220kV City Power Grid Maximum Loadability Determination with Static Security-constraints

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Abstract. City grid connecting urban residents to the electric power source via high-voltage transmission grid is a critical part of the power grid. As the urban electricity demand grows, how many electrical load city grids can supply is becoming a widely concerned problem. Various kinds of loadability problems in power system have been widely studied by several researchers during the past decades, including voltage stability, available transfer capability and power supply capability of distribution system, etc. But the existing studies and methods are not so applicable for the maximum loadability (ML) determination of 220kV city grid. In order to improve the calculation accuracy and to accelerate the calculation speed, a simplifying scheme is proposed to decouple 220kV city grids into different kinds of typical connection forms after comprehensive analysis of its connection structure. Then a non-linear mathematical model for maximum loadability determination of typical connection forms of 220kV city grid considering static security is derived from the optimal power flow (OPF) theories. According to the optimization model, a self-adaptive differential evolution algorithm with built-in Newton-Raphson (N-R) method is presented for searching the optimal solution. Finally, a case study is conducted to test the validity of the optimization model and the preciseness and efficiency of optimization algorithm.

Introduction

The maximum loadability of 220kV city grid refers to the max level of electrical load that 220kV city power grid can supply for long term operation consideration while satisfying all the equipment limitations and operating constraints. Nowadays the 220kV city grid which having a relatively complicated structure is becoming the trunk part of city grids. The precise determination of the maximum loadability is of great importance to the department of power system designing and operating of power supply bureaus for the indication of load margin and the active constraint. Base on the determination of ML, power grid designers are able to make enhancement of reliability of city grids by making up the bottleneck problem. For another aspect, power grid operators can also make amelioration to the utilization of existing resources.

Various kinds of loadability problems in power system have been widely studied by several researchers [1-7]. For instance, voltage stability is a popular issue of loadability problem that focuses on identifying the load level at the bifurcation point that leads to voltage collapse [1-2]. Actually, it is a caution margin rather than a suitable operating point for power system. Another related topic is the Available Transfer Capability (ATC) problem, which emphasizes the max apparent power that can be transferred between a source-sink pair area [3-4]. It is a valued index in real time trading in the electricity market environment but makes much less sense when the focus turns to designing and operation of city grid. In recent years, certain studies of maximum loadability have been conducted specifically for distribution systems. F.Luo et al. [5] have proposed a practical, approximate method for Power Supply Capability (PSC) calculation based on N-1 contingency analysis of interconnected
transformers. J.Xiao et al. [6-7] have put forward a new concept named Total Supply Capability (TSC) for distribution system and proposed a mathematical model with a coordinated algorithm to evaluate the maximum permissible load rate of all the transformers.

As is partially mentioned above, existing studies and methods are not so applicable for the maximum loadability determination of 220kV city grid. Because of the general use of double-circuits transmission lines in 220kV city grids, it is necessary to pay more attention to the static security analysis on transmission lines. Comparing to radial distribution systems, the connection form and structure of 220kV city grids are much more complicated. Moreover, the scale of ultra-high-voltage city power grids is becoming larger and larger with the increment of urban electrical loads. It will be a rather burdensome work to solve the maximum loadability problem if complete model and precise calculating method are adopted with N-1 static security analysis. On the other hand, the solution of various simplified models is unrealistic for the practical city power grid, limiting the potential applications of these methods.

Confronted with these difficulties, this paper develops a practical method to calculate 220kV city grid maximum loadability. Firstly, the target 220kV grid is decoupled into several small networks consisting of several typical connection forms or their combinations. Then, a simplified model is proposed to calculate the maximum loadability of each small network. The problem of ML calculation is formulated as an optimization problem, and a self-adaptive differential evolution algorithm with built-in Newton-Raphson (N-R) method is presented for searching the optimal solution. This model takes N-1 static security constraints into consideration, and it is solved using accurate AC power flow model. Due to the proposed simplifying scheme, the calculation complexity is greatly reduced and the convergence performance is improved. Case study of a test network was conducted to verify the effectiveness and efficiency of the simplifying scheme and optimization method.

**Simplifying Scheme of 220kV City Grid**

With the development of 500kV ultra-high-voltage power grids, 220kV city grids are becoming less closely connected, which are usually divided into several supplying districts in daily operation. Different districts supplied by different 500kV transformer substations have a relatively weak interaction with each other, showing relative independence. Variation of the load level in one supplying district will not lead to observable fluctuation of power flow in other supplying districts. Hence, it is possible to conduct simplifying analysis on 220kV city grids by decoupling them into several kinds of typical connection forms.

