Robot Trajectory Tracking Control Based on Look-up Table Method

Qiang YANG*, Jun ZHAO, Fu-cheng SONG and Guo HE
Control Engineering College, Chengdu University of Information Technology, Chengdu 610225, China
*Corresponding author

Keywords: Robot, Trajectory tracking, Looking-up table, Optional control.

Abstract. This paper proposed a simple and applicable look-up table based trajectory tracking control scheme for robot. Based on the robot’s kinematics model and the saturation constraint of the control inputs, a control-trajectory table was established using the European numerical calculation method. The optimal control values can be determined under the look-up table criteria. Finally, the simulation results of trajectories with different characteristics of straight line, sine line and circle show the effectiveness of the proposed scheme.

Introduction

With the development of science and technology, mobile robots are widely used in many fields, such as agriculture and industry. Mobile robots are the attract research filed in recent years [1-5]. The control problem of wheeled mobile robots attracts a large number of researchers due to their theoretical challenges and their wide range of practical applications. Among the reported results, trajectory control of a mobile robot, especially a trajectory of a series of coordinate sequences whose specified reference trajectories have strong nonlinearities or do not satisfy the single curve description, is not effective in trajectory tracking.

The objective of mobile robot control can be expressed as: i. Move between two locations, ii. track a given trajectory curve, iii. track a given geometry path. Based on adaptive control [6-9], sliding mode control [10-13], back-stepping design [14-15], PID control, fuzzy control and high gain control, many trajectory tracking controller were obtained for mobile robot [16-19]. A smooth linear or non-linear function is often used to describe the desired trajectory when presenting and analyzing problems. And with constraints, such as the first and the second derivative of the desired trajectory are bounded.

In practical application, there are several deficiencies for the mentioned trajectory control strategy:

A. In fact, it is difficult to give an exact function description model of the desired trajectory. Users tend to focus only on where the robot expect to arrive, but they do not care about what functions are used to describe these locations because it is a very difficult task, especially for the desired trajectories described by discrete sequences of irregularities.

B. In view of the expected trajectory of different characteristics, the existing methods have poor robustness and large trajectory tracking control errors. With the development of artificial intelligence, the expected trajectory of mobile robot cannot be fixed, but tends to randomness and variability. Due to different tasks at different times (task is temporary, random), yesterday, today and tomorrow, the robot's expected trajectory is not exactly the same. Therefore, the appropriate trajectory tracking control method should be selected, which is undoubtedly a difficult task for the user.

C. The control algorithm has complex structure, high cost and poor real-time performance. In many literatures, in order to improve the performance in control accuracy and robustness, a large number of uncertainties are added into the model analysis, such as parameter uncertainty, load uncertainty, modeling error, etc. The trajectory controller has a complicated structure and large calculation. In some state feedback trajectory controllers, it is necessary to detect the attitude information such as velocity, angular velocity and position of the mobile robot in real time.
sensor needs to be installed and communicates with the main controller. There is a large amount of data flow transmission and processing, which make the high cost, real-time poor of the robot.

For the situation that a series of coordinate sequences with strong nonlinearity or unsatisfied single curve description, the reported literature does not provide a simple and effective method to control the trajectory of wheeled mobile machines.

In this paper, a simple and applicable look-up table [20] based trajectory tracking method is proposed for mobile robot. This scheme can be applied to not only the line or nonlinear trajectory curve but also the trajectory that a series of coordinate sequences with strong nonlinearity or unsatisfied single curve description. Based on the robot’s kinematics model and the saturation constraint of the control inputs, a control-trajectory table was established using the European numerical calculation method. The optimal control values can be determined under the look-up table criteria. Because the matching table are calculated offline, the proposed scheme is robust, less computationally intensive and real-time.

**Problem Description**

Consider the simplified model of the wheeled mobile robot shown in figure 1.

![Figure 1. Simplified model of wheeled mobile robot.](image)

Where, \((x, y)\) is the robot's centroid coordinates in the moving plane, \(\theta\) is the robot's attitude angle, \(v\) is the velocity of the robot, \(\omega\) is the angular velocity of the robot respectively. The kinematics equation of a wheeled mobile robot can be described as

\[
\begin{align*}
    x &= v \cdot \cos(\theta) \\
    y &= v \cdot \sin(\theta) \\
    \theta &= \omega
\end{align*}
\]  

where \((v, \omega)\) is the control input vector satisfying the saturation constraints

\[
\begin{align*}
    \left|v(t)\right| &\leq v_{\text{max}}, \quad t \geq 0 \\
    \left|\omega(t)\right| &\leq \omega_{\text{max}}, \quad t \geq 0
\end{align*}
\]  

where \(v_{\text{max}}\) and \(\omega_{\text{max}}\) are two positive constants.

Objective: Given any reference trajectory \((x_r, y_r)\), which is linear or nonlinear curve or a series of trajectory coordinate sequences that do not satisfy the single curve description, we need to design...
the control input \((v, \omega)\) to make the robot's trajectory \((x, y)\) tracking the specified reference trajectory \((x_0, y_0)\).

**Trajectory Control Based on Look-up Table**

In this paper, a fast looking-up table control algorithm is proposed. First, we establish the relationship table between the control input \((v, \omega)\) and the robot's trajectory \((x, y)\). Second, determine the control input based on table look-up criteria. Finally, the determined control input acts on the robot to make the robot's moving trajectory tracking the desired trajectory.

