Research on Electromagnetic Transient Simulation of Transformer based on Controlled Source Theory

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Abstract. This paper studies the basic principle of electromagnetic transient simulation in power system, and digital simulation method, derives by the electromagnetic transient simulation model of power system components. According to component model, integrated with control principle and the damping trapezoidal integral method, it establishes the electromagnetic transient simulation model for the different connection mode of double winding transformer and three winding transformer. And it describes the solving methods of transformer electromagnetic transient simulation, adopts interior point shrinkage method for three winding transformer, which reduces the dimension of operation, improves the efficiency of simulation. Each simulation is verified by the simulated use case.

Introduction

Power transformer is one of the important equipment in the power system, it is often under long-term continuous charged operation in the operation of power system. Therefore, the transformer will inevitably occur over voltage, short circuit fault and inrush current and other transient process. There is a great difference between the transient operation and the stable state operation of the transformer. It is necessary to study transient process of transformer. It can effectively prevent accidents damage to further expand and transformer's damage, also can improve the design and manufacturing level of power transformer, provide theoretical basis for the transformer manufacturing and various the mechanism of operation and fault condition \cite{1}. Electromagnetic transient simulation is important means to study the transient process of transformer.

At present, there are mainly two methods on the transformer transient simulation , one is the application of the existing simulation software PSCAD, EMTP and MATLAB, etc., set up the simulated circuit modeling simulation, and study the transient process of transformer \cite{2,3}.Another is set up simulation based on the transformer physical structure and the basic principle of transient process.

The first method is simple and easy to grasp, but the limitation is it is unable to research and analyze on the internal principle of transformer. The second method is required a deep understanding of the transformer transient process principle, and methods of power system digital simulation. It needs a good theoretical knowledge on the researchers. However, the researchers can design the transformer transient model according to the requirements, which can help the depth study of transformer transient simulation. This article adopts the second simulation method.

This paper studies the basic principle of electromagnetic transient simulation in power system, and digital simulation method, proposes electromagnetic transient simulation method Based on the principle of controlled source and damping trapezoidal method, respectively establishes three-phase double-winding and three winding transformer electromagnetic transient simulation, validates the simulation based on simulation results.
Theoretical Basis of Electromagnetic Transient Simulation

Power system electromagnetic transient simulation essentially solve the time domain response of voltage and current of the system. It establishes mathematical model and the corresponding model calculation method. Mathematical model of electromagnetic transient simulation is mainly composed of constraint equations of power system network and components’ volt-ampere characteristic equations of the two parts [4]. Network constraint equation is commonly algebraic equation, while the element of volt-ampere characteristic equation is probably the algebraic equation and differential equation.

By solving with the constraint condition and components; volt-ampere characteristic equation simultaneously, and electromagnetic transient simulation of the node voltage equation can be formed. The node voltage equation can be represented as:

\[ YU = I \] (1)

However, the components' the differential equation can not be directly solved simultaneously with node equation to form network, it need to apply numerical integration method for its differentiation, algebraic form of differential equation is obtained, and network equations are simultaneous. Element difference equation can be considered as equivalent circuit with equivalent admittance and equivalent current source in parallel form, so the basic equation for electromagnetic transient simulation of components can be represented as:

\[ Gu = i \] (2)

Considering on the current vector at above formula, external network node injection current depends on the structure of the external network. For independent of a single component model, ignore the external influence first, then add this injection current after connecting with external network. So the independent components of the electromagnetic transient simulation equation is expressed as:

\[ Gu = J \] (3)

This simulation method can add or remove system components based on the requirements, which makes the simulation calculation is more simple and flexible. Overlaying each element node admittance matrix and the right item, then solving simultaneously with network equation, we get the state equation of whole system [5]. Update node admittance matrix and the right item after each step calculation, it gets next node voltage and current, and completes the electromagnetic transient simulation of the system.

