Understanding Permeability and Pore Throat Radius in Sandstone Reservoirs

Chun-mei XU*, Li-ping MA, Xiao-lin QIU, Jian-ke REN, Fan TANG, Rong ZHANG, Xiao-rong LI and Hai-en YANG

National Engineering Laboratory for Exploration and Development of Low Permeability Oil and Gas Field, China

Oil & gas technology research institute of Changqing Oilfield Company, Shaanxi Province 710018, China

*Corresponding author

Keywords: Permeability, Capillary pressure curve, Pore radius, Throat radius.

Abstract. The pore throat radius in sandstone reservoirs is an important factor in considering oilfield development and project design. In this paper, the core test results of sandstone reservoir with different permeability and casting thin-section analysis are used for explaining the variations of pore throat radius and permeability, simulating the relation expression of permeability with average pore radius and throat radius. Consequently, the pore throat size of sandstone reservoir with certain permeability can be determined conveniently.

Introduction

In sandstone reservoirs, pore throat size is such a crucial factor in determining the ability of oil and gas flow in porous media. Moreover, it is emphasized in designing injection medium indicators, such as water injection index, profile control and water shutoff program, polymer flooding program, etc.

There are various techniques to study pore throat radius[1]. However, researches on the relations between permeability of large range and throat radius are almost absent, especially on a certain reservoir. In this paper, the capillary pressure curve which is measured by mercury injection method with 51 pieces of sand reservoir cores and analyzing data of 25 casting slices are incorporated into researches of sand permeability and pore throat radius.

Relations between Sand Reservoir Permeability and Average Pore Radius

Casting thin-section analytic statistic is the main method of studying pore size distribution. In other word, the rock with colored resin squeezing is cut into slices and then analyzes under a microscope. It can observe the pore, pore throat and their interactive two-dimensions easily. Meanwhile, pore type, shape and average diameter can also be determined[1].

25 pieces of core with different permeability were analyzed by casting thin-section method. The corresponding data between permeability and average pore radius are listing in Table 1. The relation curve is listing in Figure 1.

The under charts show that average pore radius increased with the increase of permeability. However, in different permeability range, the pore size varies significantly different with permeability. When permeability is less than 40×10^{-3} \mu m^2, the average pore size increases rapidly with the increase of the permeability. When permeability is more than 100×10^{-3} \mu m^2, the pore size varies slightly with the variation of permeability. The expression of relations between pore radius and permeability in sand reservoirs can be correlated by the data form Table 1. The formula is:

\[ \bar{r}_p = 6.4092 \ln k + 19.867 \]  (1)
Relations between Sand Reservoir Permeability and Throat Radius

The capillary pressure curve measurement is a classic method to study the distribution of pore throat size. Physical parameters for quantitatively describing throat size distribution are including displacement pressure, capillary pressure with saturation median, minimum unsaturated pore percentage and pore sorting coefficient, etc. Displacement pressure is corresponding to the core’s maximal communicating throat radius; and capillary pressure with saturation median is corresponding to the median throat radius, almost to average throat radius. The analysis results of capillary pressure curve from 51 pieces of sand cores by mercury intrusion method are illustrated in Table 2.

Figure 2, which is derived from the data in Table 2, clearly explains that in extra-tight reservoirs of permeability less than $10 \times 10^{-3} \mu m^2$ and middle-permeability reservoirs with perm. of $100-400 \times 10^{-3} \mu m^2$, the max throat radius varies greatly but the average throat radius stays relatively stable in all levels of permeability. According to the data of Table 2, we can correlate the expressions of permeability with max/average pore throat radius as follows:
\[ r_{p,\text{max}} = 2.145 \ln k + 3.075 \]  
(2)

\[ \overline{r_p} = 0.3896k^{0.4977} \]  
(3)

