A Calculation Method and Analytical Formula of the Earth High Frequency Zero Sequence Impedance

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Abstract. This paper puts forward a method of calculating the high-frequency zero sequence impedance, and deduces the earth high frequency analytic calculation formula for the zero sequence impedance. In order to verify the validity of analytical formulas, this paper combined with a large field trial data. Analytic calculation formula in this paper, with simple forms and convenient application, will contribute to the analysis and calculation of lightning and transient characteristics of overhead transmission lines.

Introduction

High voltage overhead transmission line is an important part of power system. The transmission line transient process analysis and calculation plays an important role of the transmission line state estimation and fault diagnosis, and the high frequency of the earth zero sequence parameters of the transmission According to the actual overhead line, this paper adopts an axial line directly below the infinite long rectangle soil, to calculate the sine ac through the equivalent resistance of the earth. Computing field is shown in figure 1: line. This paper proposes a method of calculating the high-frequency zero sequence impedance, deduces the analytic calculation formula of high frequency zero sequence impedance. The analytic calculation formula, form is simple, convenient application, help to transient analysis and calculation of the transmission line.

Carson theory can effectively solve the magnetic field problem which is produced by the distribution current in the earth. Because of Carson theory of the zero sequence ac resistance is a series expression [1-3], Carson proposed the further improvement method to calculate the earth resistance, but the calculation formula is still a series expression, it is not convenient enough.

In order to simplify the calculation of high frequency ac zero sequence resistance, this paper studies the calculation method of the high frequency zero sequence impedance and analytical formula.

1. The existing calculation method and formula

Carson theory provides method for calculating the equivalent resistance of current flows through the earth. The related computation formula is as follows:

$$R_e = 4\omega \cdot 10^{-7} \left[ \frac{\pi}{8} - b_1 a \cos \phi + b_2 \left( c_2 - \ln a \right) a^2 \cos 2\phi + \phi a^2 \sin 2\phi \right] + b_3 a^3 \cos 3\phi - \ldots$$

(1)

where, $b_1 = \frac{\sqrt{2}}{6}$, $b_2 = \frac{1}{16}$, $b_3 = b_{i+2} \left( \frac{\text{sign}}{i(i+2)} \right)$, and so on. Parameter a is depended on the following formula

$$a = 4\pi \sqrt{5} \cdot 10^{-3} \cdot \frac{D}{\sqrt{\rho}}$$

(2)
where, \( D = 2h \), the unit is \( m \); \( \rho \) is earth resistivity, the unit is \( \Omega \cdot m \); \( f \) is frequency, here only discuss 50 Hz. Most of these formulas can be got by carson theory and method.

2. The high-frequency zero sequence impedance calculation method and the analytic calculation formula derivation

According to the actual overhead line, this paper adopts an axial line directly below the infinite long rectangle soil, to calculate the sine ac through the equivalent resistance of the earth. Computing field is shown in Fig. 1.

![Figure 1. Calculating the equivalent resistance Schematic diagram.](image)

The cuboid is the calculation field in Fig. 1. Let the current only has a component in the direction of \( z \) with infinite length, and the field quantity unchanged in the direction of \( z \). The current is maximum in the surface and weakens gradually with depth (in the direction of \(-y\)), owing to the AC skin effect. When the current will disperse when flow into the earth. In general, the iron tower foundation is equivalent to hemisphere grounding. The current density for distance of ground is given as Eq. 3

\[
J = \frac{I}{2\pi r^2} \tag{3}
\]

If \( r \) is large enough, it can be considered as \( J \approx 0 \). Comprehensive considering the skin effect and the dispersed current of grounding grids, this paper define \( J_2 \) as

\[
J_2 = J_0 e^{-k|y|}(A/m^2) \tag{4}
\]

The ball grounding body of current density: \( J = \frac{I}{2\pi r} \). In general, assuming that when \( r = 1m \), \( J_1 = \frac{I}{2\pi} \); \( r = d_1 \) (far away), \( J_2 = \frac{I}{2\pi d_1} \). When \( \frac{J_2}{J_1} = 10^{-6} \), \( d_1 = 1000m \); when \( x > d_1 \), can be considered \( J \approx 0 \). Considering the skin effect, if \( \alpha = \text{real}(k) \), in general, \( J_{21} = J_0 \), \( J_{22} = J_0 e^{-|y|} \) \( \big|_{y \to -d} = J_0 e^{-\alpha d} \), \( \frac{J_{22}}{J_{21}} = 10^{-6} \), \( d_2 = -\frac{1}{\alpha} \ln(10^{-6}) \) when \( |y| > d_2 \), can be considered \( J \approx 0 \). From the Fig. 1 calculation field can be obtained, left and right sides is symmetrical, and we can only count on the right half. Eq.4 can be rewritten with

\[
J_2 = J_0 e^{-k(x-y)} \tag{5}
\]

By the relationship between \( \vec{E} \) and \( \vec{J} \), \( \vec{E} = \frac{\vec{J}}{\gamma} \), so

\[
\dot{E}_2 = \frac{J_0 e^{-k(x-y)}}{\gamma} \tag{6}
\]

Where, \( \dot{E}_2 \) and \( \dot{J}_2 \) are phasor.