The numerical integration methods of electromagnetic transient simulation mainly includes Euler method, implicit trapezoid method, damping trapezoidal method, etc. Backward Euler method have good stability, do not produce the numerical oscillation problem, but has low accuracy. Implicit trapezoidal integral method has high precision, but will produce the numerical oscillation problems. Damping trapezoidal method is an integration method of backward Euler method and implicit trapezoidal method, the precision of the damping trapezoidal method are equally between the two methods. By setting the damping coefficient, it can make the numerical oscillation decrease gradually. In the process of electromagnetic transient simulation, the reasonable choice of damping coefficient can improve the accuracy of the simulation, different damping coefficient can be set according to the requirements of simulation research. In this paper, the simulation calculation adopts the damping trapezoidal integration method.

Transformer electromagnetic transient simulation

In power system, the large capacity three-phase transformer is usually connected by independent single-phase transformers, whose magnetic circuit is completely separated without any coupling relationship. This paper mainly focuses on this kind of three-phase transformer. According to the basic theory of the electromagnetic transient simulation and numerical method, the electromagnetic transient simulation model of transformer can also be represented by R (resistance), L (inductance), C
Three-phase double winding transformer model. This paper will take Y0-d1 connection of double winding
transformer as an example for analysis. In the electromagnetic transient simulation, on either side of the transformer voltage current radio relations is presented by controlled source. Primary side
of the controlled current source is controlled by the secondary side of current while the secondary side
of the controlled voltage source is controlled by a voltage on primary side. The double winding
transformer excitation branch is moved forward to the left side of controlled current source on primary
side, exciting branch contains excitation resistance and excitation inductance which are in parallel. The
winding resistance and leakage resistance on primary side is converted to the secondary side, and
merged to the secondary winding resistance and reactance. Applying controlled source principle and
adjoint model of electromagnetic transient simulation, it gets the equivalent circuit of transformer
model, shown in Figure 1.

Figure 1. Y0-d1 connection of double winding transformer electromagnetic transient simulation.

Referring A phase of transformer electromagnetic transient simulation as an example, it solves the
comprehensive adjoint simulation. The leakage reactance branch in transformer d1 is:

\[ u_T = L_T \frac{di_T}{dt} + R_T i_T \]  

In the formula, \( u_T \) is the voltage on leakage reactance, \( L_T \) is leak inductance of transformer, and
\( R_T \) is leak resistance of transformer. For the convenience of description, Leakage reactance two nodes
are defined as k and j, therefore two voltage respectively are \( u_k \) and \( u_j \).

\[ \frac{i_k}{k} = L_T u_k - u_j \] 

\[ i_k = \frac{j - u_j}{k} \] 

Figure 2. Transformer leakage resistance branch adjoint simulation.

With application of damping trapezoidal differencing method, it gets adjoint model conductance
and injection current source, and obtain the equivalent adjoint model in above figure. Similarly, B, C
phase transient model equations can be got. Finishing of figure 1 each parameter expressions are:

\[
\begin{align*}
G_T &= \frac{h(1 + \alpha)}{2\Delta t} \frac{1}{R_T} \\
J_{T+x} &= \frac{h(1 - \alpha)}{2\Delta t} u_x + \frac{1}{R_T} i_x \\
J'_{T+x} &= \frac{h(1 - \alpha)}{2\Delta t} u_x + \frac{1}{R_T} i_x
\end{align*}
\]  

(capacitance) combination. In this paper, based on the principle of the controlled source it sets up
transformer electromagnetic transient model by the equivalent resistance and current source in parallel
[6].
Where, $G_T$ is equivalent conductance of leakage reactance; $J_{T1,n}, J_{T2,n}, J_{T3,n}$ are equivalent current source of each phase's n step simulation; $i_{1,n}, i_{2,n}, i_{3,n}$ are each phase current of d1 side n step simulation; $u_{1,n} \sim u_{6,n}$ are node voltage of node 1-6 shown in figure 1; $R'_T$ is transformer excitation resistance, $L'_T$ is transformer excitation inductance, $G'_T$ is equivalent conductance of excitation inductance; $J'_{T4,n}, J'_{T5,n}, J'_{T6,n}$ are equivalent current source of each phase's excitation inductance n step simulation; $i'_{4,n}, i'_{5,n}, i'_{6,n}$ are each phase exciting current of Y0 side n step simulation; $\alpha$ is damping coefficient.

