Speed Sensorless Vector Control Based on Full-order Adaptive Observer for Induction Motor Drives at Low Speed

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Abstract
This paper presented a speed sensorless vector control method based on adaptive full-order state observer for induction motor. The rotor speed estimated by an adaptive full-order observer is fed back to the vector control system as a feedback signal to form a closed-loop control of speed. In addition, the stator resistance is also recognized and updated in real time, so the effect of change of stator resistance on accuracy of speed estimation can be eliminated. The parameter identification system is achieved according to the Popov’s hyperstability theory, therefore the system is stable enough even at very low speed. The simulation and experimental results illustrated the correctness and effectiveness of the presented control algorithm.

Keywords: full-order observer, Induction motor drives, parameter identification, sensorless control

Nomenclature

Abbreviation
SMO Sliding Mode Observer
EKF Extend Kalman Filter
MRAS Model Reference Adaptive System
AFO Adaptive Full-order State Observer
FOC Field Oriented Control

1. Introduction
Speed closed-loop is indispensable in the vector control system due to the magnetic field orientation. At the initial stage, the information of rotor speed was obtained mainly by installing a speed sensor in motor shaft. However, the adoption of speed sensor raises cost and has strict requirement to its work environments. So speed sensorless technology has gradually been a hot research. The vector control was combined with speed sensorless technology first by R.Joetten in 1983 [1]. Nowadays, speed sensorless vector control technology of induction motor gets more and more development for its low cost, simple structure and functional reliability [2]. Various speed estimation algorithms have been put forward to enhance the performance of the sensorless drives. These methods can be roughly classified into two categories: fundamental-excitation methods [3] [4] and signal injection methods [5].

The fundamental-excitation methods mainly include SMO, EKF and MRAS [6-10]. The MRAS has remarkable merits of its relative simplicity and easiness of utilization [11]. The MRAS based on rotor linkage has been extensively studied and it has been testified that these methods can work well in the medium to high-speed range [12]. AFO is one of the MRAS in essence. The AFO is popular due to its good versatility and high accuracy of steady-state, etc. Compared with MRAS, the motor body is regarded as reference model in AFO, so zero drift error that caused by the pure integral in the voltage model is eliminated [13]. In addition, the observation of rotor flux is a closed loop in AFO. In [14], the AFO based on the induction motor fundamental model is utilized to achieve a good performance. However, the effect caused by the variations of motor parameters is not taken into account when estimating motor speed. Especially, the change of stator resistance has a great influence on the accuracy of estimated speed.

This paper introduced a method based on an adaptive full-order observer, which can simultaneously identify the motor rotor speed and stator resistance online. The rotor speed is estimated by the AFO in real time and is utilized as a feedback signal for the vector control system. At the same time, the stator resistance, which is used in the observer, is also identified. So the rotor speed obtained by the proposed estimation system is accurate in low speed region. Furthermore,
the proposed full-order adaptive observer eliminates pure integral part and internal flux is a closed loop, thus, there is no zero drift error and the accuracy of parameter identification is higher. The drive system with the proposed adaptive full-order observer can function well in a relative wide range of speed, and have a strong robustness to the changes of stator resistance. The effectiveness of the proposed parameter estimation approach was verified by simulation and experiment results.

2. ADAPTIVE FULL-ORDER STATE OBSERVER

2.1 Full-order observer of induction motor

As for induction motor, when the stator voltage is chosen as the system input, the stator current is selected as the system output, state variables are composed of stator current and rotor flux linkage, the state equation can be expressed as (1) under the α-β reference frame. The output can be expressed as (2).

\[
\frac{d}{dt} \hat{I}_{s} = [A_{11} A_{12}] \hat{I}_{s} + [B_{1}] \hat{I}_{r} + [A_{21} A_{22}] \hat{I}_{r} + \hat{B}_{1} \hat{I}_{r} + Ax + Bu
\]

\[
y = C[\hat{I}_{s} \hat{I}_{r}]^{T} = C x
\]

