Strength Design and Analysis of the Grouting while Drilling Equipment’s Sleeve

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Abstract. According to structure and working-condition of the grouting while drilling equipment, sleeve’s compressive strength and torsion strength are calculated with elastic theory, which are simulated and verified with FEA Software. The results indicate that its maximum internal stress exceeds the elastic yield limit at 30 MPa external pressures, and that the sleeve is working with risk in 3000 m underground soft coal. With Prandtl stress function method, the calculated shear strength in the sleeve’s small end is far more than the material’s shear strength due to 1500 N·m torque, the small end is stress concentration area and it can’t meet the requirement. Finally, sleeve’s theoretical work external pressure is calculated as 29.8 MPa and its axial deformation is about 0.2 mm, with radial deformation level at 0.03 mm, which can be referenced to tolerance design in the assembly.

Introduction

As an unconventional natural gas, coal-bed methane has such characteristics as cleanness and efficiency, etc., and it has gradually attracted extensive attention all over the world. Coal-bed methane is rich in China, with about 3.681×10^{13} m^3 in the depth of less than 2000 m (according to the Ministry of Land and Resources assessment of China’s coal-bed methane resources in 2006). This amount is as much as that of conventional gas resources[1,2]. Extracting and developing coal-bed methane in China began in the 1980s, but the process is slow due to outdated technology and immature supporting policy[3]. The design and development of equipment and tools to exploit coal-bed methane is the first step to address this problem. In China, Soft coal seam is a considerable proportion of minable seam, so it is vital to design and research technology and equipment to extract soft coal-bed methane[4].

Under-balanced drilling (UBD) with mid-pressure wind has been the most promising technology to coal-bed methane, but it easily causes collapse of the hole-wall while drilling in soft coal seam. Thus UBD technology seems to be incapable here. As to conventional hydraulic slag drilling, due to its’ strong water scouring effect, it causes hole-wall collapse severely and sticking accident frequently in the process of drilling. It has been proved practically that it is suitable for drilling in hard coal seam, but not suitable in soft coal seam[5]. To issue of hole-wall instability when drilling in soft coal seam, a grouting while drilling equipment is designed, which will inject an appropriate drilling fluid to consolidate the well wall in loose sections, and implement the process of grouting and consolidating the well wall while drilling [2,6].

Structure of the Grouting while Drilling Equipment

In drilling engineering field, grouting while drilling technology is a process in which appropriate drilling fluid is grouted into the strata for consolidating the well-wall while drilling in collapsed strata or loose stratum (such as soft coal seam). The grouting while drilling equipment is a needed tool to implement the process efficiently in that its performance determines the success of the process. Axial cross-sectional structure of the equipment designed is shown as Fig. 1. While drilling in a coal seam which is not soft, such as Fig. 1 (a), the well wall does not need protection, then this tool recognizes a
normal drilling signal and it makes a normal drilling with atomizing air as cycling medium. While drilling in soft coal seam, such as Fig. 1 (b), this tool identifies a start-grouting signal and it begins to inject into the soft coal seam a certain amount of drilling fluid, which shortly condenses to consolidate and protect the drill-hole wall so as to ensure a smooth drilling.

Figure 1. Structure of a designed drilling along grouting equipment.

The casing, such as in Fig. 2, is an important part in the equipment. It consists of three sleeves, with the second one’s structure shown in Fig. 3. Assuming that drill-sticking or burying accident just happens around the sleeves while drilling in soft coal seam, outer surface of the sleeve is subject to the 30 MPa earth stress while supposing 3000 m as the maximum drilling depth, and the inner surface of the sleeves is subject to the fluid grouting pressure or air discharge pressure. Generally, mud-pup’s output pressure is between 2 MPa and 5 MPa, and air compressor’s output pressure between 0.5 MPa and 1.2 MPa. According to the working condition and for the sake of safety, the sleeve’s inner surface pressure is taken as 0.1 MPa. Sleeve material is 304 stainless steel (0Cr18Ni9Ti). Thus the yield strength $\sigma_{0.2}=205$ MPa, and the elastic modulus $E=206$ GPa.

