An Interference Fit Algorithm for Multi-layer Thick-Walled Cylinders

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Keywords: Interference fit, Multi-layer thick-walled cylinders, Contact pressure.

Abstract. Interference fit is an important contact mode used for torque transmission, which exists widely in engine design, such as the contacts between the main bearing seat and the engine body, the engine bearing liner and the camshaft bearing seat, the cam and the hollow camshaft and so on. To prevent trackslip, a certain magnitude of interference has to be ensured; meanwhile, the interference needs to be controlled to avoid failure of the mechanical components caused by high pressure. Hence, to transmit torque effectively, the interference fit should be within a reasonable range. Although each country has its own national standard for interference fit, they are either too coarse or too confined for application. An easier interference fit algorithm for concentric cylinders with more flexible and popular application is established in this study. Furthermore, the algorithm is adopted to design a new electronically controlled diesel engine.

Introduction

Key pins and spline are commonly used as the connect device to transmit torque from transmission shaft to gear, pulley, belt pulley and the chain wheel. Besides, the torque can also be transferred by interference fit [1, 2] with lower costs, since no connecting devices, such as keyway, pin hole and spline tooth, are in need. In addition, interference fit has the advantages of high bearing capacity, and able to withstand composite loads, and has been widely applied in heavy machinery, ship engine, general machinery and engines [3].

In interference fit, the assembly bore of gear or sprocket is smaller than the axle diameter, and the transmission device is usually set on the shaft by compress fit or shrinkage fit [4]. The application of interference fit on transferring torque involves complex fits among multi-coaxial components, the stress calculation of which is relatively complicated. Finite element method (FEM) can be used to analyze the stress [4, 5], while the computational cost of FEM involving contact algorithm is relatively high [6], and likely to come across convergence problems.

The interference fit problems in engineering machinery can be simplified to interference fit contacts between the Multi-layer Thick-walled Cylinders (MLTWC). In this study, a general analysis method for Multi-layer Thick-walled Cylinders (MLTWC) with multi-contact pairs is developed according to the solution from elastic theory of axisymmetric thick-walled cylinder.

Methods

Interference Quantity Design of MLTWC

A certain amount of interference has to be assured to prevent interference surface skid in mechanical transmission device. Meanwhile, the interference quantity cannot be too big to avoid that the component stress falls into plastic zone and lose efficacy. Therefore, the size and tolerance of components must be designed in a reasonable range to realize a befitting interference quantity and thus the effective torque transmission. Figure 1 shows the interference fit of MLTWC with $n$ thick-walled cylinders.
The designed transmission torque $M_{\text{design}}$ can be determined according to design requirements, which is the maximum torque that can be transmitted by the interference surfaces. The actual maximum torque needs to be larger than the designed transmission torque by a rational interference quantity, i.e., $M_i > M_{\text{design},i}$. There exists simple quantitative relationships between fit pressure $P$ and the actual maximum torque $M$, i.e., $M = g(P, \mu, S, R)$. Thus the relationship between the interference $\delta$ and the maximum transmission torque $M$ can be found if the quantitative description of $\delta$-$P$ is determined. Thus, the lower limit of the interference quantity $\delta_{\text{min}}$ can be determined by the designed torque.

An overlarge interference will lead to such high fit pressure that there will be stress concentration inside the components. If the stress exceeds the plastic yield stress, i.e., $\sigma_{\text{max},i} > \sigma_s$, the MLTWC will lose efficacy due to local unrecoverable deformation. Hence, there exists an upper limit $\delta_{\text{max}}$ resulting from the limit of the maximum stress.

### Relationships between Interference and Tolerance

Suppose the axial diameter is $\phi D_{\omega_1}^+$, and the bore diameter is $D_{-\omega_2}^+$. There exists relationships between maximum interference $\delta_{\text{max}}$ and the tolerance for a certain pair of interference fit, as shown in Eq. (1):

$$\delta_{\text{max}} = 0.5 \times (\omega_1 + \omega_4)$$  \hspace{1cm} (1)

The minimum interference $\delta_{\text{min}}$ also has relationship with the tolerance, as in Eq. (2):

$$\delta_{\text{min}} = 0.5 \times (\omega_2 - \omega_3)$$  \hspace{1cm} (2)

The deviation value of axis and bore in the interference pair can be totally determined with known tolerance zone, i.e., the value of $\omega_1 + \omega_2$ and $\omega_3 + \omega_4$.

### Relationships between Interference and Temperature

The interference fit for torque transmission often works within certain temperature range. The fit pressure changes when the temperature changes if the coefficients of linear expansion of the contacted pair disagree, and leads to sliding failure. Therefore, the temperature effect has to be taken into account when designing the tolerance system of the MLTWC. Suppose the interference of the contact surface is $\delta_0$, and the working temperature is $\Delta T$ higher than the design temperature. If the linear expansion coefficient of inner cylinder wall is larger than that of outer cylinder wall, the interference increases; otherwise, the interference decreases. The relationship between the interference and the temperature rise is shown in Eq. (3),

$$\delta = \delta_0 + \delta_T = \delta_0 + (\alpha_i - \alpha_o) \times R \times \Delta T$$  \hspace{1cm} (3)
where \( \delta_T \) is interference change resulting from temperature rise; \( \alpha_i \) and \( \alpha_o \) represent the coefficients of inner and outer cylinder walls, respectively; and \( R \) is the radius at the contact surface.

