Rayleigh Wave Depth-Frequency Analysis

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Keywords: Rayleigh waves, Near-surface, 2D Fourier transform, Depth-frequency analysis, Geological information extraction.

Abstract. Rayleigh wave survey is an important technique to solve the near-surface geological problems. In this paper, we developed a new method of Rayleigh wave depth-frequency analysis, which is based on 2D Fourier Transform theory. We establish the equation of Rayleigh wave depth-frequency analysis to demonstrate the relationship among frequency, wave number, phase velocity, wavelength, and penetration depth. It is more convenient to obtain a relevant depth-frequency spectrum by converting $f$-$k$ spectrum into the depth-frequency ($H$-$f$) spectrum with this relationship. We can arrive at better results by extracting the frequency and depth information from $H$-$f$ spectrum. We used the new method to test an actual dataset. The results show that the method has the ability to extract the near-surface geological information with high accuracy and strong capabilities of de-noising.

Introduction

Rayleigh waves has been obtained a wide range of applications on studying the Earth's internal structure, especially in near-surface stratigraphic architecture and attribute characteristics [1-4] since Rayleigh [5] found Rayleigh waves. Currently, there are mainly two kinds of methods for Rayleigh waves processing that are Multi-channel Analysis of Surface Waves (MASW) [3, 6] and Spectral Analysis of Surface Waves (SASW) [1-2, 7]. The core idea of them is that take the seismic records from the time-space ($t$-$x$) domain convert to frequency-wave number ($f$-$k$) domain by 2D Fourier transform to extract the dispersion curves of Rayleigh waves, and then extract the corresponding strata structure, velocity parameter and other information based on the dispersion curve characteristics. The information obtained by traditional methods is limited because they only create the relationship between propagation velocity and wavelength of Rayleigh waves, which affects the capacity to solve practical geological problems for Rayleigh waves.

In fact, the information implicit in Rayleigh wave field is very rich. As there is existed dispersion characteristic for Rayleigh waves, that the velocity is a function of the frequency, and only the velocity slightly changes that will lead to the frequency change [8]. Therefore, frequency is also a sensitivity parameter to detect geological abnormal body for Rayleigh wave survey. Furthermore, different physical properties of media have the role of frequency-selective absorption to wave motion, in the underground propagation process for seismic waves, the greater the medium of the physical property difference, the more the wave absorption. When there is geological anomaly in stratain, such as faults, fracture zone, gap, fluid or gas, and so on, which will speed up the energy attenuation effect for seismic waves, accompany by frequency attenuation or changes. So the geological anomalies body can be extracted based on the changes in energy and frequency. In order to expand the space of the existing methods and techniques to solve the actual geological problem, we proposed a Rayleigh surface waves depth-frequency ($H$-$f$) analysis method, which may directly obtain the variation law of the frequency of Rayleigh waves in depth domain, and we may directly solve practical geological problems through in-depth analyzing the reasons of frequency change and energy attenuation of Rayleigh waves.
Methodology

Spectrum Information of Rayleigh Waves in f-k Domain

The surface wave of different amplitudes, frequencies and wave numbers can be clearly identified in f-k domain. Also, the phase velocity is revealed by the ratio of frequency to wave number,

\[ v_R = \frac{f}{k}, \]

where \( v_R \) is phase velocity, \( f \) is frequency, and \( k \) is wave number.

It needs to further build up the relationship of velocity with depth in solving practical geological problems. Since there is a relationship between wavelength (\( \lambda_R \)), frequency (\( f \)) and phase velocity (\( v_R \)), and the energy of surface wave is mainly focused on the range of a wavelength [5], so the penetration depth is related to wavelength,

\[ \lambda_R = \frac{v_R}{f}, \]

(2)

\[ H = \beta \lambda_R, \]

(3)

where, \( H \) is penetration depth, \( \beta \) is the correction factor between wavelength and penetration depth.

Based on Equation 3 and 4, a relationship between phase velocity and penetration depth can be further established,

\[ H = \frac{\beta v_R}{f}. \]

(4)

Heukelom and Foster [9], Abbiss [10] took \( \beta \) equals 0.5, which called half-wavelength method. Error of the half-wavelength method has been proved to be great from practical case studies [11], which indicate that the correction factor \( \beta \) of penetration depth is related to Poisson's ratio of medium [11-12]. Based on the propagation characteristic of surface wave and the relationship between seismic energy and media properties, Yang [13] and Chen et al. [11] determined the correction factors \( \beta \) of different media (Figure 1). For media existing in nature, Poisson's ratio is from 0.1 to 0.48, and the correction factor \( \beta \) is from 0.53 to 0.875. For general rock, Poisson's ratio equals 0.25, correction factor \( \beta \) is approximately 0.65. The Poisson’s ratio of Quaternary soil is from 0.4 to 0.45, and correction factor \( \beta \) is from 0.79 to 0.84.