Figure 2. Structure of the casing.

Figure 3. Cross-sectional view of the second sleeve.
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Compressive Strength Analysis

The grouting while drilling equipment is installed between drill bit and drill pipe. It should transfer torque mainly through the casing part during drilling. In addition, ratchet transposition mechanism in the tool should rotate flexibly, thus there should be enough gap between the sleeve and the ratchet transposition mechanism to ensure perfect performance of the mechanism. Especially, the casing part’s deformation induced by earth stress shall not affect the equipment’s performances once accidents happen in the drill-hole. In other words, sleeve deformation should be within a permissible range. Therefore, it is necessary to conduct strength analysis of the sleeve in two aspects, namely, compressive strength analysis and torsion strength analysis.

To the second sleeve, ratio of the outer diameter and the inner diameter is 38/30=1.27>1.2, so it can be analyzed as a thick-walled cylinder. Its aspect ratio is 340/38=8.95. In order to simplify analysis, this space problem can be turned into a plane problem along length dimension, and its force diagram is shown in Fig.4. [7,8].

Figure 4. Force diagram of the sleeve. Figure 5. Stress distribution of the sleeve.

Where \( P_1 \) and \( P_2 \) are internal pressure and external pressure, respectively; \( a \) and \( b \) are sleeve’s inner diameter and outer diameter, respectively. According to assumption of elasticity mechanics, because of the axial symmetry, hollow cylinder’s displacements only include \( u(r) \) – component of \( r \) direction, and \( \omega(z) \) – component of \( z \) direction. The equilibrium equations and compatibility equations are Eq. 1 shown as follows:

\[
\frac{\partial \sigma_r}{\partial r} + \frac{\sigma_r - \sigma_\theta}{r} = 0, \quad \frac{\partial \sigma_\theta}{\partial z} = 0 = \frac{d^2 \omega}{dz^2}, \quad \frac{d\sigma_\theta}{dr} - \nu \frac{d\sigma_r}{dr} = \frac{1+\nu}{r} (\sigma_r - \sigma_\theta) \tag{1}
\]

Where \( \sigma_r, \sigma_\theta, \sigma_z \) are internal stress components in \( r \) direction, \( \theta \) direction, and \( z \) direction, respectively, and \( \nu \) is Poisson’s ratio of the material. By solving Eq. 1, we can get Eq. 2 as follows:

\[
\sigma_r = A - \frac{B}{r^2}, \quad \sigma_\theta = A + \frac{B}{r^2}, \quad \sigma_z = 2\nu A + E \varepsilon_z \tag{2}
\]

Where \( A, B, \) and \( C \) are constants which can be determined by the boundary conditions; \( E \) is material elasticity modulus, \( G \) is material shear modulus, and \( \varepsilon_z \) is component of internal strain in \( z \) direction.

The boundary condition is such as Eq. 3:

\[
\sigma_r = -p_1 \quad \text{while} \quad r = a; \quad \sigma_r = -p_2 \quad \text{while} \quad r = b. \tag{3}
\]

The casing is installed between the drill bit and drill pipe, thus the sleeve can be regarded as a model closed at both ends and constrained with axial stiffness. Therefore, \( \sigma_z = \text{const} \) and \( \varepsilon_z = 0 \). Based on boundary condition (Eq.3) and Lame’s formula, we can get stress solution as Eq. 4:
are displacement components in \( r = 0.3 \). Therefore, the stress at \( z = 0 \) is calculated as Eq. 8:
\[
\sigma = \frac{p_1 a^2 - p_2 b^2}{b^2 - a^2} \quad \text{and} \quad \sigma = \frac{(p_1 - p_2) a^2 b^2}{(b^2 - a^2)r^2} \quad \sigma_z = 2\nu \frac{p_1 a^2 - p_2 b^2}{b^2 - a^2}
\]  

Correspondingly, according to Hooke’s law, the displacements can be obtained in Eq. 5, where \( u, v \) and \( \omega \) are displacement components in \( r \) direction, \( \theta \) direction, and \( z \) direction, respectively.