Node injection current equations are obtained from the parameters equation and volt-ampere characteristic of transformer electromagnetic transient adjoint model.

\[
\begin{align*}
    i_{1,n+1} &= i_{1,n} - 2G_T u_{1,n} - G_T u_{2,n} + kG_T u_{3,n} + kG_T u_{4,n} + J_{T3,n} - J_{T1,n} \\
    i_{2,n+1} &= i_{2,n} - 2G_T u_{2,n} - G_T u_{1,n} + kG_T u_{3,n} + kG_T u_{4,n} + J_{T2,n} - J_{T1,n} \\
    i_{3,n+1} &= i_{3,n} - 2G_T u_{3,n} - G_T u_{1,n} + 2G_T u_{4,n} + kG_T u_{5,n} + G_T u_{6,n} + J_{T2,n} - J_{T3,n} \\
    i_{4,n+1} &= i_{4,n} - G_T u_{4,n} + kG_T u_{5,n} + k^2 G_T u_{6,n} + J_{T2,n} - J_{T1,n} \\
    i_{5,n+1} &= i_{5,n} + G_T u_{4,n} + kG_T u_{5,n} + k^2 G_T u_{6,n} + J_{T1,n} - J_{T3,n} \\
    i_{6,n+1} &= i_{6,n} - (kG_T u_{4,n} - G_T u_{1,n} + kG_T u_{5,n} + k^2 G_T u_{6,n} + J_{T1,n} - J_{T3,n})
\end{align*}
\]  

(6)

Where, $i_{w1,n+1} - i_{w6,n+1}$ are node injection current from external network to the transformer, those are dependent on the external network structure and component. Disregard this part temporarily. The actual calculation, it can be added by increasing injection current after system connection. When to ignore the node injection current from external network, transformer matrix equation is:

\[
\begin{bmatrix}
2G_T & -G_T & -G_T & -G_T & 0 & kG_T \\
-G_T & 2G_T & 0 & 2G_T & -kG_T & 0 \\
-G_T & G_T & 2G_T & 0 & -kG_T & 0 \\
-kG_T & 0 & 0 & kG_T & 0 & 0 \\
0 & -kG_T & 0 & 0 & kG_T & 0 \\
kG_T & 0 & 0 & 0 & 0 & k^2 G_T + G_T
\end{bmatrix}
\begin{bmatrix}
    u_{1,n} \\
    u_{2,n} \\
    u_{3,n} \\
    u_{4,n} \\
    u_{5,n} \\
    u_{6,n}
\end{bmatrix}
= \begin{bmatrix}
    J_{T1,n} - J_{T1,n} \\
    J_{T2,n} - J_{T1,n} \\
    J_{T3,n} - J_{T2,n} \\
    J_{T4,n} - J_{T3,n} \\
    J_{T5,n} - J_{T4,n} \\
    J_{T6,n} - J_{T5,n}
\end{bmatrix}
\]  

(7)

Pay attention to the elements of the node admittance matrix is increment the original node admittance, that is the increase in the original node admittance matrix corresponding admittance and injection current, and getting a new node admittance matrix.

