An Integrated Short-Term Demand Forecasting and Optimal Pricing for Rail Freight Transportation

JIN ZENG, XIANG CHAO and CHUNJIAO DONG

ABSTRACT

In the freight rail industry, profit maximization relies heavily on the integration of freight transport activities with an improved pricing management. The pricing policies chosen by the carrier have an important impact on the enterprise revenue. This paper bridges the gap between short-term demand forecasting and pricing management. We propose a new mathematical formulation which encompasses pricing decisions and short-term demand prediction on the base of considering the prediction deviation. The revenue from the optimal pricing model has a more substantial increase than the uniform pricing based on the results of the case.

INTRODUCTION

Since 1990s, Chinese road transportation, railway transportation, water transportation and air transportation mode have developed rapidly. Water transport and air transport had gone to the market pricing. Especially, airlines had adopted advanced revenue management system to manage pricing dynamically. In the railway aspect, because the price specialty of China railway transport service is shown among the public welfare, singleness and externality. The price could not be made by the enterprise’s cost completely. In Feb 2016, the China Railway Corporation began to get pricing power on the high speed railway. The China railway freight uniform transport rate increased from 0.1451 Yuan to 0.1551Yuan. As a benchmark price, the price was allowed to move up not more than 10% and there is no limitation in discount. The marketization has already sped up.

The China rail freight e-business platform threw in operation on May 2015. We adopt the service demand pricing in the e-business freight platform according to the spot market demand forecasting in order to make the spot market price more predictable and meet the needs of the increasing market competition.

The railway freight pricing model is a topic that has not received much attention

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in the empirical literature. Kraft (1998) [1] has focused on finding a way to increase the reliability of rail in order to compete with trucks. He develops ‘the train segment pricing’ model, a stochastic model that uses a bid-price approach to determine the acceptance of future orders. Similar work is also available in Kraft (2002) [2]. Li et al. (2005) [3] develop a “medium-term” tactical planning model for intermodal rail transport that jointly considers operations planning and pricing decisions. Brotcorne et al. (2008) [4] have presented a combined network design and pricing model that they formulate as a mixed-integer bi-level program. Amerstrong et al. (2010) [5] gave an extensive overview of railway RM and its models for both passenger and freight transportation. They have suggested switching existing process to an on-line learning model would benefit both car scheduling and revenue decision. Crevier et al. (2012) [6] have developed a model integrating operations planning and pricing optimization for carriers.

Comparing the real transport demand of the spot market with the forecasting transport demand, we have analyzed the reason why the transport demand curve is a family of demand curves in the specific interval. And the short term demand pricing model of rail freight transport is developed on the base of the transport market, demand curve and the distribution of demand probability.

METHODOLOGY

In this section, we develop a railway freight pricing model of the short-term demand based on the freight market demand curve and the distribution of demand probability. We demonstrate that the model solved the optimal pricing problem.

RESEARCH ASSUMPTIONS

The demand curve is difficult to be forecasted exactly in advance. There existed three situations where the amount of the actual spot market demand is equal to or less than or more than the amount of the demand forecasting. The demand curve is a series of curves in an uncertain interval. We assume the transportation market, the demand curve and the probability demand distribution as follows while developing the pricing model of the short-term demand.

(1) Market hypothesis: We assume the rail enterprise has rights to independent pricing and not take account of social impacts of the public welfare. The pricing object is to maximize the expected revenue of the enterprise.

(2) Demand curve hypothesis: We assume the market demand curve as a line to treat the tangent line of the intersection point of a carrier's supply curve S and a demand forecasting curve M as a carrier's demand forecasting curve D. And we suppose that the probability of the excess demand is equal to the probability of the demand shortage, the deviation of the forecasting demand determined by a carrier is $\pm k$ ($0 < k < 1$), $k$ value is based on the carrier’s ability of grasping the market.

In order to establish the proposed model in the rail freight market, we need to define in great detail our notation. We therefore define the following sets.
$M$: Demand forecasting curve. $M_l$: Possible minimum demand curve. $M_h$: Possible maximum demand curve. $D$: The freight demand quantity. $D_m$: Simplified M curve M to $D_m$ curve. $D_l$: Simplified minimum demand curve. $D_h$: Simplified maximum demand curve. $D_{om}$: $D_m$ is calculated to get the demand quantity, while the freight rate is zero. $D_{ol}$: $D_l$ was calculated to get the transport demand quantity, while the rate is zero. $D_{oh}$: $D_h$ was calculated to get the demand quantity, while the rate is zero. $S$: The supply quantity of a carrier. $P$: Rail freight rate. $p_0$: The price determined by the cross of the carrier’s supply curve S and the demand forecasting curve. $p_l$: The price decided by the cross of the carrier’s supply curve S and the minimum demand curve. $p_h$: The price decided by the cross of the carrier’s supply curve S and the maximum demand curve. $p_3$: The freight rate obtained by calculating $D_l$, while the demand quantity is zero. $a$ : $-1/a$ shows the slope of the line $D_{om}$.