Where \( \hat{u} = [u_{ai} u_{ir}]^{T} \) represent stator voltage, \( \hat{i}_{s} = [i_{ai} i_{ir}]^{T} \) represent stator current, \( \hat{\psi}_{s} = [\psi_{ai} \psi_{ir}]^{T} \) represent rotor flux, and

\[
A_{11} = -\left( \frac{R_{s}}{L_{s}} + \frac{1 - \sigma}{\sigma T_{r}} \right) \mathbf{I}, \quad A_{12} = \frac{L_{m} \omega_{r}}{\sigma L_{L} L_{T}} \mathbf{I} - \frac{L_{m}}{\sigma L_{L} L_{T}} \mathbf{J}, \quad A_{21} = \frac{L_{m}}{T_{r}} \mathbf{I},
\]

\[
B_{1} = \frac{1}{\sigma L_{L}}, \quad C = [1 \ 0]^{T}, \quad \mathbf{J} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},
\]

where \( R_{s} \) and \( R_{r} \) represent stator resistance and rotor resistance respectively, \( L_{s}, L_{r}, \) and \( L_{m} \) represent stator, rotor and mutual inductance respectively, \( \sigma = 1 - L_{m} / (L_{s} L_{r}) \) is the leakage coefficient, \( T_{r} = L_{r} / R_{r} \) is the rotor time constant, \( \omega_{r} \) is the rotor angular velocity.

Based on (1), the state equation of the observer can be expressed as (3).

\[
\frac{d}{dt} \hat{\psi}_{s} = \mathbf{A}_{o} \hat{\psi}_{s} + \hat{\mathbf{B}}_{1} \hat{\mathbf{I}}_{r} + \mathbf{G} (\hat{\mathbf{I}}_{s} - \hat{\mathbf{I}}_{r})
\]

\[
\mathbf{G} = \begin{bmatrix} g_{1} & g_{2} & J \end{bmatrix}^{T}
\]

Where the symbol \( \hat{\mathbf{A}}_{o} \) means the estimated value, \( \omega_{r} \) and \( \mathbf{R} \) are replaced by \( \hat{\omega}_{r} \) and \( \mathbf{\hat{R}} \) respectively in the matrix \( \hat{\mathbf{A}}_{o} \). \( \mathbf{G} \) is the output feedback gain matrix of the state observer. It can be used to change the pole position of the AFO. As shown in Fig.1, the poles of the motor itself computed by Matlab are situated on the left side of the s-plane when the rotor speed is in the range of 0 to 200\( \pi \) rad/s, so the induction motor itself is stable in this range. Provided the pole position of the observer is assigned as \( k (k \geq 1) \) times of the motor poles, then the observer is stable and unknown state variables can converge to the corresponding actual value of the motor with a fast rate. So, the elements in \( \mathbf{G} \) can be expressed as:

\[
\begin{align*}
g_{1} &= -(k - 1) \frac{\hat{R}_{L} + R_{L}}{L_{m} L_{r} - L_{m}^{2}} \quad (k - 1) \hat{\omega}_{r}, \\
g_{2} &= -(k - 1) \hat{\omega}_{r}, \\
g_{3} &= (k^{2} - 1) \frac{\hat{R}_{L} - (k - 1) \hat{R}_{L} + R_{L}}{L_{m} L_{r} - L_{m}^{2}} \quad k \geq 1 \\
g_{4} &= (k - 1) \hat{\omega}_{r} \left( L_{s} L_{r} - L_{m}^{2} \right) / L_{m}
\end{align*}
\]

In general, the greater the \( k \), the faster the convergence rate of the observer. However, the steady-state error will increase with the increase of \( k \). Usually, \( k \) is set to 1.5.

The poles of observer can be obtained by (6).

\[
|\lambda - (A - GC)| = 0
\]

It can be seen from Fig.1 that the poles of the AFO are proportional to situate on the left side of the pole of the induction motor. Therefore, the AFO can be considered to be convergent and stable, and the convergence speed of the observer is faster than the motor itself. Motor speed estimation is obtained by adaptive system. The adaptive system will be discussed in the next section.

![Figure 1 The pole distribution](image)

2.2 Adaptive identification of parameters

According to the MRAS, the motor itself and the AFO can be treated as the reference model and the adjustable model, respectively. Then, the output errors between the reference model and the adjustable model could be utilized in driving the adaptive identification system of the rotor speed \( \hat{\omega}_{r} \). In addition, the change of the stator current also reflects indirectly the variation of the rotor flux linkage, so the observation error of the current can be utilized to judge whether the state variables converge. If the estimation error of the stator
current converges stably, it indicates that the estimated flux is accurate. This means that the estimated speed is almost as same as the actual motor speed.