In 220kV city grids, the capacity of 500kV transformer stations equipped with reactive power compensation devices serving as the power source nodes are much greater than that of 220kV substations, the magnitude and angle of the voltage vector of 500kV source stations can nearly stay constant when the load level changes on 220kV substations. For this reason, the 500kV source station can be considered as a power injection node with its voltage given. On the other hand, with the widely use of terminal substations in 110kV power grid, the topology of power grids of 110kV or lower voltage levels is converted to radial structure, whose power flow is simplified to single direction(from higher voltage side to lower voltage side only). Thus, the load power of lower voltage level can be summarized up to the 220kV level without conducting complicated equivalence work.

According to the above analysis, we hold the assumption in this paper that the 500kV source stations are regarded as power injection nodes with given voltage value and the 220kV substations are treated as gathered load nodes. From this perspective, 3 kinds of typical connection forms can be discerned from the 220kV city grid after comprehensive analysis of the interconnecting relationship of those 220kV substations.
Figure 1 shows the single source radial network which is similar to the distribution system. Figure 2 shows the self-cure ring network which normally has one 500kV transformer station serving as the power source node while connecting a few 220kV substations successively as load nodes, with the last 220kV substation linking back to the 500kV source station. Figure 3 shows the double sources hand-in-hand network. Compared to the single source radial network, hand-in-hand network contains two 500kV stations at both end of the 220kV load chain, making the power flow more complicated.

**Optimization Model for Maximum Loadability Problem**

After the simplification, the problem size of estimating the maximum loadability of 220kV city grids has been greatly reduced to less than one tenth compared to the original scale. The focus turns from the entire grid system to different kinds of concise typical connection forms. In this section, a precise non-linear programming model is derived from operational research theories to formulate the problem of determining the maximum loadability of typical connection forms of 220kV city grid as an optimal power flow problem. In the power flow calculation, 220kV load substations are set to be PQ buses, and 500kV source stations to be slack buses. Based on the proposed simplifying scheme, the optimization model can be described as follows:

\[
\max P(x_c) = \sum_{i=1}^{m} P_i
\]

subject to:

\[
\begin{align*}
0 & \leq g^i(x_c, x_s) = \theta^i \quad k = 0, 1, 2, \ldots, n_c \\
0 & \leq h^i (x_c, x_s) \leq \bar{h}^i \quad k = 0, 1, 2, \ldots, n_c
\end{align*}
\]

(Eq. 1)

The objective function is aiming to maximize the summarized active power of all the 220kV load substations, where \( m \) is the number of 220kV load substations; \( x_c = \{S_{ii} | i = 1, 2, \ldots, m\} = \{P_{ii}^0, Q_{ii}^0 | i = 1, 2, \ldots, m\} \) is the control variable vector, \( S_{ii} \), \( P_{ii} \) and \( Q_{ii} \) are the apparent power, active power and reactive power of bus \( i \), respectively; \( x_s \) is the state variable vector, including voltage magnitude \( V_i (i = 1, \ldots, m) \) and phase-angle \( \theta_i (i = 1, \ldots, m) \) of bus \( i \), along with the active power \( P_{G}^i \) and reactive power \( Q_{G}^i \) of slack bus; \( n_c \) is the number of contingencies. As we consider the 500kV source stations as power injection nodes with given voltage ignoring their internal connection and operation patterns, so the contingency set includes only N-1 contingencies of 220kV transmission lines.

\( g^i(x) \) refers to the power flow equations. When the superscript \( k=0 \), it means the normal state with no contingency happening; when \( k>0 \), \( k \) refers to the \( k \) th contingency state, which means the outage of one circuit of the \( k \) th transmission line. \( g^i(x) \) is formulated as follows:

\[
\begin{align*}
\Delta P_i^k &= P_{li}^0 - S_{ii}^0 \cos \phi^l - \sum_{j=1}^{i} V_j^i (G_{ij}^i \cos \theta_j^i + B_{ij}^i \sin \theta_j^i) \\
\Delta Q_i^k &= Q_{li}^0 - S_{ii}^0 \sin \phi^l - \sum_{j=1}^{i} V_j^i (G_{ij}^i \sin \theta_j^i - B_{ij}^i \cos \theta_j^i)
\end{align*}
\]

\[
\begin{align*}
V_i^k &= \sqrt{P_i^k^2 + Q_i^k^2} \\
\theta_i^k &= \arccos \frac{P_i^k}{V_i^k} \\
\phi^l &= \arctan \frac{Q_i^k}{P_i^k}
\end{align*}
\]

(Eq. 2)
where, $\Delta P^i$ and $\Delta Q^i$ are the unbalance of active and reactive power of bus $i$ in the $k$th state, respectively; $P^i_0$ and $Q^i_0$ are the active and reactive generation power of bus $i$ in the normal state, respectively; $\varphi$ is the power factor angle; $G^i_{\theta}$ and $B^i_{\theta}$ are the real and imaginary part of the element of the node admittance matrix at row $i$ and column $j$ in the $k$th state; $\theta^i_{ij} = \theta^i_i - \theta^j_j$.

