$$

Statement 2. The control (3) asymptotically stably outputs the object (1) to a given target manifold $\psi_i = 0,\ i \ge 1$ with a minimum of the control quality functional Φ_1, while the parameter of the control system ω is interpreted as a quantity proportional to the duration of the transition process.

Setting Problems and Designing Vector Control Systems

Vector Control Problem

Let us look at a controlled object described by the following system of equations:

$$
\begin{cases}
X_{i+1} = X_i\left(\alpha C_0 - \mu\beta_{X,i} X_i Y_i\right), \\
Y_{i+1} = A\left(\alpha C_0 - \mu\beta_{Y,i} X_i Y_i\right), \\
\beta_{X,i+1} = c_1\beta_{X,i} + u_1, \\
\beta_{Y,i+1} = c_2\beta_{Y,i} + u_2,
\end{cases} \tag{4}
$$

where X_{i+1}, Y_{i+1} are expected volumes of regular sales of the same type of products during the *i*-th period, sold by private and state enterprises, respectively; X_i, Y_i are volumes of previous sales; C_0 is an average buyer's income in the region; $\beta_{X,i}$, $\beta_{Y,i}$ are variables of prices for goods from private and state enterprises, respectively; *A* quantitative characteristics of state needs in this type of product; α, μ, c_1, c_2 are proportionality factors; u_1, u_2 are the vector variables of control.

It is required to carry out control in the state space of the object transferring this object (1) from its given initial state (X_0, Y_0) into the neighborhood of the target manifold (target attractor) $\psi(X,Y) = \left(\psi_1(X,Y), \psi_2(X,Y)\right)$, where $\psi(X,Y)$ is a certain vector function meeting the desired physical properties of the target state, and minimizing the quality functional Φ, taking into account 2-dimensional control variable

$$
\Phi_2 = \sum_{i=0}^{\infty}\sum_{j=1}^{2}\left(\omega_j^2\psi_{j,i}^2 + \Delta\psi_{j,i}^2\right),\ \omega_j > 0,\ j = 1, 2.. \tag{5}
$$

Let's look at the target set, described by the following system of equations:

$\psi_{1,i} = Y_i - Y_i^0,$

$\psi_{2,i} = X_i - \rho Y_i, \rho \in (0;1),\ i \geq 1,$

where Y_i^0 is expected value of variable Y_i during the *i*-th period.

Solution of the Discrete Vector Control Problem

We shall derive formulas for control variables u_1, u_2 in the form of the following algorithm.

1. Determine the first auxiliary vector macro-variable $\psi^{(1)} = \left(\psi_1^{(1)}, \psi_2^{(1)}\right)$:

$\psi_1^{(1)} = \beta_{X,i} - \varphi_1(X_i),$

$\psi_2^{(1)} = \beta_{Y,i} - \varphi_2(Y_i).$

In this step the functions $\varphi_1(X_i)$, $\varphi_2(Y_i)$ are unknown.

2. Formulate a variational problem given by $\left(\Phi_2^{(1)}, \psi^{(1)}\right)$ as follows:

$$
\Phi_2^{(1)} = \sum_{i=0}^{\infty}\sum_{j=1}^{2}\left(\left(\omega_j^{(1)}\psi_{j,i}^{(1)}\right)^2 + \left(\Delta\psi_{j,i}^{(1)}\right)^2\right) \to \min,\ \omega_j^{(1)} > 0,\ j = 1, 2;\ \psi^{(1)} = 0.
$$

A solution of $\left(\Phi_2^{(1)}, \psi^{(1)}\right)$ yields a system of equations for vector control $u = (u_1, u_2)$ (in accordance with statement 1):

$$
\psi_{1,i+1} + \lambda_1\psi_{1,i} = Y_{i+1} - Y_{i+1}^0 + \lambda_1\left(Y_i - Y_{i+1}^0\right) = 0,\ |\lambda_1| < 1,
$$

$$
\psi_{2,i+1} + \lambda_2\psi_{2,i} = X_{i+1} - \rho Y_{i+1} + \lambda_2\left(X_i - \rho Y_i\right) = 0,\ |\lambda_2| < 1,\ i \geq 1.
$$

Solve this functional equation for variable u_i:

We solve these equations for a variables u_1, u_2 taking into account the form of system (4):

$$u_1 = -c_1\beta_{X,i} + \varphi_1(X_{i+1}) + \omega_1^{(1)}(\beta_{X,i} - \varphi_1(X_i)),$$
$$u_2 = -c_2\beta_{Y,i} + \varphi_2(Y_{i+1}) + \omega_2^{(1)}(\beta_{Y,i} - \varphi_2(Y_i)),\ i \ge 1. \quad (6)$$