Firstly, considering that the deviation between estimated speed and actual speed will cause the deviation between estimated and actual state variables. Secondly, the accuracy of estimated speed of adaptive full-order observer depends on whether the motor parameters are accurate. The error between preset and actual stator resistance is a main influence factor, especially at low speed. So, the variation of the stator resistance will also be considered in this speed estimation system. The state error equation can be expressed as (7) by subtracting (3) from (1).

\[
\frac{d}{dt} \begin{bmatrix} \dot{\hat{e}}_1 \\ \dot{\hat{e}}_2 \end{bmatrix} = (A - GC) \begin{bmatrix} \dot{\hat{e}}_1 \\ \dot{\hat{e}}_2 \end{bmatrix} + \Delta A \begin{bmatrix} \hat{e}_1 \\ \hat{e}_2 \end{bmatrix} = (A - GC) \tilde{e} - W (7)
\]

Where \( W \) is a nonlinear time-varying feedback module, the error matrix \( \Delta A \) is defined as follows.

\[
\Delta A = A - \hat{A} = \begin{bmatrix} 0 & 1/J \left(\omega_s - \dot{\omega}_s\right) + \frac{1}{\sigma L_s} I & 0 \\ 0 & J & 0 \end{bmatrix} \begin{bmatrix} \hat{R}_s - \hat{R} \end{bmatrix} (8)
\]

Equation (7) can be equivalent to the nonlinear feedback system, as shown in Fig.2. It includes a linear time-invariant forward channel and a nonlinear time-varying feedback channel.

\[
\int_0^1 e^T W dt \geq -\gamma^2, \gamma = \text{constant} (9)
\]

The transfer function \( [sI - (A - GC)]^{-1} \) of the observer system is strictly positive, since the poles of the observer are situated on the left side of the s-plane as shown in Fig.1. For the nonlinear time-varying feedback part, both of \( e \) and \( W \) are substituted into the inequality shown as (9). Then the left part of the inequality can be rewritten as (10).

\[
\int_0^1 e^T W dt = -\int_0^1 \begin{bmatrix} \dot{\hat{e}}_1^T \\ \dot{\hat{e}}_2^T \end{bmatrix} (\Delta A) \begin{bmatrix} \dot{\hat{e}}_1 \\ \dot{\hat{e}}_2 \end{bmatrix} dt
\]

\[
= -\int_0^1 (L_m + 1/\rho) \Delta \omega \begin{bmatrix} \dot{\hat{e}}_1 \\ \dot{\hat{e}}_2 \end{bmatrix} \dot{\hat{e}}_1 \dot{\hat{e}}_2 dt - \int_0^1 \Delta R_s / (\sigma L_s) \dot{\hat{e}}_1 \dot{\hat{e}}_2 dt (10)
\]

**Figure 2** Equivalent structure of error state equation

\( \Phi_1 (e) \) and \( \Phi_2 (e) \), as shown in Fig.2, denote the adaptive identification functions of rotor speed and stator resistance respectively. Based on the Popov’s hyperstability theory, as long as the transfer function of the linear regular forward part is strictly positive, then the necessary and sufficient condition of the system’s asymptotic stability is that its nonlinear time-varying feedback part satisfies the Popov inequality shown as (9).

\[
\int_0^1 e^T W dt \geq -\gamma^2, \gamma = \text{constant} (9)
\]

The condition \( |\dot{\hat{e}}_1| \leq L_m |\hat{e}_1| \) is assumed and \( \dot{\hat{e}}_2 \) is replaced by \( L_m \dot{\hat{e}}_1 \) in (10). According to (11), if the adaptive laws of the two unknown variables are shown as (12) and (13) respectively, the result of integral operation in (10) can satisfy the Popov inequality shown as (9). Moreover, the rotor speed and stator resistance have slower dynamics as compared to the other electrical state variables, so we can regard the two variables as constant during a period and the differential of them can be considered as zero.

**Figure 3** The schematic of parameter identification

**Figure 4** The schematic of vector control system

The condition \( |\dot{\hat{e}}_1| \leq L_m |\hat{e}_1| \) is assumed and \( \dot{\hat{e}}_2 \) is replaced by \( L_m \dot{\hat{e}}_1 \) in (10). According to (11), if the adaptive laws of the two unknown variables are shown as (12) and (13) respectively, the result of integral operation in (10) can satisfy the Popov inequality shown as (9). Moreover, the rotor speed and stator resistance have slower dynamics as compared to the other electrical state variables, so we can regard the two variables as constant during a period and the differential of them can be considered as zero.
\[ \int_{0}^{t} k \cdot \frac{df(t)}{dt} \cdot f(t) \, dt = \frac{k}{2} \left( f^2(t) - f^2(0) \right) \geq -\frac{1}{2} k \cdot f^2(0) \] (11)