\[
u = 1 - v \frac{p_1 a^2 - p_2 b^2}{b^2 - a^2} r + \frac{1 + v (p_1 - p_2) a^2 b^2}{b^2 - a^2} r, \quad v = 0, \quad \omega = -2 \nu \frac{p_1 a^2 - p_2 b^2}{b^2 - a^2} z
\]

Based on Eq. 4, the sleeve’s internal stress distribution in three dimensions can be got as Fig. 5. As for accidents of pipe-sticking or drill burying that may happen in sleeves during drilling in soft coal seam, the resultant internal stress \( \sigma_w \) at \( z=0 \) is calculated as Eq. 8:
\[
\sigma_w = \sqrt{\sigma_r^2 + \sigma_\theta^2 + \sigma_z^2} = \sqrt{\sigma_r^2 + \sigma_\theta^2 + \text{const}} = \frac{-p_2 b^2}{b^2 - a^2} \sqrt{2 + 2 \frac{a^4}{r^4} + 4v^2}
\]

From Eq. 8, \( \sigma_w \) will get the maximum modulus value at \( r=a \), thus the yield of material starts from sleeve’s inner surface. At sleeve’s small end, when \( a=15 \text{mm}, b=19 \text{mm}, v=0.3, p_1=0, \) and \( p_2=30 \text{MPa} \) are substituted into Eq. 8, the maximum resultant stress at \( z = 0 \) is reached as Eq. 9:
\[
\sigma_{w1} = \frac{-p_2 b^2}{b^2 - a^2} \sqrt{2 + 2 \frac{a^4}{r^4} + 4v^2} = -166.3 \text{MPa}
\]

At sleeve’s big end, when \( a=30.75 \text{mm}, b=36.5 \text{mm}, v=0.3, p_1=0, \) and \( p_2=30 \text{MPa} \) are substituted into Eq. 8, the maximum resultant stress at \( z = 0 \) is obtained as Eq. 10:
\[
\sigma_{w2} = \frac{-p_2 b^2}{b^2 - a^2} \sqrt{2 + 2 \frac{a^4}{r^4} + 4v^2} = -215.8 \text{MPa}
\]

Yield strength of 304 stainless steel is \( \sigma_{0.2} = 205 \text{MPa} \), it obvious that \( \left| \sigma_{w2} \right| > \sigma_{0.2} \). Therefore, the sleeve should turn into an elastic-plastic one from the big end and could not work at 30 MPa external pressure. Through Tresca yield condition, the elastic limit stress \( p_\text{et} \) is calculated as Eq. 11:
\[
p_\text{et} = 0.5 \times \sigma_{0.2} \times \left(1 - \frac{a^2}{b^2}\right) = 29.8 \text{MPa}
\]

Obviously plastic zone in the sleeve should have been caused by this external pressure. Taking \( r_s \) as radius of the boundary surface which divides elastic zone and plastic zone, taking \( q \) as uniform pressure on the boundary surface, and taking \( p_\text{pw} \) as its plastic limit stress, the force diagram is shown as Fig. 6.
In plastic zone, where \( r_2 \leq r \leq r_1 \), equilibrium equation \( \frac{\partial \sigma_r}{\partial r} - \frac{\sigma_r - \sigma_\theta}{r} = 0 \) and Tresca yield condition \( \sigma_r - \sigma_\theta = \sigma_{0.2} \) should be satisfied, thus \( \sigma_r = \sigma_{0.2} \ln r + c \) can be gotten, where \( c \) is a const determined by boundary conditions. Substituting boundary conditions \( \sigma_r \bigg|_{r=r_2} = -q \) and \( \sigma_r \bigg|_{r=r_1} = 0 \), we can get Eq. 12.

\[
\sigma_r = \sigma_{0.2} \ln r - \sigma_{0.2} \ln \frac{a}{r} = \sigma_{0.2} \ln \frac{r}{a}, \quad \sigma_\theta = \sigma_r + \sigma_{0.2} = \sigma_{0.2} \left(1 + \ln \frac{r}{a}\right) \quad (12)
\]