### Table 2. Mercury injection experiment data.

<table>
<thead>
<tr>
<th>No.</th>
<th>Perm. (10^{-3} \mu \text{m}^2)</th>
<th>Max. Throat Radius (\mu \text{m})</th>
<th>Avg. Throat Radius (\mu \text{m})</th>
<th>No.</th>
<th>Perm. (10^{-3} \mu \text{m}^2)</th>
<th>Max. Throat Radius (\mu \text{m})</th>
<th>Avg. Throat Radius (\mu \text{m})</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.056</td>
<td>0.2013</td>
<td>0.0621</td>
<td>27</td>
<td>0.736</td>
<td>1.608</td>
<td>0.4394</td>
</tr>
<tr>
<td>2</td>
<td>0.103</td>
<td>0.4229</td>
<td>0.1152</td>
<td>28</td>
<td>0.781</td>
<td>1.697</td>
<td>0.4296</td>
</tr>
<tr>
<td>3</td>
<td>0.118</td>
<td>0.3861</td>
<td>0.1164</td>
<td>29</td>
<td>0.821</td>
<td>2.32</td>
<td>0.5373</td>
</tr>
<tr>
<td>4</td>
<td>0.15</td>
<td>0.4191</td>
<td>0.1567</td>
<td>30</td>
<td>0.838</td>
<td>1.687</td>
<td>0.4783</td>
</tr>
<tr>
<td>5</td>
<td>0.153</td>
<td>0.4099</td>
<td>0.1221</td>
<td>31</td>
<td>0.862</td>
<td>2.144</td>
<td>0.434</td>
</tr>
<tr>
<td>6</td>
<td>0.183</td>
<td>0.3032</td>
<td>0.0918</td>
<td>32</td>
<td>0.927</td>
<td>1.554</td>
<td>0.3843</td>
</tr>
<tr>
<td>7</td>
<td>0.22</td>
<td>0.2825</td>
<td>0.0917</td>
<td>33</td>
<td>1.056</td>
<td>2.8654</td>
<td>0.6056</td>
</tr>
<tr>
<td>8</td>
<td>0.247</td>
<td>0.4174</td>
<td>0.1232</td>
<td>34</td>
<td>1.44</td>
<td>3.398</td>
<td>0.7632</td>
</tr>
<tr>
<td>9</td>
<td>0.263</td>
<td>1.2448</td>
<td>0.2111</td>
<td>35</td>
<td>1.76</td>
<td>3.116</td>
<td>0.5567</td>
</tr>
<tr>
<td>10</td>
<td>0.274</td>
<td>0.5638</td>
<td>0.2293</td>
<td>35</td>
<td>1.76</td>
<td>3.116</td>
<td>0.5567</td>
</tr>
<tr>
<td>11</td>
<td>0.277</td>
<td>0.5752</td>
<td>0.1817</td>
<td>36</td>
<td>2.25</td>
<td>4.058</td>
<td>1.017</td>
</tr>
<tr>
<td>12</td>
<td>0.3</td>
<td>0.7637</td>
<td>0.2301</td>
<td>37</td>
<td>2.3</td>
<td>4.582</td>
<td>0.8274</td>
</tr>
<tr>
<td>13</td>
<td>0.306</td>
<td>1.048</td>
<td>0.2814</td>
<td>38</td>
<td>4.08</td>
<td>4.9968</td>
<td>1.1093</td>
</tr>
<tr>
<td>14</td>
<td>0.323</td>
<td>0.9231</td>
<td>0.265</td>
<td>39</td>
<td>7.43</td>
<td>7.668</td>
<td>1.404</td>
</tr>
<tr>
<td>15</td>
<td>0.336</td>
<td>0.7622</td>
<td>0.2259</td>
<td>40</td>
<td>12.9</td>
<td>8.05</td>
<td>1.242</td>
</tr>
<tr>
<td>16</td>
<td>0.352</td>
<td>0.9555</td>
<td>0.278</td>
<td>41</td>
<td>15.9</td>
<td>7.956</td>
<td>1.818</td>
</tr>
<tr>
<td>17</td>
<td>0.395</td>
<td>0.7119</td>
<td>0.0621</td>
<td>42</td>
<td>22.9</td>
<td>7.88</td>
<td>1.854</td>
</tr>
<tr>
<td>18</td>
<td>0.483</td>
<td>1.299</td>
<td>0.4244</td>
<td>43</td>
<td>26.2</td>
<td>8.174</td>
<td>1.94</td>
</tr>
<tr>
<td>19</td>
<td>0.485</td>
<td>1.294</td>
<td>0.3393</td>
<td>44</td>
<td>29.9</td>
<td>10.48</td>
<td>2.055</td>
</tr>
<tr>
<td>20</td>
<td>0.49</td>
<td>0.8164</td>
<td>0.2778</td>
<td>45</td>
<td>35.8</td>
<td>10.32</td>
<td>2.196</td>
</tr>
<tr>
<td>21</td>
<td>0.492</td>
<td>0.738</td>
<td>0.2417</td>
<td>46</td>
<td>40.8</td>
<td>9.6</td>
<td>2.529</td>
</tr>
<tr>
<td>22</td>
<td>0.532</td>
<td>0.8431</td>
<td>0.293</td>
<td>47</td>
<td>78</td>
<td>10.85</td>
<td>2.915</td>
</tr>
<tr>
<td>23</td>
<td>0.551</td>
<td>0.8212</td>
<td>0.1912</td>
<td>48</td>
<td>103</td>
<td>13.56</td>
<td>3.601</td>
</tr>
<tr>
<td>24</td>
<td>0.665</td>
<td>1.541</td>
<td>0.3137</td>
<td>49</td>
<td>164</td>
<td>17.31</td>
<td>4.057</td>
</tr>
<tr>
<td>25</td>
<td>0.689</td>
<td>1.167</td>
<td>0.2765</td>
<td>50</td>
<td>311</td>
<td>16.91</td>
<td>3.978</td>
</tr>
<tr>
<td>26</td>
<td>0.702</td>
<td>1.853</td>
<td>0.5143</td>
<td>51</td>
<td>382</td>
<td>20.92</td>
<td>5.883</td>
</tr>
</tbody>
</table>