(3) Probability density function assumptions: All the corresponding market demand quantities at each price is sophisticated variables which are in a special range. We conclude that the range of the demand variable is $[D_{ol} - ap, D_{oh} - ap]$.

ESTABLISHMENT OF SHORT-TERM DEMAND PRICING MODEL

Railway freight companies need to determine a reasonable transport price in the case of freight supply $S$, in order to maximize the profits of enterprises. The expected revenue $ER$ is the product of $P$ and expected transport volume $E(q)$.

$$ER = p 	imes E(q)$$  \hspace{1cm} (1)

When the rate $p$ is less than $p_3$, the expected market demand volume $E(d)$ corresponding to any price can be described by:

$$E(d) = \int_{p_o}^{p_3} \frac{4}{(D_{ol} - D_o)^2} (D - D_{ol})DdD + \int_{p_3}^{p_h} \frac{4}{(D_{oh} - D_o)^2} (D_{oh} - D)DdD - ap = \frac{D_{ol} + D_{oh} - 2ap}{2}$$  \hspace{1cm} (2)

Therefore, the expected transport demand curve of railway freight companies is:
The key to solving the expected revenue function of the railway freight companies is to determine the expression of the expected transportation volume $E(q)$. Therefore, this section will discuss the $E(q)$ expression of the three kinds of cases according to the size of the freight rate $P$ that is less than $p_1$, the price $P$ is higher than $p_2$, the freight rate $P$ is between $p_1$ and $p_2$.

1. Freight rate $P$ is less than $p_1$: The freight demand is greater than the supply capacity $S$ of rail freight companies. The $E(q)$ is equal to the supply $S$.

$$E(q) = S \quad p \leq p_1$$ (4)

2. Freight rate $P$ is greater than $p_2$: The demand volume is less than that $S$. The expected transportation volume $E(q)$ is equal to the market demand $E(d)$.

$$E(q) = E(d) = \frac{D_{oh} + D_{dl}}{2} - ap$$ (5)

3. Freight rate $P$ is in the range of $p_1$ and $p_2$: The demand volume is likely to be greater or less than the transportation supply capacity $S$. The $E(q)$ is that

$$E(q) = \begin{cases} \int_{0}^{p_1} \frac{4(D - D_{dl})}{(D_{oh} - D_{dl})^2} DdD + \int_{p_1}^{p_2} \frac{4(D - D_{dl})}{(D_{oh} - D_{dl})^2} SdD + \int_{p_2}^{p_3} \frac{4(D - D_{dl})}{(D_{oh} - D_{dl})^2} DdD \quad p_1 \leq p \leq p_2, \\ \int_{0}^{p_1} \frac{4(D - D_{dl})}{(D_{oh} - D_{dl})^2} DdD + \int_{p_1}^{p_3} \frac{4(D - D_{dl})}{(D_{oh} - D_{dl})^2} DdD + \int_{p_2}^{p_3} \frac{4(D - D_{dl})}{(D_{oh} - D_{dl})^2} SdD + \int_{p_2}^{p_3} \frac{4(D - D_{dl})}{(D_{oh} - D_{dl})^2} DdD \quad p_3 \leq p \leq p_2. \end{cases}$$ (6)

Then, we obtain the carriers’ optimal rate according to the optimization problems.

$$\max ER(p) = p \times E(q) \quad \text{s.t.} \quad p > 0$$ (7)

**SOLUTION OF SHORT-TERM DEMAND PRICING MODEL OF RAILWAY FREIGHT COMPANIES**

We know that the size of $E(q)$ is related to the size of the freight rate. There are three kinds of situations, and these three cases are analyzed as follows.

1. Freight rate $P$ is less than $p_1$: The $ER$ function can be presented by:

$$ER = p \times S$$ (8)

The derivative of the function $ER$ at the freight rate $P$ can be expressed as:

$$\frac{dER}{dp} = S > 0$$ (9)

It shows that the derivative is greater than zero, indicating that the expected profit function on freight rate increases monotonically. So rising rates can improve the expected revenue. Therefore, from the point of view of railway freight companies, the freight rate cannot be less than $p_1$. 

$$D_m = D_{out} - ap = E(d) = \frac{D_{oh} + D_{dl}}{2} - ap$$ (3)
(2) Freight rate $P$ is greater than $P_2$: The expected revenue function can be calculated by combining the formula (3) and formula (7).

$$ER = p \times \left( \frac{D_{ol} + D_{oh} - ap}{2} \right)$$

(10)

The second derivative of the function $ER$ at the freight rate $P$ is that

$$\frac{d^2ER}{dp^2} = -2a < 0$$

(11)

It shows that the second derivative is less than zero, stating the $ER$ is a convex function. We can obtain the maximum value of the function while the first derivative of the $ER$ at the freight rate $P$ is equal to zero and the optimal rate is that

$$p^*_1 = \frac{D_{ah} + D_{ol}}{4a} = S/a = p_0$$

(12)

Jointly analyzing the condition 1 and condition 2, we know carriers make the rate only in the range of $p_1$ and $P_2$. The optimal rate exists in the range of $p_1$ and $P_2$.