The forms of control (6) ensure a transfer of the object (4) into the neighborhood of the target manifold $\psi^{(1)} = \left(\psi_1^{(1)}, \psi_2^{(1)}\right) = (0,0)$, on which the following relations are fulfilled:

$$\psi_1^{(1)} = 0 \Rightarrow \beta_{X,i} = \varphi_1(X_i),$$
$$\psi_2^{(1)} = 0 \Rightarrow \beta_{Y,i} = \varphi_2(Y_i).$$

3. Decompose the initial system of equations (4) on manifold $\psi^{(1)} = 0$:

$$X_{i+1} = X_i\left(\alpha C_0 - \mu\varphi_1(X_i)X_iY_i\right),$$
$$Y_{i+1} = A\left(\alpha C_0 - \mu\varphi_2(Y_i)X_iY_i\right). \quad (7)$$

4. Assume the second macro-variable $\psi_i^{(2)} = \psi_i = \left(\psi_{1,i}, \psi_{2,i}\right) = \left(Y_i - Y_i^0, X_i - \rho Y_i\right)$ in order to define functions $\varphi_1(X_i),\ \varphi_2(Y_i)$, and as in the above case formulate a variational problem given by $\left(\Phi_2^{(2)}, \psi^{(2)}\right)$: $\Phi_2^{(2)} = \sum_{i=0}^{\infty}\sum_{j=1}^{2}\left(\left(\omega_j^{(2)}\psi_{j,i}^{(2)}\right)^2 + \left(\Delta\psi_{j,i}^{(2)}\right)^2\right) \to \min,\ \omega_j^{(2)} > 0,\ j = 1,2;\ \psi^{(2)} = 0.$

As above, we solve the equations $\psi_{j,i+1} + \lambda_{j+2}\psi_{1,i} = 0,\ \left|\lambda_j\right| < 1,\ j = 1,2,\ i \ge 1$ for variables $\varphi_1(X_i),\ \varphi_2(Y_i)$ taking into account the form of system (7):

$$\varphi_1(X_i) = \left(\mu X_i^2 Y_i\right)^{-1}\left(X_i\alpha C_0 + \lambda_4\psi_{2,i} - \rho Y_{i+1}^0 + \lambda_3\rho\psi_{1,i}\right),$$
$$\varphi_2(Y_i) = \left(A\mu X_iY_i\right)^{-1}\left(A\alpha C_0 - Y_{i+1}^0 + \lambda_3\psi_{1,i}\right). \quad (8)$$

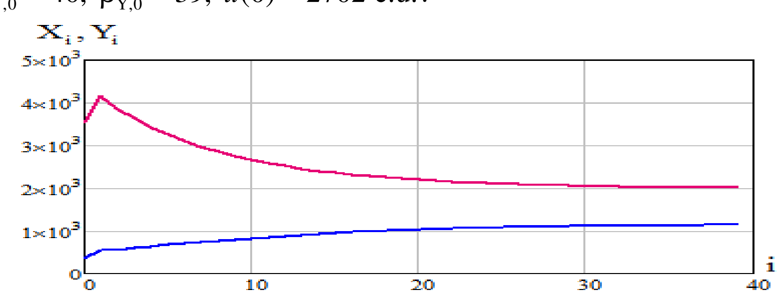
Statement 3. The vector control (6), (8) asymptotically stably outputs the object (4) to a given vector target manifold $\psi(X,Y) = 0$ with a minimum of the control quality functional Φ_2, while the parameters of the control system ω_1, ω_2 are interpreted as a quantities of proportional to the duration of the transition process.

Simulation of Control Systems

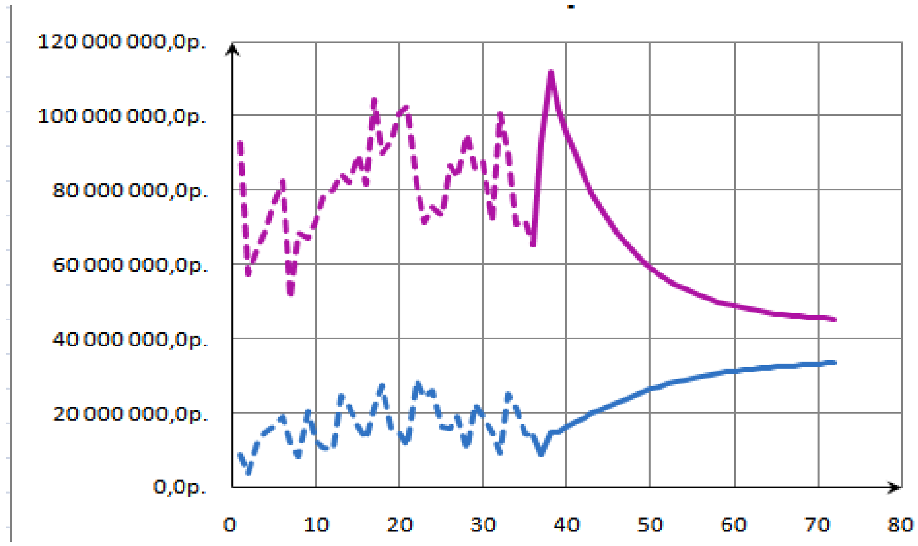
The plots of controlled coordinates presented in Fig. 1, 2 suggest an acceptable quality of control over the objects (1), (4) which ensures that the goal is achieved $\psi(X) = 0$ in terms of the global minimum of functional Φ_1, Φ_2 and the control object is sustained in the neighborhood of the desired manifold $\psi(X) = 0$.