\[ \hat{\omega}_r = \left( k_{a \alpha} + k_{a \beta} \right) / s \hat{\psi}_r^T \hat{J}_e \]

\[ = \left( k_{a \alpha} + k_{a \beta} \right) / s(e_{\alpha \alpha} \hat{\psi}_{r \alpha} - e_{\alpha \beta} \hat{\psi}_{r \beta}) \] (12)

\[ \hat{R}_s = -\left( k_{a \alpha} + k_{a \beta} \right) / s e_{\alpha \alpha} \hat{\psi}_{r \alpha} \]

\[ = -\left( k_{a \alpha} + k_{a \beta} \right) / s(e_{\alpha \alpha} \hat{\psi}_{r \alpha} + e_{\beta \beta} \hat{\psi}_{r \beta}) \] (13)

Where \( 1/s \) is the integral operation, \( k_{a \alpha} \), \( k_{a \beta} \) and \( k_{a \alpha} \), \( k_{a \beta} \) represent the PI parameters of speed estimator and stator resistance estimator, respectively.

The final schematic diagram of estimated system is shown in Fig. 3, which mainly includes the full-order observer module and adaptive identification module. Both the speed and stator resistance can converge to the actual value, since the two adaptive laws are obtained by the Popov’s hyperstability theory.

<table>
<thead>
<tr>
<th>TABLE 1</th>
<th>PARAMETERS OF THE INDUCTION MOTOR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Symbol</td>
<td>Quantity</td>
</tr>
<tr>
<td>( P_n )</td>
<td>rated power</td>
</tr>
<tr>
<td>( U_n )</td>
<td>rated voltage</td>
</tr>
<tr>
<td>( N_n )</td>
<td>rated speed</td>
</tr>
<tr>
<td>( R_S )</td>
<td>stator resistance</td>
</tr>
<tr>
<td>( R_r )</td>
<td>rotor resistance</td>
</tr>
<tr>
<td>( L_{s\alpha} )</td>
<td>stator inductance</td>
</tr>
<tr>
<td>( L_{s\beta} )</td>
<td>rotor inductance</td>
</tr>
<tr>
<td>( L_m )</td>
<td>mutual inductance</td>
</tr>
<tr>
<td>( \rho )</td>
<td>pole number</td>
</tr>
<tr>
<td>( J )</td>
<td>inertia</td>
</tr>
</tbody>
</table>

2.3 Vector control system

Using the estimated speed \( \hat{\omega}_r \) and flux linkage \( \hat{\psi}_r \), we can build a speed closed loop and calculate the flux position to compose the vector control system. As for the vector control system, there are two kinds of FOC methods. One is the indirect field orientation method, and the other is direct FOC. In terms of the speed sensorless vector control system, since the integral operation is needed in the indirect field orientation control, the flux linkage position could produce zero drift because of the estimation error of rotor speed, as shown as (14). Adopting direct field orientation control, shown as (15), can avoid the problem.

\[ \dot{\theta} = \int_{0}^{t} (\hat{\omega}_r + \omega_r ) \, dt = \int_{0}^{t} (\omega_r + \omega_r ) - (\omega_r - \omega_r ) \, dt \] (14)

\[ \begin{Bmatrix}
\cos \hat{\theta} = \hat{\psi}_{r \alpha} / \psi_r \\
\sin \hat{\theta} = \hat{\psi}_{r \beta} / \psi_r \\
\psi_r = \sqrt{\hat{\psi}_{r \alpha}^2 + \hat{\psi}_{r \beta}^2}
\end{Bmatrix} \] (15)

So the direct field orientation is chosen. And flux and electric torque adopt indirect closed-loop method which is achieved by adjusting corresponding current component. The overall block diagram of the proposed control system of induction motor is depicted in Fig. 4.

3. SIMULATION

A series of simulation results are shown and discussed in this section for proofing the feasibility and effectiveness of the proposed system. The specific parameters of motor utilized in this system are shown in Table 1.

The speed following result is illustrated in Fig.5 when the estimated speed obtained by the proposed observer is applied to the closed-loop control system. The step speed is added at 1s, and the step load is added at 3s. The estimated speed error is within 2r/min and both the estimated and actual can return to the target value quickly.