In elastic zone, where \( r_2 \leq r \leq b \), according to Eq. 4, we can get Eq. 13.

\[
\sigma_r = \frac{q r_2^2 - p_{pu} b^2}{b^2 - r_2^2} - \frac{(q - p_{pu}) r_2^2 b^2}{(b^2 - r_2^2)^2}, \quad \sigma_\theta = \frac{q r_2^2 - p_{pu} b^2}{b^2 - r_2^2} + \frac{(q - p_{pu}) r_2^2 b^2}{(b^2 - r_2^2)^2} \quad (13)
\]

On the boundary surface between plastic area and elastic area, \( \sigma_r \bigg|_{r=r_2} \) should be equal to \( \sigma_\theta \bigg|_{r=r_2} \). So calculating \( \sigma_\theta \bigg|_{r=r_2} \), we get a solve as follows:

\[
\sigma_\theta \bigg|_{r=r_2} = \frac{r_2^2 \cdot \sigma_{0.2} \ln \frac{r_2}{a} - p_{pu} b^2}{b^2 - r_2^2} + \frac{\left(\sigma_{0.2} \ln \frac{r_2}{a} - p_{pu}\right) r_2^2 b^2}{(b^2 - r_2^2)^2} = \sigma_{0.2} \left(1 + \ln \frac{r_2}{a}\right) \quad (14)
\]

Thus plastic limit stress \( p_{pu} \) can be calculated as in Eq. 15.

\[
p_{pu} = \frac{\sigma_{0.2}}{2b^2} \left( b^2 - r_2^2 \left(1 + \ln \frac{r_2}{a}\right) + (b^2 + r_2^2) \ln \frac{r_2}{a} \right) = 35.14 \text{MPa} > 30 \text{MPa} \quad (r_2 = b) \quad (15)
\]

It is obvious that the casing designed can only be used in theoretical maximum external pressure up to 29.8 MPa. In order to ensure its normal function deeper than 3000 m underground, measures such as increasing wall thickness or setting stiffeners should be taken.

Substituting \( E = 206 \text{ GPa} \), \( z=680 \text{ mm} \), \( p_1=0 \), \( p_2=29.8 \text{ MPa} \), and \( v=0.3 \) into Eq. 7, the maximum amounts of displacement component at the sleeve’s big end are obtained as in Eq. 16 and Eq. 17, respectively:

\[
\begin{align*}
\omega_w &= -\frac{2v}{E} \left( -\frac{p_2 b^2}{b^2 - a^2} \right) z = 0.291 z \times 10^{-3} = 0.197 (z = 680) \\
\end{align*}
\]

It is clear that the minimum clearance is just induced by the elastic deformation. Theoretically, the gap between the casing and the internal parts, such as rotary valve, etc, should be greater than 0.03
mm in radial direction and should be greater than 0.2 mm in axial direction, thus ensuring that its internal parts can be installed in a flexible manner.

**Torsion Strength Analysis**

Suppose accidents of pipe-sticking or drill burying just happen around the drill bit during drilling in soft coal seam. In this case, assuming the drill bit is a fixed end and the sleeve is a free end, and torque is transferred through drill-pipe to the sleeve without loss. Then the sleeve’s stress issue can be simplified as a uniform cross-section bar’s torsion problem. It can be solved with Prandtl stress function method to calculate in Cartesian coordinate system shown in Fig. 7. [8].

Stress components in the sleeve are $\sigma_x = \sigma_y = \sigma_z = \tau_{xy} = 0$. To hollow cylinder with a cross-section of multiply connected domains, the stress function can be attained such as Eq. 18:

$$\tau = \sqrt{\tau_{xx}^2 + \tau_{yy}^2} = \frac{2Tr}{\pi(b^2-a^2)} \quad \tau_{\text{max}} \big|_{b-a} = \frac{2Tb}{\pi(b^2-a^2)}$$

The limit shear strength of 304 steel is $\tau_c = (0.5\cdot0.6)\sigma_{0.2}/n_s$, $\sigma_{0.2} = 205\text{MPa}$ is limit yield strength, and $n_s = (2 - 2.5)$ is safety factor. Finally we calculate and reach a $\tau_c = 41 - 61.5\text{MPa}$.