**Multiple Contact Analysis of the MLTWC**

The interference fit between the \( i^{th} \) layer and \((i+1)^{th}\) layer of the MLTWC is shown in Figure 1, which is composed of the outer surface of the \( i \)-th thick-wall cylinder and the inner surface of the \( (i+1) \)-th cylinder. The radial displacement \( u \) of these two surface needs to satisfy the displacement coordination described in Eq. (4), so that there is no gap or penetration between these two surfaces.

\[
\delta_{i,i+1}^n - \delta_{i,i+1}^{out} = \delta_{i,i+1} \quad (i = 1, 2 \ldots n - 1)
\]

where the superscript represents the inner and outer surface, respectively.

Under the hypothesis of small elastic deformation, the interference fit of \( n \) MLTWC satisfies superposition principle, which can be equivalent to \( n \) axisymmetric thick-wall cylinders with uniformly distributed interior and exterior pressure. According to the elastic theory solution, the radial displacement of the outer surface of the \( i \)-th cylinder is shown in Eq. (5).

\[
u^{out}_i = \frac{R_i(P_{i-1}(R_{i-1} - \delta_{i-1}))^2 - P_iR_i^2(1 - \nu_i)}{E_i(R_i^2 - (R_{i-1} - \delta_{i-1})^2)} + \frac{(P_{i-1} - P_i)(R_{i-1} - \delta_{i-1})^2R_i(1 + \nu_i)}{E_i(R_i^2 - (R_{i-1} - \delta_{i-1})^2)}
\]

Since \( \delta_{i-1,i} \) is small compared to \( R_{i-1} \), Eq. (5) can be simplified as in Eq. (6).

\[
u^{out}_i = \frac{R_i[P_{i-1}R_i^2 - R_i^2 + R_i^2(1 - \nu_{i+1})](1 - \nu_i)}{E_{i+1}(R_i^2 - R_i^2)} + \frac{(P_{i-1} - P_i)R_i^2R_i^2(1 + \nu_i)}{E_{i+1}(R_i^2 - R_i^2)}
\]

Similarly,

\[
u^{in}_i = \frac{R_i[P_{i}R_i^2 - R_i^2 + R_i^2(1 - \nu_{i+1})](1 - \nu_{i+1})}{E_{i+1}(R_i^2 - R_i^2)} + \frac{(P_{i} - P_i)R_i^2R_i^2(1 + \nu_i)}{E_{i+1}(R_i^2 - R_i^2)}
\]

Put Eq. (6) and Eq. (7) into Eq. (4), then

\[
\delta_{i,i+1} = \frac{R_i^2(2P_{i-1}R_i^2 - P_i(-R_i^2(1 + \nu_{i+1}) + R_{i-1}^2(1 - \nu_i))}{E_i(R_i^2 - R_i^2)} + \frac{2P_{i+1}R_i^2 - P_i(R_i^2 + R_i^2) + P_i(R_i^2 - R_i^2)\nu_{i+1}}{E_{i+1}(R_i^2 - R_i^2)}
\]

Expand it and group terms, Eq. (9) can be derived.

\[
\delta_{i,i+1} = \frac{2R_i^2(P_{i-1} - P_i)}{E_i(R_i^2 - R_i^2)} + \frac{R_{i}R_{i-1}^2 - P_iR_i^2 + P_iR_i^2 + P_iR_i^2}{E_{i+1}(R_i^2 - R_i^2)} + \frac{R_i^2R_i^2 + R_i^2}{E_{i+1}(R_i^2 - R_i^2)} - P_{i+1}
\]

It can be seen from Eq. (9) that \( P_{i-1}, P_i \), and \( P_{i+1} \) all have their own contributions to \( \delta_{i,i+1} \), which is determined by the parameters of material and geometrical characteristics. Represent the coefficients of \( P_{i-1}, P_i \), and \( P_{i+1} \) in Eq. (9) by \( S_{i,i-1}, S_{i,i}, \) and \( S_{i,i+1} \), respectively, and Eq. (9) can be rewritten as:

\[
\begin{align*}
\delta_{i,i+1} &= S_{i,i-1}P_{i-1} + S_{i,i}P_i + S_{i,i+1}P_{i+1} \\
S_{i,i-1} &= \frac{2R_i^2(P_{i-1} - P_i)}{E_i(R_i^2 - R_i^2)} \\
S_{i,i} &= \frac{R_{i}R_{i-1}^2 - P_iR_i^2 + P_iR_i^2 + P_iR_i^2}{E_{i+1}(R_i^2 - R_i^2)} + \frac{R_i^2R_i^2 + R_i^2}{E_{i+1}(R_i^2 - R_i^2)} \quad (i = 1, 2 \ldots n - 1)
\end{align*}
\]