![Figure 5 Simulation results of speed identification](image1)

Figure 5 Simulation results of speed identification

![Figure 6 stator current and estimated rotor flux](image2)

Figure 6 stator current and estimated rotor flux

The dynamic changes of stator current and estimated flux are illustrated in Fig.6. The simulation condition is same as in Fig.5. The estimated flux linkage can fast reach the given value and the stator current has an ideal waveform. Fig.7 illustrates that dynamic following performance of the resistance adaptive module when the actual stator resistance changes dynamically. The simulation condition is that the stator resistance, which is utilized in the motor model, is increased to 1.445 ohms at 1s,
that is to say it's 1.3 times the initial value, and reduced by 30% of the initial value at 2s, respectively. The other conditions are same as in Fig.5. It can be seen that the estimated value can follow the real one in 0.1s and the steady state error is near the zero. It can be concluded that the proposed speed sensorless vector control system works very well and stably. Also, the estimated values both rotor speed and stator resistance obtained from the full-order state observer could be well qualified to meet the dynamic requirement of the system.

Fig.8 illustrates the results of speed estimation with the identification of stator resistance, when the stator resistance of the motor is set to 0.7 times of the rated value and a step load is added at 3s. It can be seen the estimated speed is almost as same as the actual motor speed even at extremely low speed, and the error is near the zero. In addition, the estimated speed can reconverge to the given value in 0.2s after a load step.

![Figure 7 Simulation results of resistance identification](image)

**Figure 7** Simulation results of resistance identification

![Figure 8 Simulation results with resistance identification](image)

**Figure 8** Simulation results with resistance identification

Fig.9 illustrates the results of rotor speed estimation and rotor flux estimation without the resistance identification. The simulation condition is that the stator resistance of the motor is set to 0.7 times of the rated value and the other conditions are same as in Fig.5. We can see that both estimated and real rotor speed all have a very strong jitter and the system shows a poor performance compared with Fig.8. The simulation result shows that the inaccurate stator resistance greatly influences the performance of speed sensorless system in the low speed region. Therefore, stator resistance identification-online is very important to the speed identification based on the adaptive full-order observer.

![Figure 9 Simulation results without resistance identification](image)

**Figure 9** Simulation results without resistance identification

### 4. EXPERIMENTS

The experimental platform, shown in Fig.10, was used to test and verify the proposed estimator. The parameters of the asynchronous motor in Fig.10 (a) are the same as those in Table 1. A magnetic powder dynamometer in Fig.10 (b) is utilized as a load, and the load torque can be adjusted by a separate controller. The real motor torque and speed can be obtained by an external torque/speed sensor.

The proposed speed sensorless control system has been implemented in a DSP28335 floating-point microcontroller. The inverter switching frequency is 10 kHz and dead-time period is set to 1μs.

![FIGURE 10 (a) Motor and motor controller (b) Magnetic powder dynamometer](image)

**FIGURE 10** (a) Motor and motor controller (b) Magnetic powder dynamometer

![FIGURE 11 Experimental results of speed identification](image)

**FIGURE 11** Experimental results of speed identification

Fig.11 shows the responses of control system when the given speed and the load torque change. Target speed changes from 20 to 35rpm and the full load is
added at 2s. It can be seen that both the estimated speed and the measured value can follow the given one quickly and accurately. And the steady state error is within 2rpm.

Fig.12 shows the responses of the estimated stator resistance. We can see that the estimated stator resistance could follow the real value quickly, it takes about 0.7s and the steady-state error is small enough. It can be seen that the experimental results are similar to the simulation results, so the feasibility and validity of the proposed method are certified.

![Experimental results of resistance identification](image)

**FIGURE 12** Experimental results of resistance identification

5. CONCLUSION

This paper proposed a speed sensorless vector control system based on full-order adaptive state observer, which can identify rotor speed and stator resistance at the same time. Through utilizing the proposed full-order state observer, the control system can work without the influence of variation of stator resistance, especially at low speed. Therefore, the system can operate well in a low speed. Simulation and experimental results have validated the proposed method.

In consideration of that the proposed sensorless system is mainly used in the Electro-Hydraulic Power Steering system, the induction motor usually operations under rated conditions. Therefore, inductance parameters can be regarded as constant. At the same time, regenerating-mode will be discussed when the system is applied to other occasions.

Acknowledgement

The research work was supported by National Natural Science Foundation (51105032).

Reference