**Appendix**

In sandstone reservoir, the average pore radius increases with the increase of permeability. However, the pore size varies slightly with permeability in different permeability scales. When the perm. is less than \(40 \times 10^{-3} \mu \text{m}^2\), the average pore radius increases sharply with the increase of permeability. When the perm. is greater than \(100 \times 10^{-3} \mu \text{m}^2\), the average pore radius various slightly with permeability.

The expression of variation between average pore radius and permeability in sandstone reservoir is as follows:

\[ \overline{r_p} = 6.4092 \ln k + 19.867 \]  
(4)
In the extra-low perm reservoirs of permeability under $10 \times 10^{-3} \mu m^2$ and middle-perm reservoirs with permeability of $100 \sim 400 \times 10^{-3} \mu m^2$, the max. pore throat radius has a greater difference but the average throat radius with every permeability is relatively stable.

![Diagram of permeability and pore throat radius.](image)

The expressions of sandstone reservoir permeability with the maximum/average pore throat radius are as follows:

$$r_{p\text{max}} = 2.145 \ln k + 3.075 \quad \text{and} \quad \bar{r}_p = 0.3896 k^{0.4977} \tag{5}$$

**References**


Calculation of Time-Varying Meshing Stiffness of Helical Gear Based on ANSYS

X.C. Shi, D. Liu and L.J. Shan
School of Mechanical Engineering, Dalian Jiaotong University, Dalian 116028, China

Keywords: Helical gear, Time-varying mesh stiffness, Finite element method, Modification.

Abstract. The time-varying mesh stiffness is an important basic parameter of the gear system, directly affect the accuracy of the results of the dynamics. As the theoretical method is not accurate in the calculation of time-varying mesh stiffness. For a model of the gear system, the finite element method to calculate the stiffness is more accurate. In this paper, the finite element contact analysis model of the gear system is established by using ANSYS Parametric Design Language (APDL), the stiffness of the gear teeth is calculated, and compared with the model of non-modification, profile modification, axial modification and comprehensive modification.

Introduction

Gear transmission is one of the most important mode of transmission of modern mechanical transmission, is widely applied to various types of machinery and equipment, with the gear transmission system to develop in the direction of high speed, overloading, large-scale, relying solely on traditional statics model of tooth profile modification, such as spiral Angle modification has been far from enough to meet the use requirements, therefore, vibration, shock and noise of the gear system dynamic problems become the focus of the current study. The time-varying meshing stiffness as an important basic parameters of vibration differential equation of gear system, directly affects the accuracy of gear system dynamics calculation results, has very important significance on the research of dynamics.

The calculation method of time varying gear mesh stiffness of main material mechanics method, simplified square wave method and finite element method. The method of material mechanics, the general use of Ishikawa formula or improved Ishikawa equation, because of the simplification of the tooth, and to simulate the real load, the calculation result has certain deviation. A simplified method of square wave is relatively simple, but its results and the actual stiffness of large deviations, the deflection is not applicable to solving the gear system dynamics. Finite element method (FEM) can be divided into two kinds, one kind is to use a single gear finite element model of the single tooth load were obtained, then the tooth stiffness, this method is similar to the material mechanics method, the model is more accurate than the method of material mechanics, but also can not simulate real load; The second is using contact analysis model, simulation of a pair of gears the real tooth contact state and loading condition, deformation of each tooth is obtained, calculated deformation of a meshing period different meshing gear position, and then obtain a period of engagement of tooth meshing stiffness.