The simulation of scalar control system was performed for the following parameter values:

C_0=32000 c.u., α=6.8·10^{-5}, μ=1.296·10^{-8}, A=2702 , c_1=0.5, c_2=0.1, ρ=0.16, $Y_{i+1}^0 = Y^0 = 3500$, ω=0.9, k=0.6, $\beta_{X,0} = 40$, $\beta_{Y,0} = 39$, $u(0) = 2702$ c.u..



a)



b)

Figure 1. Behavior of controllable variables: X, Y a) and a graph of the profit change curves according to the specified object control goal b) (blue and red chart, respectively).

The simulation of vector control system was performed for the following parameter values:

C_0=32000 c.u., α=6.8·10^{-5}, μ=1.296·10^{-8}, A=2702, c_1=0.5, c_2=0.1, ρ=0.16, $Y^0_{i+1} = Y^0 = 3500$, λ_1=0.4, λ_2=0.01, λ_3=0.35, λ_4=0.01, $\beta_{X,0} = 40$, $\beta_{Y,0} = 39$.

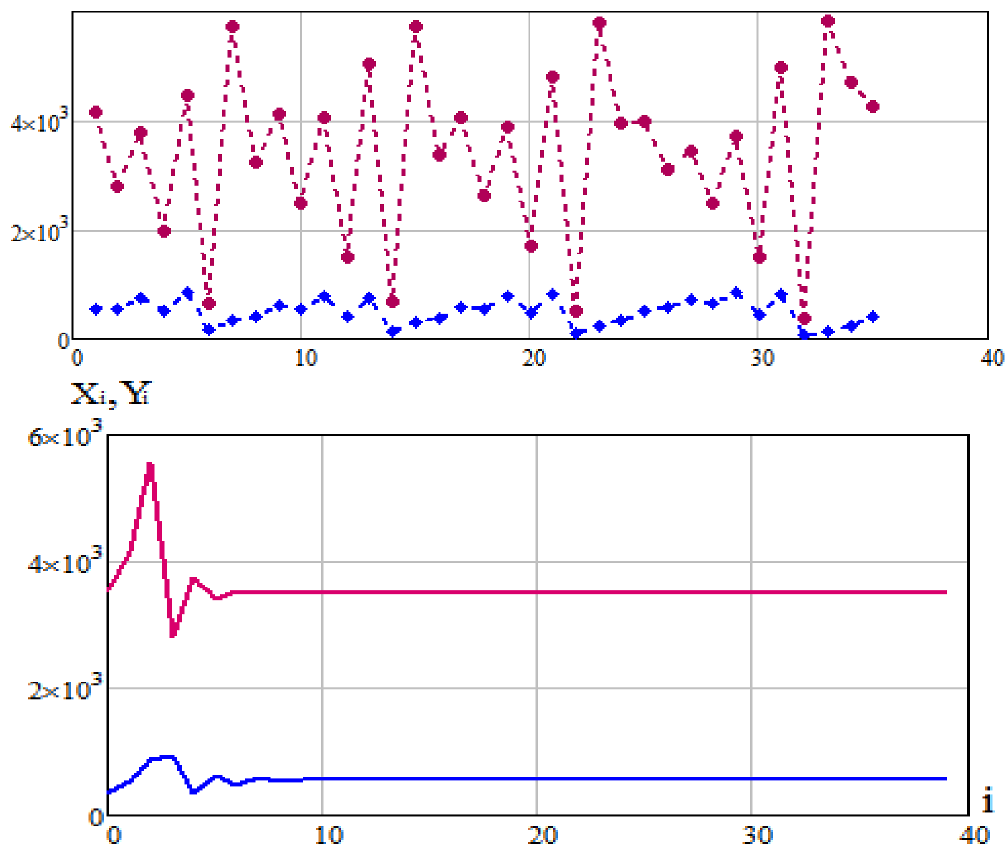


Figure 2. Behavior of uncontrollable a) and controllable b) variables: X, Y (blue and red chart, respectively).

For a numerical illustration, we will consider the production of two manufacturers of dairy products.

Summary

In this study we have constructed two control systems for two discrete economic models of a mixed type, for example, public and private productions of the same type of products. The resulting systems of control have been tested using initial data obtained from the performance indicators of two real small business enterprise.

The results obtained can be useful for solving applied economic problems in decision-making in intelligent control systems.

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