As to sleeve’s big end, when $a=30.75\text{mm}, b=36.5\text{mm}, T=1500\text{N} \cdot \text{m}$ and substituted into Eq. 27, we get the biggest shear stress $\tau_{\text{max}} = 39.57\text{MPa} < \tau_c$, which means that the sleeve designed meets torsion strength requirement. As to sleeve’s small end, where $a=15\text{mm}, b=19\text{mm}, T=1500\text{N} \cdot \text{m}$, again we get $\tau_{\text{max}} = 227.66\text{MPa} > \tau_c$, which means that the sleeve’s torsion strength in the small end is not enough and the small end is the stress concentration area.

To satisfy both the compressive strength and torsion strength requirements, the sleeve’s minimum inner diameter and outer diameter can be calculated from the simultaneous equations $P_e = 0.5\times\sigma_{0.2} \times (1 - a^2/b^2) \quad \tau_{\text{max}} \big|_{b-a} = \frac{2Tr}{\pi(b^2-a^2)}$. Taking $P_e = 30\text{MPa}, \tau_{\text{max}} = \tau_c = 41\text{MPa}$, the result is $a = 23.4\text{mm}, b = 27.8\text{mm}$, which is the minimum size of the sleeve’s small end. It is clear that increasing the wall-thickness is one of the effective measures.

**Simulation about the Sleeve’s Stress and Strain**

To check and verify calculations above, FEA software is used to analyze the sleeve’s stress and deformation. The stress distribution in 30 MPa outer pressure is shown in Fig. 8 (a). Maximum stress calculated with theory is 215.8 MPa > $\sigma_c = 205$ MPa, and maximum stress simulated with FEA software is 205.74 MPa > $\sigma_c$, both at the sleeve’s big end. We can calculate that the maximum stress is 204.37 MPa while elastic limit takes 29.8 MPa. Moreover, the sleeve’s radial maximum displacement is 0.027668 mm which occurs on the large end of the inner cylinder surface, and the maximum axial displacement is 0.282 mm. These results simulated are shown in Fig. 8 (b), Fig. 8 (c) and Fig. 8 (d). Results calculated with theory are consistent with the simulation ones.

To the sleeve’s torsion strength analysis, the hollow straight rod with the small end fixed and the big end free, due to 1500 N·m torque, the stress distribution is shown as Fig. 9, the maximum shear stress is $\tau_c = 242.02\text{MPa} > \tau_c$, which occurs on small end of the outer surface, which means the sleeve designed does not meet the torsion strength requirement. This result is consistent with the one calculated with theory.
Elastic theory is utilized to analyze the stress and deformation of the second sleeve in grouting while drilling tool. As regards compressive strength analysis, both the calculation with theory and the simulation with FEA software show that the maximum internal stress exceeds the material elastic yield limit in 30 MPa external pressure. According to Tresca yield condition, theoretical external work pressure is calculated as 29.8 MPa, the radial deformation is about 0.03 mm, and the axial deformation is about 0.2 mm. To ensure the sleeve’s normal performance more than 3000 m underground, its wall thickness should be increased or stiffeners should be added. As for sleeve’s torsion strength analysis, according to both calculation with Prandtl stress function method and simulation with FEA software, conclusions can be taken that shear strength on the sleeve’s small end is far more than the material’s limit shear strength in the role of 1500 N·m torque, the small end is stress concentration area and the sleeve can’t meet the requirement.

Figure 8. (a) Stress in 30MPa external pressure.

Figure 8. (b) Stress in 29.8MPa external pressure.

Figure 8. (c) Radial displacement in external 29.8MPa.

Figure 8. (d) Axial displacement in external 29.8MPa.

Figure 9. Stress due to 1500N·m torsion.

Conclusion
The analysis above only considers the sleeve’s static analysis and simulation, and treatment has been simplified in accordance with the actual situation. Therefore, the conclusions above should be discussed in depth and be verified in practice.

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