The inequality constraints vector $h^k(x)$ including load power limits, bus voltage limits and branch power flow limits is given by:

$$
h^k(x) = \begin{cases} 
S_i^l \leq S_i^k \leq S_i^u, & i = 1, 2, \ldots, m + 1 \\
V_i^l \leq V_i^k \leq V_i^u, & i = 1, 2, \ldots, m + 1 \\
S_{ij}^l \leq S_{ij}^k \leq S_{ij}^u, & (i, j) \in B \end{cases}
$$

(Eq. 3)

where, $S_i^k$ is the apparent power of bus $i$ in the $k$th state; $S_{ij}^k$ is the apparent power of branch connecting bus $i$ and bus $j$, $B$ is the branch set; $S_i^l, V_i^l$ and $S_{ij}^l$ are lower limits, $S_i^u, V_i^u$ and $S_{ij}^u$ are upper limits, respectively.

The optimization method applied to solve the mathematical model present above is a self-adaptive differential evolution algorithm (DE) with built-in Newton-Raphson (N-R) method, which is a population-based optimization algorithm widely used for various kinds of optimization problems for its simple principle and highly-strong robustness. The basic introduction of DE can be found in ref [9,10]. In this paper, we implant an iterative N-R method suitable for multi slack buses systems into DE algorithm to ensure the accuracy of the result of optimization. The population generated by DE algorithm will be sent into the N-R module to calculate the power flow and then it is checked, making sure that there is no any violation of equipment limitations and operating constraints.

The flow chart of determining ML problem using the presented optimization method is shown in Figure4:

![Figure 4. Flow chart of determining maximum loadability.](image-url)
Case Study

Case overview and parameter settings

In this paper, a composite network of two kinds of typical connection forms is adopted to test the proposed mathematical model and optimization method. The network connecting structure is shown in Figure 5:

![Figure 5. Connecting structure of a composite network.](image)

As shown in Figure 5, the composite network can be regarded as the combination of self-cure ring network and hand-in-hand network, which includes five 220kV substations numbered #1-#5 and two 500kV source stations numbered #6 and #7 and eight branches. All the branches are double-circuit lines. The bus data and branch parameters of the composite network are shown in Table 1 and Table 2, respectively.

In Table 1, the maximum load of a 220kV substation refers to the summarized power of the remaining transformers overloading to 120% when the transformer with the largest capacity is out of service. And because we are seeking the maximum loadability base on the current load level, load power at each 220kV substation should not be lower than their current value. From this point of view, the minimum load is set to the same level as the current load in the substation.

![Table 1. Bus data of test composite network.](image)

In this paper, the base voltage is 220kV and the base capacity is 100 MVA. During the process of optimization, we set and for all the PQ buses, and the voltages of slack bus 7 and 8 are set to 1.036 and 1.054, respectively. The phase-angle difference between two slack buses is 1.0°. For briefness, the power factor of all the nodes is set to 0.98. For the DE algorithm, we set the maximum iterative
number \( G_n = 600 \), the population size \( N_p = 20 \), the scale factor \( F_{\text{min}} = 0.5 \) and \( F_{\text{max}} = 0.9 \), the crossover rate \( C_{\text{Rmin}} = 0.5 \) and \( C_{\text{Rmax}} = 0.9 \).

Maximum loadability determination and analysis

After implementing the optimization method, the optimal result can be obtained and shown in Table 3–Table 4. Table 3 shows the maximum loadability of the test composite network under all the constraints shown in Table 1–Table 2 and the load level of each 220kV substation respectively.