In this paper a locomotive traction gear system as the research object, established a pair of gears by using APDL, calculate a mesh cycle different meshing position of gear tooth deformation, get a time-varying meshing stiffness of gear meshing period.

Establish a Precise Finite Element Model

Because of the complexity of the helical gear transmission structure and contour curve, the distortion of the model which through external data import will be more obvious. This paper use design language APDL of ANSYS for helical gear modeling, design language APDL not only make the whole process of modeling and analysis has good flexibility and reusability, and it has
prominent advantages in such aspects as on the sensitivity study, parametric design, and topology optimization, can bring great convenience for solving and post-processing.

Model of helical gear in the ANSYS environment, according to the relevant parameters of the gear, with the tooth profile equations of coordinate transformation under the transverse tooth profile curve of single tooth, then get the complete gear. The tooth profile of the gear face is composed of four parts of the profile top arc, standard involute curve, root transition curve and fillet.

**The Formation of Standard Involute**

Standard involute form as shown in figure 1, when a line of NK for rb along the radius of the circle make pure roll, the line for a little bit of K trajectory is the circle involute, involute tooth is part of it.

We can get the principal coordinate equation of involute:

\[
\begin{align*}
    x &= \frac{rb}{\cos \alpha_k} \cos (\varphi_k + \theta_k) \\
    y &= \frac{rb}{\cos \alpha_k} \sin (\varphi_k + \theta_k)
\end{align*}
\]

Among them, rb is the radius of base circle; \( \theta_k \) is at the K point of the involute angle; \( \alpha_k \) is at the K point of the involute angle of pressure; the relationship between angle and pressure angle for: \( \theta_k = \tan \alpha_k \cdot \alpha_k \), \( \varphi_k \) is the polar angle corresponding to point B (the main starting point of involute) on the basic circle, \( \varphi_k \) is corresponding to any point on the main K involute angle, it can be seen from the figure: \( \varphi_k = \varphi_k + \theta_k \). Among them, \( \varphi_k = \frac{\pi}{2} - \frac{\varphi}{2} \), \( \varphi \) represents the base thickness corresponding to the circumferential angle, the calculation formula of tooth thickness can be obtained. According to the equation of involute, with \( \alpha_k \) as independent variables (in the range from 0 to \( \alpha_k \) tooth top circle pressure angle) set the key step, obtained a series of key points with a smooth curve connecting forms the main involute.

**The Formation of the Tooth Root Transition Curve**

For high speed, heavy load and large modulus of the locomotive traction gear, the root will have great bending stress when it works. Therefore, in order to improve the tooth root bending fatigue strength and ensure the tip clearance, as possible to increase tool tooth top round radius, so it use the single round head gear hob for processing.

The relationship between gear tooth root transition curve part and hob cutter tip arc is shown in figure 2~3.
Figure 2. Normal gear shape of a round-head hob.

Figure 3. Coordinate transformation of Tooth root transition curve.

The equation is:

\[
\begin{align*}
\begin{aligned}
x &= r \sin \phi - \left( \frac{a_1}{\sin \alpha'} + r_0 \right) \cos(\alpha' - \phi) \\
y &= r \cos \phi - \left( \frac{a_1}{\sin \alpha'} + r_0 \right) \sin(\alpha' - \phi)
\end{aligned}
\end{align*}
\]

In the equation: \( r \)—Gear pitch radius; \( x \)—modification coefficient; the normal line of the contact point; \( \alpha' \) is a variable parameter, its value changes within the range \( \alpha \sim 90^\circ \), set up step, then obtained a series of key points with smooth curve connecting to form the tooth root transition curve.

For helical cylindrical gear drive, due to the existence of the helix angle, making it with end of coincidence degree and axial coincidence degree, in the working process, the load is often formed by a plurality of pairs of teeth sharing, therefore establishing finite element model, based on the coincidence degree established on tooth meshing model. This paper establishes three tooth model as shown in figure 4.

Figure 4. A three-dimensional finite element model of gear.

The Calculation of Time-Varying Meshing Stiffness

The meshing stiffness of gear tooth is needed to define 1um deflection in 1mm tooth width on the meshing line load. As the gear contact ratio generally is not an integer, in gear meshing process,
while participating in the meshing tooth pairs changes periodically with time, each pair are involved in producing the meshing elastic deformation also periodic variation, in addition, tooth from the tip of the tooth to tooth in meshing process, elastic deformation is not the same, causing meshing stiffness with time as cyclical changes, the gear system becomes a nonlinear time-varying stiffness system.