(3) Freight rate $P$ is in the range of $p_1$ and $P_2$: When freight rate $P$ is in the range of $p_1$ and $P_2$, we can conclude that

$$\frac{d^2ER}{dp^2} \leq -\frac{4a^2 p}{(D_{on} - D_{ol})^2} (1 - 3k) S$$

(13)

The formula (14) shows that when $k \leq 1/3$, $d^2ER_2/dp^2 < 0$. Therefore, $d^2ER_2/dp^2 < 0$ is right. It shows that expected revenue function $ER_2$ is a convex function in the interval $[p_0, P_2]$. The first derivative of the freight rate $P$ with respect to the expected revenue function can be expressed as:

$$\frac{dER_2}{dp} = 32a^3 p^3 - 36(D_{ah} - S)a^2 p^2 + [3(D_{ah} - 3D_{ah} - 12S)D_{ah} + 3(D_{ah} - D_{al})D_{ah} + 4(3D_{ah} - S)S^2]$$

(14)

The optimal rate to make the expected revenue to reach the maximum value is that

$$P^* = \left\{ \begin{array}{ll} p^*_2 & \text{if } ER(p^*_2) > ER(p^*_3) \\ p^*_3 & \text{if } ER(p^*_2) < ER(p^*_3) \end{array} \right.$$  

(15)

Finally, the optimal price in the interval $[p_1, p_0]$ is $p^*_2$, while the optimal price in the interval $[p_0, P_2]$ is $p^*_3$. The local optimal rate $p^*_2$ or $p^*_3$ makes the carrier’s revenue reached the maximum.

**RESEARCH CASE STUDY**

This section takes coal transportation as an example in China from the H1 station of H railway bureau to the B station for the H Railway Bureau to determine a reasonable price. The average rate of H railway bureau transporting coal mine from H1 station to B station is 19.5258 cent/ton-km. The coal transportation of H railway bureau is mainly using C80 type gondola car whose capacity is 80 tons, the data obtained from the official website of China. Every day in April we assume the freight forecast demand curve of the H Railway Bureau for the coal transport market is an
ideal hyperbolic. We suppose that the H railway bureau determine the scheduling arrangement and the freight rate of the last month according to the forecasting curve. The average rate 19.5258 cent/ton-km and the freight supply is a point on the forecasting curve. Therefore, all A values of the demand forecasting curve are shown in Table 1.

<table>
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<th>Date</th>
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Taking the $k$ as 0.1, 0.2 and 0.3, the results in Table 2 shows the optimal prices in different situations of forecast error. $p^*_j$ is the optimal price.

<table>
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<tr>
<td>$p_j$ (cent/ton-km)</td>
<td>17.7501</td>
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<td>$p_o$ (cent/ton-km)</td>
<td>19.5258</td>
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<td>$p_2$ (cent/ton-km)</td>
<td>21.6953</td>
<td>24.4072</td>
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<td>$p^*_j$ (cent/ton-km)</td>
<td>21.3540</td>
<td>23.9227</td>
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<tr>
<td>$ER(p_j)$ (cent)</td>
<td>7.3495E3</td>
<td>6.6805E3</td>
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<td>$ER(p_o)$ (cent)</td>
<td>1.3360E4</td>
<td>1.2301E4</td>
<td>1.1880E4</td>
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<tr>
<td>$ER(p^*_j)$ (cent)</td>
<td>1.6963E4</td>
<td>1.8767E4</td>
<td>2.0627E4</td>
</tr>
<tr>
<td>$p^*$ (cent/ton-km)</td>
<td>21.3540</td>
<td>23.9227</td>
<td>26.6030</td>
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</table>

Table 2 shows when the value of the prediction error increases, the optimal rate gradually increases. We raise the freight rate with the increase of the prediction error to avoid the pricing risk. As the variation rule is consistent with the qualitative analysis of the actual problem, the optimal pricing model is reasonable.

CONCLUSION

The paper has analyzed the actual freight demand and the forecasting demand of rail freight companies. On the base of considering the prediction error, the demand forecasting curve of the freight market is defined as a curve family of the demand
curve in a certain interval. The short-term demand pricing model of railway freight is
developed on the simplified nonlinear demand curve and the hypotheses of the
probability distribution of the freight demand. The mathematical theory is used to
prove that the model has a maximum value. Taking H1 railway station to B station as
a case, we analyze the influence of forecasting error factors on the optimal freight rate
in the short-term railway freight demand model and demonstrate that the freight
pricing model for short-term demand is reasonable. In addition, using the short-term
demand pricing model proposed, there is an optimal rate to be found for H Railway
Bureau. According to the market research data and combining with the geographical
features and time to predict the market demand curve and dynamic adjustment will be
a next research direction.

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