<table>
<thead>
<tr>
<th>Substation</th>
<th>Voltage [kV]</th>
<th>Active power [MW]</th>
<th>Apparent power [MVA]</th>
<th>Load rate [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>#1</td>
<td>226.55</td>
<td>285.05</td>
<td>290.87</td>
<td>53.864</td>
</tr>
<tr>
<td>#2</td>
<td>226.53</td>
<td>352.81</td>
<td>360.01</td>
<td>50.001</td>
</tr>
<tr>
<td>#3</td>
<td>227.57</td>
<td>264.61</td>
<td>270.01</td>
<td>50.003</td>
</tr>
<tr>
<td>#4</td>
<td>230.08</td>
<td>211.65</td>
<td>215.96</td>
<td>59.990</td>
</tr>
<tr>
<td>#5</td>
<td>229.96</td>
<td>564.44</td>
<td>575.96</td>
<td>79.994</td>
</tr>
<tr>
<td>ML</td>
<td>—</td>
<td>1678.56</td>
<td>—</td>
<td>—</td>
</tr>
</tbody>
</table>

From Table 3, it can be seen that #1, #4 and #5 substations get an increment of load level, reaching 53.864%, 59.990% and 79.994%, respectively. #2 and #3 substations remain the same load rate as the current situation. The maximum loadability of the entire test composite network is 1678.56MW.

<table>
<thead>
<tr>
<th>Line State</th>
<th>Normal</th>
<th>“N-1”</th>
<th>“N-1”</th>
<th>“N-1”</th>
<th>“N-1”</th>
<th>“N-1”</th>
<th>“N-1”</th>
<th>“N-1”</th>
</tr>
</thead>
<tbody>
<tr>
<td>1(#6-#1)</td>
<td>56.29</td>
<td>81.18</td>
<td>53.49</td>
<td>61.96</td>
<td>65.10</td>
<td>55.62</td>
<td>55.95</td>
<td>57.23</td>
</tr>
<tr>
<td>2(#1-#2)</td>
<td>19.60</td>
<td>6.05</td>
<td>33.37</td>
<td>24.62</td>
<td>28.45</td>
<td>18.38</td>
<td>19.25</td>
<td>20.42</td>
</tr>
<tr>
<td>3(#2-#3)</td>
<td>28.96</td>
<td>44.14</td>
<td>31.40</td>
<td>46.31</td>
<td>22.04</td>
<td>28.83</td>
<td>29.25</td>
<td>27.99</td>
</tr>
<tr>
<td>4(#6-#3)</td>
<td>71.71</td>
<td>82.41</td>
<td>73.70</td>
<td>68.30</td>
<td>100.00</td>
<td>68.18</td>
<td>70.60</td>
<td>74.41</td>
</tr>
<tr>
<td>5(#3-#4)</td>
<td>30.72</td>
<td>29.87</td>
<td>30.32</td>
<td>30.68</td>
<td>26.69</td>
<td>39.28</td>
<td>30.05</td>
<td>30.90</td>
</tr>
<tr>
<td>6(#4-#5)</td>
<td>27.96</td>
<td>26.50</td>
<td>27.69</td>
<td>28.45</td>
<td>24.14</td>
<td>26.75</td>
<td>44.22</td>
<td>42.73</td>
</tr>
<tr>
<td>7(#5-#7)</td>
<td>48.24</td>
<td>49.68</td>
<td>48.48</td>
<td>47.69</td>
<td>51.64</td>
<td>48.23</td>
<td>53.63</td>
<td>64.87</td>
</tr>
<tr>
<td>8(#7-#4)</td>
<td>46.46</td>
<td>49.14</td>
<td>46.87</td>
<td>45.36</td>
<td>52.57</td>
<td>45.46</td>
<td>43.65</td>
<td>56.74</td>
</tr>
</tbody>
</table>

Table 4 shows the load rates of all the transmission lines under normal and all the contingency states. Intuitively, we can find that the load level of Line4 hits the upper limit of 100% when the N-1 contingency occurs on it. Obviously, this is the active constraint limiting the loadability to get extra increment. Taking further investigation, it can be seen from the 5th column in Table 4 that when Line4 is fully loaded, Line1, Line2 and Line3 are still showing certain redundancy on capacity. This phenomenon indicates that there are certain defects in the existing structure of the test composite network, causing the imbalance of power flow distribution among the transmission lines. To relieve this problem, updating or expanding the transmission lines to restructure the network is one of the practicable measures to be done.

Summary

This paper has derived a practical simplifying scheme and a non-linear optimization model with N-1 static security constraints aiming at the 220kV city grid maximum loadability determination. And a self-adaptive DE algorithm with built-in N-R method is adopted to search the optimal solution. The results of case study have confirmed the validity of the optimization model and the preciseness and efficiency of DE algorithm. The maximum loadability, load levels of the 220kV substations as well as the bottleneck that limiting the loadability can be all clearly identified by implementing the optimization method. The optimal result can be applied for long term operation. Besides, targeted
adjustments and updates can be taken for debottlenecking according to the information provided by the optimization method presented in this paper. Detailed models such as the internal interrelationship of transformer substations can be added conveniently in future studies if needed.

Acknowledgements

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References