**Finite Element Method for Calculating Time-Varying Meshing Stiffness**

With the deepening research of the dynamics of the gear system, the value accuracy requirements of time-varying meshing stiffness are also getting higher and higher, in this paper, to use ANSYS parametric language APDL to build a precise contact finite element model of the first and check the accuracy of the model, then calculating a meshing period each meshing position meshing stiffness based on reliable finite element model, considering the extracted tooth deformation amount and calculates the position of engagement of the length of thread engagement is complicated, this paper uses the APDL program calculating the length of thread engagement and extraction of contact deformation, improved the efficiency of the finite element method calculation of time-varying meshing stiffness.

**The Total Length of the Contact Line Calculation**

In helical gear meshing process, with the rotation of the gear, the total length of the contact line in the actual meshing state changes over time. The longer the actual contact line, the uniform load distribution on the surface of the tooth, tooth resulting from load deformation smaller, at this time of tooth meshing stiffness is bigger. Therefore, the total length of the contact line calculation of helical gear at any time on the analysis of the time varying meshing stiffness is significant.

In Figure 5, N1N2 is the length of theoretic meshing line, AD is the length of actual meshing line, in Figure 8 ADEF is meshing plane, LN, Ln+1, Ln+2, Ln+3 is contact line, in the gear rotating, the meshing plane remain motionless, the contact line moving down (or up), the contact line in meshing plane ADEF (shown in the solid part of the contact line) is actually in meshing state. The calculation formula to the total length of contact line L by the formula:
By the formula (3) to calculate the total length of the contact line are given in figure 10.

The Calculation Results of Time-Varying Meshing Stiffness

Based on the locomotive traction gear finite element model, in a meshing cycle each corner after loading to solve respectively, under the gear respectively in a mesh period of gear tooth deformation, according to the tooth stiffness calculation formula:

\[ k_0 = \frac{F_n}{r_b \cdot B \cdot \delta} \] (4)

In the equation: \( T \) — The torque of driving wheel; \( r_b \) — The radius of base circle; \( B \) — Tooth width; \( \delta \) — The tooth deformation along the meshing line.

Substituted the gear parameters and gear tooth deformation amount into the equation, calculated the time-varying meshing stiffness of the tooth without modification, profile modification, axial modification and comprehensive modification of tooth surface. The calculation results are shown in Figure 7~10.

Figure 7. Profile modification of meshing stiffness.   Figure 8. Profile modification of meshing stiffness.  
Figure 9. Profile modification of meshing stiffness.   Figure 10. Profile modification of meshing stiffness.

By the meshing stiffness curve before and after modification can be seen, the meshing stiffness roughly in 10.2~10.8N/(μm·mm), is smaller than theoretical formula calculated value. This is due to the non-solid gear blank structure coefficient can not accurately reflect the shape of the gear, and
finite element calculation includes shaft deformation and tooth surface contact deformation, leading the deformation larger than theoretical methods deformation, converted into stiffness is smaller.

From Figure 7~9, we can see that after modification the meshing stiffness fluctuation more smooth, in gear meshing process, the impact of varying meshing stiffness of gear system will be smaller, the effect of modification is obvious. And the comprehensive modification is the best.

From Figure 10 can be seen, time-varying mesh stiffness is positively related to the total length of the line of gear, meshing stiffness does not appear obvious alternating single and double tooth stiffness mutation, This is because the helical gears in meshing process, by the double teeth gradually into the single tooth, the length of meshing line is gradually reduced, the load is smoothly by double tooth load change to single tooth load, so the stiffness changes smoothly.

**Conclusion**

The gear of locomotive traction time-varying meshing stiffness is calculated by finite element method, and calculates the time-varying meshing stiffness of profile modification, axial modification and comprehensive modification, comparison of the results, it can conclude that:

(1) Because of considering the deformation of shaft and tooth surface contact deformation, the stiffness calculated by finite element method is smaller than it by the theoretical method.

(2) After the gear tooth modification, time-varying meshing stiffness fluctuation is smaller, in the gear meshing process, the impact of varying meshing stiffness will be smaller, the effect is obvious after modification. Finite element method for calculating time-varying meshing stiffness could feedback the effect of modification accurately.

(3) The time-varying meshing stiffness of helical gear has no obvious mutation, the meshing stiffness and the contact line length is positive related at any moment, the contact line is longer, the meshing stiffness is larger.

**References**