Fig. 1 is in the infinite length, \( \frac{\partial}{\partial z} = 0 \); current only has Z component, \( \vec{H}_2 = 0 \). From Maxwell’s first equation (Maxwell):
\[ \nabla \times H = \dot{J} + \frac{\partial D}{\partial t} \]  
\( \left[ \begin{array}{ccc} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \hat{e}_x & \hat{e}_y & \hat{e}_z \end{array} \right] = J_0 e^{-k(x-y)} (1 + j \frac{\omega \varepsilon}{\gamma}) \hat{e}_z \]  
(8)

\[ \frac{\partial \hat{H}_x}{\partial x} - \frac{\partial \hat{H}_y}{\partial y} = J_0 e^{-k(x-y)} (1 + j \frac{\omega \varepsilon}{\gamma}) \]  
From Maxwell's second equation (Maxwell) 
\[ \nabla \times E = -\frac{\partial B}{\partial t} \]  
(10)

\[ \left[ \begin{array}{ccc} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \hat{e}_x & \hat{e}_y & \hat{e}_z \end{array} \right] = -j \omega \mu (\hat{H}_x + \hat{H}_y \hat{e}_x) \]  
(11)

\[ \frac{\partial E_x}{\partial x} - \frac{\partial E_y}{\partial y} = -j \omega \mu (\hat{H}_x + \hat{H}_y \hat{e}_x) \]  
(12)

In above equations, \( \hat{e}_x, \hat{e}_y, \hat{e}_z \) are unit vector of each coordinate axis, \( \hat{H}_x, \hat{H}_y, \hat{E}_z \) are all the phasors. Put Eq.6 into Eq.12, obtaining 
\[ \dot{H}_x = -\frac{J_0 e^{-k(x-y)} k}{j \omega \mu} \]  
(13)

\[ \dot{H}_y = -\frac{J_0 e^{-k(x-y)} k}{j \omega \mu} \]  
(14)

\[ \frac{\partial}{\partial x} \frac{\partial \hat{H}_x}{\partial x} = \frac{J_0 e^{-k(x-y)} k^2}{j \omega \mu} \]  
(15)

\[ \frac{\partial}{\partial y} \frac{\partial \hat{H}_y}{\partial y} = -\frac{J_0 e^{-k(x-y)} k^2}{j \omega \mu} \]  
(16)

Put Eq.15 into Eq.16, obtaining 
\[ \frac{2 J_0 k^2 e^{-k(x-y)}}{j \omega \mu} = J_0 e^{-k(x-y)} (1 + j \frac{\omega \varepsilon}{\gamma}) \]  
(17)

\[ j \omega \mu (1 + j \frac{\omega \varepsilon}{\gamma}) \]  
(18)

\[ k^2 = \frac{2}{\sqrt{2} j \omega \mu (1 + j \frac{\omega \varepsilon}{\gamma})} \]  
(19)

Take Eq.4 a surface integral, can get the total current: 
\[ \dot{I} = 2 J_0 \int_{0}^{b} \int_{d_y}^{0} e^{-k(x-y)} dy \]  
(20)

\[ \dot{I} = 2 J_0 (1 - e^{-kd_y})(1 - e^{-kd_z}) \]  
(21)

\[ \dot{I} = 2 J_0 (1 - e^{-kd_y})(1 - e^{-kd_z}) \]  
(22)

Because of \( e^{-kd_1}, e^{-kd_2} \ll 10^{-6} \), the equation above can be simplified as the type
\[
\dot{I} = \frac{2J_0}{\omega \mu (\omega \varepsilon - j\gamma)}
\]

The valid values of \( \dot{I} \) is

\[
I = \frac{2J_0}{\omega \mu \sqrt{\omega^2 \varepsilon^2 + \gamma^2}}
\]

The complex poynting vector in the earth is

\[
\vec{S}_p = \vec{E}_z \times (\hat{H}_x \hat{e}_x + \hat{H}_y \hat{e}_y)
\]

\[
\vec{S}_p = \vec{E}_z \vec{H}_x \hat{e}_y - \vec{E}_z \vec{H}_y \hat{e}_x
\]

\[
\vec{E}_z \vec{H}_x = \frac{J_0^* k e^{-(k+k)(x-y)}}{j \omega \mu y^2}
\]

\[
\vec{E}_z \vec{H}_y = \frac{J_2^* k e^{-(k+k)(x-y)}}{j \omega \mu y^2}
\]

