Research on Greedy Reconfiguration Algorithm of Compressed Sensing Based on Image

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Abstract. Compressed sensing theory is a subversion of the traditional theory. The main content of this thesis is reconstruction algorithm. It’s the key of the compressed sensing theory, which directly determines the quality of reconstructed signal, reconstruction speed and application effect. In this paper, we have studied the theory of compressed sensing and the existing reconstruction algorithms. On the basis of summarizing the existing algorithms and models, we analyze the results such as PSNR, relative error, matching ratio and running time of them from image signal respectively. The convergence speed of CoSaMP algorithm is faster than that of the OMP algorithms, but it depends on sparsity K quietly. StOMP algorithm on image reconstruction effect is the best, and the convergence speed is also the fastest. Sadly, its accuracy is not as good as that of the OMP algorithm.

Introduction

Signal reconstruction algorithm is the core section of compressed sensing theory. The researchers proposed a series of algorithm to obtain sub optimal solution, such as minimum L1 regularization, Greedy iterative matching pursuit algorithm, Iterative threshold method, the minimum total variational method which is specially processed for the two dimensional images, and so on. Greedy pursuit algorithm is one of the main algorithms in the present study[1].

In this paper, we study the greedy algorithm in the compressed sensing signal reconstruction algorithm. In actual application the greedy algorithm is more meaningful than the convex optimization algorithm, and the greedy algorithm can meet the quality requirements of signal reconstruction in most cases, and even some of the quality of the greedy algorithm is better than the convex optimization algorithm. Based on compressed sensing theory and the existing reconstruction algorithm, the reconstruction algorithm based on the matching pursuit is studied, and a new algorithm is proposed, which is the step orthogonal matching pursuit algorithm.

OMP Algorithm Principles and Simulations

Matching Pursuit Algorithm (MP) is a greedy iterative algorithm. However, because the signal projection on the vector set of selected atom i.e. measurement matrix column, which makes the results of each iteration may not be optimal, so multiple iterations are needed in order to obtain a good convergence[2].

The OMP algorithm effectively overcome this problem, uses the principle of selecting atoms in matching algorithm, and it can be used to make the selected atom set to be orthogonal, so as to ensure the optimal iteration, which reduces the number of iterations. The OMP algorithm uses a lot of probability to reconstruct the original signal accurately, and the speed is faster than the minimum L1 norm method.
The Core Idea of OMP Algorithm

In the OMP algorithm, the reconstruction of the signal is realized by using the iterative method. Firstly, we obtain the column vector of the atoms in the observation matrix by means of multiple iterations and gradual selections. Each iteration must ensure that selected atom has the largest correlation with the current residual (redundant) vector, and then subtracting the relevant part from the observation vector to update the residual vector, until the number of iterations achieve the sparsity K, then stop iteration. Since the OMP algorithm will do orthogonal processings to all searched atoms before the projection of the signal, which is largely to increase the complexity of the OMP algorithm, resulting in signal reconstruction time longer.

Simulation of Image Signal Reconstruction Based on OMP Algorithm

In processing image, we need to transform the image, such as FFT, DCT, wavelet transform, transform the image into the sparse coefficient of the corresponding base, then the coefficient matrix is processed, and finally we can get the sparse restructure of the image by transforming back the processed coefficient.

In this paper, we use DWT for image processing. Firstly, we did grayscale image processing to “a school badge”, then reconstructed the image by using DWT, observed the recovery effect in different sampling rate at the same time. Since the recovery effect is very poor when the sampling rate is less than 0.3, therefore, the paper selected sampling rate( M/N) were 0.7, 0.6, 0.5, 0.4, 0.3. The effect of image restoration is shown in Figure 1.

![Original image](image1)
(a) Original image

![M/N=0.3](image2)
(b) M/N=0.3

![M/N=0.4](image3)
(c) M/N=0.4

![M/N=0.5](image4)
(d) M/N=0.5

![M/N=0.6](image5)
(e) M/N=0.6

![M/N=0.7](image6)
(f) M/N=0.7

Figure 1. Badge grayscale results reconstructed from OMP algorithm at different sampling rates.

In order to compare the effect of OMP algorithm in different sampling rate, Table 1 shows the PSNR value and recovery time after the reconstruction of OMP algorithm in the sample rate of 0.3 ~ 0.7.

<table>
<thead>
<tr>
<th>Sampling rate M/N</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
<th>0.6</th>
<th>0.7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Image</td>
<td>PSNR</td>
<td>time</td>
<td>PSNR</td>
<td>time</td>
<td>PSNR</td>
</tr>
</tbody>
</table>

By the simulation figure and data can be seen, the higher the sampling rate, the higher PSNR, the better the effect of image reconstruction, and then the recovery time getting longer and longer, which...
shows that the reconstruction effect of OMP algorithm is closely related to the sampling value of M. However, the study of compressed sensing theory is needed to recover the original signal in less effective information. Therefore, it is necessary to control the sampling value and find a better balance between the reconstruction time and the reconstruction quality.

**CoSaMP Algorithm Principles and Simulations**

In order to improve the convergence speed and the efficiency of the algorithm, Compressive sampling matching pursuit (CoSaMP) algorithm chooses more related atoms from atomic library and eliminates some irrelevant atoms at the same time[3].

We supposed the iterative step size of the algorithm is K, and the candidate set up to have a maximum of 3K atoms, then removed the most K atoms to ensure 2K atoms in the support set.