Combining \( n \) thick-wall cylinders together, there appears \( n-1 \) pair of interference fit, and \( n-1 \) Equations can be established as in Eq. (11).
\[
\begin{align*}
\delta_{1,2} &= S_{1,0} P_0 + S_{1,1} P_1 + S_{1,2} P_2, \quad i = 1 \\
\delta_{2,3} &= S_{2,1} P_1 + S_{2,2} P_2 + S_{2,3} P_3, \quad i = 2 \\
&\vdots \\
\delta_{n-2,n-1} &= S_{n-2,n-3} P_{n-3} + S_{n-2,n-2} P_{n-2} + S_{n-2,n-1} P_{n-1}, \quad i = n - 2 \\
\delta_{n-1,n} &= S_{n-1,n-2} P_{n-2} + S_{n-1,n-1} P_{n-1} + S_{n-1,n} P_n, \quad i = n - 1 
\end{align*}
\]

Rewrite Eq. (11) to matrix form:

\[
\begin{bmatrix}
\delta_{1,2} \\
\delta_{2,3} \\
\vdots \\
\delta_{n-2,n-1} \\
\delta_{n-1,n}
\end{bmatrix} =
\begin{bmatrix}
S_{1,0} & S_{1,1} & S_{1,2} \\
S_{2,1} & S_{2,2} & S_{2,3} \\
& & \ddots \\
S_{n-2,n-3} & S_{n-2,n-2} & S_{n-2,n-1} \\
S_{n-1,n-2} & S_{n-1,n-1} & S_{n-1,n}
\end{bmatrix}
\begin{bmatrix}
P_0 \\
P_1 \\
& \ddots \\
P_{n-1} \\
P_n
\end{bmatrix}
\]

(12)

\(n-1\) values of \(P_1, P_2, \ldots, P_{n-1}\) can be derived from Eq. (12) in total.

Based on the elastic theory solution of axisymmetric thick-wall cylinders under uniformly distributed interior and exterior pressure, the principal stress of the \(i^{th}\) thick-wall cylinder is shown in Eq. (13).

\[
\begin{align*}
\sigma_r &= \frac{P_i - p_{i-1}}{R_i^2 - R_{i-1}^2} R_i^2 + \frac{1}{r^2} R_i^2 R_{i-1}^2 (P_i - p_{i-1}) \\
\sigma_\theta &= \frac{P_i - p_{i-1}}{R_i^2 - R_{i-1}^2} R_i^2 - \frac{1}{r^2} R_i^2 R_{i-1}^2 (P_i - p_{i-1}) \\
\sigma_z &= 0
\end{align*}
\]

(13)

Case Studies

When complicated MLTWC is degenerated to double-layer or triple-layer thick-walled cylinders, superposition principle of elastic mechanics can be used to obtain the relationship between contact pressure and the magnitude of interference. Taking single contact pair between double-layer thick-walled cylinders as an example, the relative error between theoretical solution and finite element analysis solution is no more than 1% for both cases we studied, as shown in Table 1. For triple-layer thick-walled cylinders, with the same size of contact surface and the same magnitude of interference, the relative error is also within 1%, shown in Table 2.

### Table 1. Interference fit analysis for double-layer thick-walled cylinders.

<table>
<thead>
<tr>
<th>Contact Pressure</th>
<th>FEA Solution (MPa)</th>
<th>Theoretical Solution (MPa)</th>
<th>Relative Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>contact pair (\phi=67)</td>
<td>9.8894</td>
<td>9.8498</td>
<td>-0.40%</td>
</tr>
<tr>
<td>contact pair (\phi=140)</td>
<td>7.5513</td>
<td>7.5756</td>
<td>0.32%</td>
</tr>
</tbody>
</table>

### Table 2. Interference fit analysis for triple-layer thick-walled cylinders.

<table>
<thead>
<tr>
<th>Contact Pressure</th>
<th>FEA Solution (MPa)</th>
<th>Theoretical Solution (MPa)</th>
<th>Relative Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>contact pair (\phi=67)</td>
<td>14.5879</td>
<td>14.5441</td>
<td>-0.30%</td>
</tr>
<tr>
<td>contact pair (\phi=140)</td>
<td>10.9596</td>
<td>11.0214</td>
<td>0.56%</td>
</tr>
</tbody>
</table>

Conclusion

This study provides a general algorithm for interference fit of MLTWC, which can be extended to the calculation of interference contact pressure under the condition of temperature rising and centrifugal force[7]. The diesel engine designed on the basis of the algorithm has completed the road test of 6500 km, and the reliability test of 750 hours. Further study is expected on the interference fit between thick-walled and thin-walled cylinders with elastoplastic deformation.
Acknowledgments
This research was supported by National Natural Science Foundation of China (Grant No.11402136).

References