Enter a closed surface complex power for

\[
- \oint_s \vec{S}_p \cdot ds = I^2 (R + jX)
\]

In Fig. 1, intercept a cuboid, and the length is 1, the six surfaces of the cuboid form a closed surface. \( \vec{S}_p \) has no \( Z \) component, before and after the two surface integral is 0; above and below the two surface integral is

\[
2 \left[ - \int_{x=0}^{x=d} \vec{E}_z \vec{H}_x \big|_{y=d} \, dx + \int_{x=0}^{x=d} \vec{E}_z \vec{H}_x \big|_{y=d} \, dx \right] + \left[ \begin{array}{c}
J_0^* k e^{-(k+k)d_1} - 1 \vspace{0.5cm} \\
J_0^* k e^{-(k+k)d_2} - 1 \vspace{0.5cm} \\
J_2^* k e^{-(k+k)d_3} - 1
\end{array} \right] \left[ \begin{array}{c}
j \omega \mu y^2 (k+k) \vspace{0.5cm} \\
j \omega \mu y^2 (k+k) \vspace{0.5cm} \\
j \omega \mu y^2 (k+k)
\end{array} \right] \]

\[
2 \left[ \begin{array}{c}
J_0^* k e^{-(k+k)d_1} - 1 \vspace{0.5cm} \\
J_0^* k e^{-(k+k)d_2} - 1 \vspace{0.5cm} \\
J_2^* k e^{-(k+k)d_3} - 1
\end{array} \right] \left[ \begin{array}{c}
1 - e^{-(k+k)d_1} \vspace{0.5cm} \\
1 - e^{-(k+k)d_2} \vspace{0.5cm} \\
1 - e^{-(k+k)d_3}
\end{array} \right]
\]

Integral to the left and right

\[
-2 \int_{-d_2}^{0} (- \vec{E}_2 \vec{H}_x) \big|_{x=d} \, dy = 2 \int_{xd_2}^{x=0} \vec{E}_2 \vec{H}_y \big|_{x=d} \, dy
\]

\[
2J_0^* k e^{-(k+k)d_1} \left[ 1 - e^{-(k+k)d_2} \right]
\]

\[
- \oint_s \vec{S}_p \cdot ds = \frac{2J_0^* k e^{-(k+k)d_1} - 1}{j \omega \mu y^2 (k+k)}
\]

As \( e^{-10^6} < 10^{-6} \), \( e^{-(k+k)d_2} < 10^{-6} \)
According to Eq.19
\[ k = \frac{\sqrt{2}}{2} \sqrt{\frac{\omega \mu}{\gamma}} \sqrt{-\omega \varepsilon + j\gamma} \]

or
\[ k = \frac{\sqrt{2}}{2} \sqrt{\frac{\omega \mu}{\gamma}} \sqrt{\sqrt{\omega^2 \varepsilon^2 + \gamma^2 + \omega \varepsilon} + j \sqrt{\omega^2 \varepsilon^2 + \gamma^2 - \omega \varepsilon}} \]

or
\[ k = \frac{\sqrt{2}}{2} \sqrt{\frac{\omega \mu}{\gamma}} \sqrt{\sqrt{\omega^2 \varepsilon^2 + \gamma^2 + \omega \varepsilon} - j \sqrt{\omega^2 \varepsilon^2 + \gamma^2 - \omega \varepsilon}} \]

or
\[ k + k = \frac{\sqrt{2}}{2} \sqrt{\frac{\omega \mu}{\gamma}} \sqrt{\omega^2 \varepsilon^2 + \gamma^2 + \omega \varepsilon} \]

Put Eq.40 and Eq.41 into Eq.36, obtaining
\[ -\int_S \tilde{P} d\tilde{s} = \frac{j^2 J_0^* k}{\omega \mu \gamma^2 (k + k)} \]

Compare Eq.29 with Eq.42, obtaining
\[ R = \frac{J_0^2 \gamma}{I^2 \omega \mu \gamma^2 (\sqrt{\omega^2 \varepsilon^2 + \gamma^2 + \omega \varepsilon})} \]

Put Eq.24 into Eq.43, obtaining
\[ R = \frac{\omega \mu (\omega^2 \varepsilon^2 + \gamma^2)(\sqrt{\omega^2 \varepsilon^2 + \gamma^2 - \omega \varepsilon})}{16 \gamma^3} \]

or
\[ X = \frac{\omega \mu (\omega^2 \varepsilon^2 + \gamma^2)}{16 \gamma^2} \]

Eq.44 and Eq.45 are the formulas of equivalent resistance and reactance per unit length.