To Build Rayleigh Wave Velocity Spectrum

On the basis of the above method and principle, we may establish a Rayleigh wave depth-frequency analysis method. There are divided into four steps to build Rayleigh wave depth-frequency (\( H-v_R \)) spectrum.

The first step: Carry out f-k transform for Rayleigh wave data to obtain f-k spectrum.

The second step: Take equations (1) and (4) further transform to form the equations (5). Then sequentially calculate each phase velocity value, and then calculate every depth value which corresponding to the frequency value according to the equations (5). Next, make the energy value at the position in f-k domain map to the corresponding position in H-f domain.

\[
\begin{align*}
  v_R(k_j, f_j) &= \frac{f_j}{k_j}, & J &= -M, -M + 1, -M + 2, \ldots, 0, 1, 2, \ldots, M \\
  H(k_j, f_j) &= \frac{\beta v_R(k_j, f_j)}{f_j}, & I &= -N, -N + 1, -N + 2, \ldots, 0, 1, 2, \ldots, N
\end{align*}
\]

(5)

where, \( J \) and \( I \) are the index of the \( k \) and \( f \) coordinate respectively, \( M \) is the total number of seismic channels after side-expanding, and \( N \) is the number of sample per trace after side-expanding. \( M \) and
$N$ are integer powers of 2.

The third step: Give the value range of frequency $f$ and depth $H$, and extract the data within the $H$-$f$ domain to grid and draw a contour map.

The fourth step: Pick the strong energy groups in the $H$-$f$ spectrum successively (determine the optimum phase velocity and penetration depth values with the maximum stacked energy in the $H$-$f$ spectrum), and extract the corresponding $(f, H)$ values to obtain $H$-$f$ curve.

So that we can get one Rayleigh wave $H$-$f$ spectrum and $H$-$f$ dispersion curve to be used to the Rayleigh wave depth-frequency analysis. The characteristics of the method are the frequency information extraction of the Rayleigh waves is very easy, and the frequency $f$ corresponding to the depth $H$ is intuitive, too.

**Application to Real Data**

We have processed a shot real seismic data which contains 280 traces. Channel spacing is 20m, 1000 sampling points per channel, and sampling rate is 4ms. The original shot records are shown in Figure 2a in which various wave field types are developed, and mainly develops Rayleigh waves, reflected waves, direct waves, and so on. As the Rayleigh waves developed in the first 85 traces, we extracted the first 85 traces seismic records to process (Figure 2b).

Figure 3b shows the $H$-$f$ spectrum (take $\beta$ as 0.79) which was mapped from $f$-$k$ spectrum in Figure 3a. Figure 3c is the extracted $H$-$f$ dispersion curves. The characteristics of the $H$-$f$ spectrum are also obvious with strong energy and concentrate. The frequency distribution range with strong energy is from 4 to 9.5Hz that corresponds to the depth distribution range is from 15 to 100 m (Figure 3b). From the inflection points information on $H$-$v_R$ curves and the energy groups in $H$-$v_R$ spectrum, we may divide three stratigraphic boundaries which corresponding to the depth at 24m with the frequency of 7.6 Hz, at 51 m with the frequency of 5.8 Hz, at 80 m with the frequency of 4.5 Hz (Figure 3c), and the frequency decreases with increasing depth, which better reflected the Rayleigh wave properties of the formation. There are two-stage strata from 20 to 27 m and from 30 to 70 m with seismic wave energy concentrated which may be soft strata leading to the formation of Rayleigh surface wave energy absorption exacerbated. These anomalies are better reflecting the response relationship between the frequency of the Rayleigh waves and the formation attribute information, and determining the corresponding relationship of the frequency with the depth of formation interface. Therefore, according to the $H$-$f$ spectrum and $H$-$f$ dispersion curves, we can effectively extract frequency response information of Rayleigh waves of strata at different depths, and thus direct access to the physical properties of the strata and structure information.
Summary

Frequency of Rayleigh waves is one of carriers of the strata property information. Frequency $f$, wave number $k$ and wavelength $\lambda_R$ of the seismic signal imply the information of the frequency $f$ and depth $H$ in $f$-$k$ domain. On the basis of 2D Fourier transform, we have formed an effective Rayleigh wave depth-frequency analysis method in pan $f$-$k$ domain after established the relationship of depth $H$ and frequency $f$, then taken $f$-$k$ spectrum map into $H$-$f$ spectrum. It may study the formation properties, structure and other aspects through the information of the extracted depth $H$ corresponding to the frequency $f$ to directly solve the actual geological problems. Real data application show that the method has the characteristics that the frequency $f$ information extraction of Rayleigh waves is simple and convenient, as well as frequency $f$ corresponding to depth $H$ is very intuitive, and the precision is higher.

Acknowledgements

This research was financially supported by the Petro China Innovation Foundation (#2014D-5006-0303), and Key Laboratory Research Project of Shaan’xi Provincial Department of Education (#13JS093).

References