**The Core Idea of CoSaMP Algorithm**

The OMP algorithm can only achieve a precise selection of individual atoms, which greatly increases the computation time. But the CoSaMP algorithm introduced the idea of backtracking is to choose a number of atoms from the atomic library, and eliminate some atoms, which greatly improve the efficiency of the algorithm. In this algorithm, a set of K optimal sets is selected in the iterative process, finally the optimal estimation of the original signal x is obtained by the least square method.

**Simulation of Image Signal Reconstruction Based on CoSaMP Algorithm**

In order to unify the OMP algorithm simulation and make sure the simulation effects can be compared, we reconstructed the school badge also in the sampling rate 0.7, 0.6, 0.5, 0.4 and 0.3, the simulation effects are shown in Figure 3.

![Image Reconstruction](image)

Figure 2. Badge grayscale results reconstructed from CoSaMP algorithm at different sampling rates.

<table>
<thead>
<tr>
<th>Sampling rate M/N</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
<th>0.6</th>
<th>0.7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Image</td>
<td>PSNR</td>
<td>time</td>
<td>PSNR</td>
<td>time</td>
<td>PSNR</td>
</tr>
<tr>
<td>School badge</td>
<td>17.7781</td>
<td>1.231s</td>
<td>19.279</td>
<td>3</td>
<td>21.719</td>
</tr>
</tbody>
</table>

Table 2 shows the PSNR value and recovery time after the reconstruction of CoSaMP algorithm in the sample rate of 0.3~0.7.
From the simulation results, the CoSaMP algorithm can recover the reconstructed image well. At the sampling rate of 0.3, the PSNR value is 17.7781, which means at a lower sampling rate, the algorithm can still select the optimal atoms set. Combined with the idea of the combination algorithm, the reconstruction efficiency is significantly improved. When the sampling rate is high, the PSNR value is higher, the image restoration effect is better, the reconstruction time is also increased, but compared with the OMP algorithm, the CoSaMP algorithm selects the optimal set of atoms. So in the same case, the computation time is significantly reduced.

**StOMP Algorithm Principles and Simulations**

The goal of the Step shift orthogonal matching pursuit (StOMP) algorithm is to convert the signal into a negligible margin by a series of operations.

Supposed initial margin \( r_0 = y \), a matched filter \( \Phi^T r_{r-1} \) can be formed in the s state, which is used to identify the coordinates of all amplitudes greater than a specially selected threshold, using these selected coordinates to do the least squares, and then subtracting the least square fitting to obtain a new margin. After a certain number of state transitions, the whole process will end. By comparing the OMP algorithm, the StOMP can add more coefficients to each state. The StOMP algorithm only needs a certain number of state transitions, but the OMP algorithm needs more state transitions. The StOMP algorithm is faster than many other solving the sparse solution methods (Such as L1 minimum norm and OMP algorithm), so has more attraction in solving large-scale problems.

**The Core Idea of StOMP Algorithm**

The purpose of StOMP algorithm is to obtain the approximate value of the original signal by \( y = \Phi x_0 \). The algorithm runs in the S phase, and the original signal is restored by eliminating a series of margins \( r_1, r_2, \ldots \) to get a series of approximations \( x_0, x_1, \ldots \).

**Simulation of Image Signal Reconstruction Based on StOMP Algorithm**

In order to unify the OMP algorithm simulation, we also reconstructed the school badge in the sampling rate 0.7, 0.6, 0.5, 0.4 and 0.3, the simulation effects are shown in Figure 3.

![Figure 3. Badge grayscale results reconstructed from StOMP algorithm at different sampling rates.](image)

Table 3 shows the PSNR value and recovery time after the reconstruction of StOMP algorithm in the sample rate of 0.3~0.7.
Table 3. Comparison of PSNR and time under different sampling rate of StOMP algorithm.

<table>
<thead>
<tr>
<th>Sampling rate M/N</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
<th>0.6</th>
<th>0.7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Image</td>
<td>PSNR time</td>
<td>PSNR time</td>
<td>PSNR time</td>
<td>PSNR time</td>
<td>PSNR time</td>
</tr>
<tr>
<td>School badge</td>
<td>17.276 2 1.073s</td>
<td>19.563 0 1.431s</td>
<td>22.110 3 1.849s</td>
<td>24.842 0 2.417s</td>
<td>27.340 0 3.31s</td>
</tr>
</tbody>
</table>

The above solution show that StOMP algorithm can not only reconstruct restoration image at lower sampling rate, but also be shorter than CoSaMP algorithm in recovery time. On the recovery results, the PSNR value reconstructed from StOMP algorithm is higher than the OMP algorithm and CoSaMP algorithm at the same sampling rate. And sparsity dependence of StOMP algorithm is small, which greatly increase the real value of the algorithm.

Summary
In this paper, the relative error, matching degree and running time of the simulation effects of OMP, CoSaMP, StOMP algorithms are analyzed in different sampling rate. It can be seen that the StOMP algorithm is the most practical reconstruction algorithm in the three algorithms, and the CoSaMP algorithm is better than the OMP algorithm in the reconstruction time, but it has to be predicted in advance of the sparsity of signal, otherwise it is necessary to debug the sparse degree of the signal to achieve the recovery effect, which makes the CoSaMP algorithm reduces the real value. Since each iteration of the OMP algorithm, the algorithm can only select one of the best atoms, which makes the algorithm to calculate the time, but it can recover the original signal accurately.

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References