3. Setting of high frequency zero sequence impedance analytical formula

Considering the sine steady-state is quasi-stationary field, can be dealt with as a Quasi-static magnetic field, namely: \( \nabla \times \vec{H} = \vec{J} \), ignore \( \frac{\partial \vec{D}}{\partial t} = \frac{\partial \vec{E}}{\partial t} = \frac{\vec{e}}{\gamma} \), so take \( \varepsilon = \varepsilon_0 \), In general, the earth is a ferromagnetic material, take \( \mu = \mu_0 \), therefore, simplify (44) as follows
\[ R = \frac{\omega \mu \varepsilon_0 (\omega^2 \varepsilon_0^2 + \gamma^2)(\sqrt{\omega^2 \varepsilon_0^2 + \gamma^2 - \omega \varepsilon_0})}{16 \gamma^3} \]

Frequency parameters were measured about same tower multi-circuit transmission lines by Henan Electric Power Research Institute and test results are shown in table 1.

<table>
<thead>
<tr>
<th>Name of the loop</th>
<th>The positive sequence impedance / Ω</th>
<th>Zero sequence impedance / Ω</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>1.146+j13.514</td>
<td>6.729+j33.593</td>
</tr>
<tr>
<td>II</td>
<td>1.147+j13.917</td>
<td>6.736+j33.630</td>
</tr>
<tr>
<td>III</td>
<td>1.210+j6.837</td>
<td>6.154+j17.750</td>
</tr>
<tr>
<td>IV</td>
<td>1.218+j6.858</td>
<td>6.259+j17.664</td>
</tr>
</tbody>
</table>
Where, the length of loop I and loop II is 58.945km, loop III and loop IV is 20.645km. As Eq.44 and Eq.45 can be established in factory frequency, so according to field experiment data for calibration. By the data in the table can work out the earth zero sequence power frequency resistance is about 0.05 Ω/km. We can determine, the correction coefficient of Eq.44 and Eq.45 are 2. Make \( \omega = \omega_0 \), \( 2\pi\omega_0 = 100\pi \), we can obtain power frequency electric resistance. According to the actual data of the scene [4], Eq.44 and Eq.45 will be changed to

\[
R = \frac{\omega\mu(\omega^2\varepsilon^2 + \gamma^2)(\sqrt{\omega^2\varepsilon^2 + \gamma^2 - \omega\varepsilon})}{8\gamma^3}
\]

(47)

\[
X = \frac{\omega\mu(\omega^2\varepsilon^2 + \gamma^2)}{8\gamma^2}
\]

(48)

Eq.47 and Eq.48 are the high frequency zero sequence impedance analytic calculation formula of the earth.

In the power system, the usual formula for earth resistance [5] is as follows

\[
R_g = \pi^2 \times 10^{-4} \times f (\Omega/km)
\]

(49)

With the change in frequency, calculation results of Eq.49 are shown in Table 2.

<table>
<thead>
<tr>
<th>frequency(Hz)</th>
<th>5 \times 10^1</th>
<th>5 \times 10^2</th>
<th>5 \times 10^3</th>
<th>5 \times 10^4</th>
<th>5 \times 10^5</th>
<th>5 \times 10^6</th>
</tr>
</thead>
<tbody>
<tr>
<td>resistance (Ω/km)</td>
<td>0.049</td>
<td>0.49</td>
<td>4.93</td>
<td>49.35</td>
<td>493.5</td>
<td>4935</td>
</tr>
</tbody>
</table>

This paper proposes a new analytical formulas earth resistance, with the change in frequency, earth resistance changes, the result are shown in Table 3.

<table>
<thead>
<tr>
<th>frequency(Hz)</th>
<th>5 \times 10^1</th>
<th>5 \times 10^2</th>
<th>5 \times 10^3</th>
<th>5 \times 10^4</th>
<th>5 \times 10^5</th>
<th>5 \times 10^6</th>
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<td>4.93</td>
<td>49.34</td>
<td>493.5</td>
<td>4837</td>
</tr>
</tbody>
</table>

As can be seen from the table, the result of the formula proposed in this paper is consistent with the result of commonly used formula in power system, especially at low frequencies.

4. Conclusion

This paper puts forward a method of calculating the high-frequency zero sequence impedance, and according to Maxwell's equations, this article deduced the earth high frequency analytic calculation formula for the zero sequence impedance (carson formula is a form of series). Analytic calculation formula proposed in this paper will contribute to the analysis and calculation of lightning and transient characteristics of overhead transmission lines.

References