Three-phase three winding transformer model. Three winding transformer structure is relatively complex, which contains three high voltage, medium voltage and low voltage winding. The third winding is always delta connection, and the other two windings are usually a Y connection. The transformer with power flow is defined to be the primary edge Y0 side, power flow side is seconday edge y side and d1 side respectively. The ratio of three winding transformer can be understood as two double winding transformer with different ratio, figure 3 is electromagnetic transient model of three winding transformer based on the principle of the controlled source.
As shown in the model in Figure 3, three winding transformer primary side Y0 side corresponding y side and d1 side with two different variable ratio relations, expressed in $k_y$ and $k_d$. While Y0 side controlled current source current relationship regard as y side and d1 side superposition.

Because of three winding transformer primary side leakage reactance can't be converted to the secondary side, which is imputed to the transformer winding leakage reactance under each voltage grade, each winding leakage reactance can be calculated according to the experimental data. Excitation branch in simulation is in parallel in three (8), (9), (10) internal nodes, which can increase the precision of the simulation, align with the actual situation. In order to speed up the calculation and make the internal node does not participate in the matrix of system, so it needs to clear the internal nodes within the transformer model internal operation, and get node admittance matrix and the voltage and current equations which contains only external node. This article adopts the method of node shrinkage that will be completed node within the operations in the model, and reduce the dimension of the system operation, and speed up the overall operation. Internal nodes shrinkage method is as follows:

1. First of all get the whole node admittance matrix;
2. Transform the internal node admittance matrix parameter to the side of the boundary of the node admittance matrix (lower right);
3. Conduct diagonalization and unit processing to the corresponding element in the internal node voltage;
4. Implement linear transformation on the matrix, and at the same time pay attention to the injection current matrix transformation;
5. Complete the shrinkage, all matrix on the upper left corner unit is external node admittance matrix, the matrix for the unit is on the bottom right hand corner. The upper left corner element is node admittance matrix for the system operation need.
6. After each external node voltage is calculated, and then internal node voltage equation can be obtained by the unit matrix, thus the internal node voltage is got, preparing for next time to solve the external node voltage.

In the process of the simulation calculation, it solves with external first node admittance matrix and the network of simultaneously, gets the external node voltage and current in the transformer, again by the internal node, obtained within the internal node voltage and current value.

**Use case analysis**

Simulation program written in MATLAB language designed in this paper, the simulation results using dotted line in the figure of examples, the solid line in the graph is shown in the simulation results is from the PSCAD simulation software.

Case one. Double winding transformer Y0-d1 connection, the simulation system circuit diagram is as follows:
Figure 4. Simulation circuit of double winding transformer.

The main parameters in the system as follows: 1 kv rated voltage 1 ohm resistance; The transformer ratio $K = 0.577$, leakage inductance $La = 0.0096$ H; For pure impedance load, each phase is 1 ohm. The simulation results are as follows:

![Simulation Results](image)

Figure 5. Simulation results of double winding transformer.

Case two. Three winding transformer Y0-yc connection, the simulation system circuit diagram is as follows:

![Simulation Circuit](image)

Figure 6. Simulation circuit of three winding transformer.

The main parameters in the system as follows: 1 kv rated voltage 1 ohm resistance; Transformer ratio $k_d = 0.577$, $k_y = 1$, leakage inductance $La = 0.0032$ H; For pure impedance load, each phase 1 ohm, grounding resistance 0.001 ohms. The simulation results are as follows:

![Simulation Results](image)

Figure 7. Simulation results of double winding transformer.

It shows from the simulation result, in this paper established the transient simulation and the design of the simulation program in accordance with the PSCAD simulation results is identical, the correctness of the simulation model is verified.

**Conclusion**

This paper analyzed the basic principle of electromagnetic transient simulation of power system, proposed integration of controlled source and damping trapezoidal integration method for transformer electromagnetic transient simulation method. The paper established a three-phase double winding transformer electromagnetic transient simulation and three winding transformer model, describe the simulation solving method and demonstrate the validation of the simulation model by simulation. In view of the three winding transformer model considering excitation branch, this paper puts forward the methods of shrinkage which can finish the node operation inside the model, and reduce the dimension of the system operation, so that to speed up the overall operation.
References