**Establish the Relationship Table between the Control Input and the Trajectory**

According to the saturation constraints, we discrete \((v, \omega)\) into \((m, n)\) different equal parts respectively e.g.

\[
\begin{align*}
V & : [v_1, v_2, v_3, \ldots, v_m] \\
\Omega & : [\omega_1, \omega_2, \omega_3, \ldots, \omega_n]
\end{align*}
\]  

Satisfying

\[
\begin{align*}
v_1 & = -v_{\text{max}}, \quad v_m = v_{\text{max}} \\
\omega_1 & = -\omega_{\text{max}}, \quad \omega_m = \omega_{\text{max}}
\end{align*}
\]  

And there is a component \(v_i = 0\), \(i \in (2, 3, \ldots, m-1)\) and a component \(\omega_j = 0\), \(j \in (2, 3, \ldots, n-1)\). According to the results of discrete division, we can obtain \(mn\) different kinds of control input.

It is assumed that the initial position of the robot is the coordinate origin. Under different input control input \((v, \omega)\), the trajectory of the robot is different. In a certain sampling time, the robot arrives at different position coordinates. The Euler scheme is used to solve the robot's motion model. The calculation process is as follows

\[
\begin{align*}
x_{k+1} &= x_k + \Delta t \cdot v \cdot \cos(\theta_k) \\
y_{k+1} &= y_k + \Delta t \cdot v \cdot \sin(\theta_k) \\
\theta_{k+1} &= \theta_k + \Delta t \cdot \omega
\end{align*}
\]  

Where \(\Delta t\) is the sample time.

Starting from the coordinate origin, under different input of control variables, different positions reached by the robot can be obtained based on the iterative numerical calculation formula (5), as shown in table 1.

<table>
<thead>
<tr>
<th>Control input</th>
<th>Robot’s position</th>
</tr>
</thead>
<tbody>
<tr>
<td>(C_i(v_i, \omega_i))</td>
<td>(M_i(x_i, y_i))</td>
</tr>
<tr>
<td>(C_j(v_j, \omega_j))</td>
<td>(M_j(x_j, y_j))</td>
</tr>
<tr>
<td>(\vdots)</td>
<td>(\vdots)</td>
</tr>
<tr>
<td>(C_{mn}(v_{mn}, \omega_{mn}))</td>
<td>(M_{mn}(x_{mn}, y_{mn}))</td>
</tr>
</tbody>
</table>

**Look-up Criteria**

The starting position of the robot is the origin of the coordinate. According to the designated reference trajectory \((x_0, y_0)\) and combined with the control input-position correspondence established in table 1, the shortest Euclidean distance between the position of the robot under certain control input and the desired trajectory tracking position is taken as the Look-up criterion. We can determine the optimal control values at each moment.
In the current moment, the position coordinate of the robot is $M_i(x_i, y_i)$, and the robot expects to reach the position $M_r(x_r, y_r)$ at the next moment, the look-up table criterion is as follows:

$$\min \left[ \sqrt{(x_{i0} + x_j - x_{ri})^2 + (y_{i0} + y_j - y_{ri})^2} \right] \Rightarrow C_{opt_i}$$  \hspace{1cm} (6)

Where $j = 1, 2, 3, \ldots, m \times n$.

For the $i$th step, the optimal control input value $C_{opt_i}$ is one of the Table 1, $[C_1(v, \omega), C_2(v, \omega), \ldots, C_{max}(v, \omega)]$. The combination of optimal control strategies at different moments forms the optimal control strategy $C_{opt}$. The robot effectively track the desired trajectory with constantly looking up Table I under the criteria (6).

**Simulation Results**

In the simulation, we chose $v_{max} = 2m/s$, and $\omega_{max} = \pi/4$. The discrete interval of velocity is $0.5m/s$ and the discrete interval of angular velocity is $\pi/8$. We obtained the tracking simulation results for straight line, sinusoidal and circle with the proposed scheme.

**Tracking Straight Line**

The equation of the given straight line is $y = x$, and the sampling time $\Delta t = 4s$. The simulation result was shown in figure 2.

![Figure 2. Straight linear trajectory tracking.](image)

**Tracking Sinusoidal**

The equation of the given sinusoidal is $y = \sin(x) + 2$, and the sampling time $\Delta t = 1s$. The simulation result was shown in Figure 3.
Tracking Round

The equation of the given circle is $x = 8\sin(t)$, $y = 8\cos(t)$. The sampling time $\Delta t = 2s$. The simulation result was shown in Figure 4.

From the simulation results shown in figure 2 to figure 4, we can see that the trajectory of the robot effectively tracked the different desired trajectories (straight line, sinusoidal and circle) with the proposed control scheme.

Conclusion

This paper presents a robot look-up table control scheme. Firstly, using the Eulerian numerical solution to the robot kinematics model, the control-trajectory relation table under saturated constraint condition is established. Secondly, the control table is determined according to the proposed look-up criterion with the shortest distance. Finally, the optional control values act on the robot to achieve trajectory tracking. Based on Matlab, the tracking simulation results of trajectories with different characteristics of straight line, sine line and circle show the effectiveness of the proposed algorithm.
Acknowledgment

This research was financially supported by the Fund of Sichuan Provincial Science & Technology Department under Grant (No.2017FZ0010) and the Scientific Research Foundation of CUIT (No.KYTZ201625).

References


