A PERT Schedule Planning Method Based on the Improved Time Parameter Estimation

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Abstract. Project Evaluation and Review Technique (PERT) is one of the main project management methods used in electrical engineering construction. Due to indeterminacy of the criteria of the time parameter estimation and neglect of the project resource constraints, scheduling error often occurs. This paper introduces a new PERT schedule planning method based on the improvement of the time parameter estimation. Given that the planned duration of a single activity obeys \( \beta \) distribution, the principle that time parameter estimation should follow is discussed, and then a method how to instead the most optimistic time by the least time with resource constraints is proposed, therefore the criteria of time parameter estimation are determined. Next, the \( \beta \) distribution is fitted according to the given time parameters. Finally, cost-time linear optimization model is established, including probabilistic constraint and the buffer constraint of the \( \beta \) distribution. The model is solved, results in the optimal planned schedule. Simulation analysis verifies the effectiveness of the improved method.

Introduction

In the management of power construction projects, progress, quality and cost are the three major management objectives, they have a relation of unity of opposites and the optimal of schedule is very important \[1\].

Project Evaluation and Review Technique (PERT) has been widely used in the progress management of power construction projects \[2\][3]. However, since the PERT was put forward, there are lots of discussion about its rationality, which mainly focus on whether the hypothesis of the distribution model is reasonable and the impact of the estimated results caused by the choice of the three time parameters (the shortest time, the longest time and the most reasonable time). In \[4\], it was proposed that the PERT duration distribution model could be one of the four different probability distribution including \( \beta \) distribution, \( \Gamma \) distribution, triangular distribution and exponential distribution, in which \( \beta \) distribution was the suggested option. Reference \[5\] emphasized that it is more reasonable to choose the \( \beta \) distribution as the model of the duration in PERT, the main reason was that distributions such as triangular distribution and \( \Gamma \) distribution, can be simulated by changing the parameters of the \( \beta \) distribution. The choice of three time parameters in PERT will greatly influence the estimation of construction period. Reference \[6\] came up with six kinds of fixed expectation and variance estimation formula of boundary value probabilities, considering the difficulty to confirm the extreme probability of the shortest and longest time, which could result in a large deviation of the estimated result of the duration. In \[7\], the variance estimation formula was improved, considering the influence of most reasonable time. When the difference between the longest time and the shortest time is the same, the variance estimation result will be determined by the most reasonable time, which solves the problem of the conflict between the variance estimation result and the reality. In \[8\], the answer was given to the question how the three time parameters should be chosen to be closer to the mean value. It was pointed that the probability of the shortest, most probable
and longest time was more satisfied to the mean estimate when they were close to 1/6, 4/6 and 5/6. In
addition, reference [9] had already proposed that progress schedule expectation of PERT is much
smaller than the actual statistical value, which had not been effectively resolved yet.

On the other hand, the resource constraints of the construction project also affect the three time
values. Reference [10] introduced the activity criticality indicators and importance degree indicators
to assess the risk of activities in the PERT, and used them as risk coefficients in resource allocation
decision, meanwhile the expected value of time was estimated by the look-up table method. But the
method which corrects results directly according to the experience does not really reflect the impact of
resource constraints on the value of time, whose reasonability worth more discussion.

In view of the above problems, this paper discusses the scale principle of three time parameters
given that PERT planning duration is subject to the $\beta$ distribution, adding the consideration of
resource constraints to it. On the basis of the simulation of $\beta$ distribution, as well as the situation that
expected value is too small, we propose a new PERT schedule planning method to obtain a more
reasonable schedule value. Simulation results show that the proposed method is effective.

**PERT and Its Problems**

**PERT**
PERT is a technique considering the uncertainty of the duration of the network planning and
evaluation. Its basic processes applied to the construction process progress management are as
follows: Draw P3 network diagram according to the logic relationship between working procedures of
the construction project; Estimate the duration of the process reasonably after clearing the power grid
project duration target, in general, the schedule assumption is subject to the $\beta$ distribution, which can
be obtained using the three-time estimation method; Finally, we get the schedule by determining the
critical path based on the expected duration or the total time difference between processes is equal to
0. It is possible to further calculate the flexible time schedule of each process on the critical path to
cope with the progress deviation in the subsequent progress of the project.

It can be seen that the most critical link of PERT applied to the project schedule development, is the
estimation of three time parameters, so as to determine the $\beta$ distribution of the planned duration.

The section headings are in boldface capital and lowercase letters. Second level headings are typed
as part of the succeeding paragraph (like the subsection heading of this paragraph).

**Problem Analysis**
The $\beta$ distribution of the PERT plan duration is related to three times, namely, the shortest time $a$, the
longest time $b$, the most reasonable time $m$. The $\beta$ distribution function is expressed as

$$
\beta(a, b, r, s) = \frac{r! s!}{(r+s)!} \int_0^\infty \frac{(x-a)^{r-1} (b-x)^{s-1}}{(b-a)^{r+s-2}} \Gamma(r) \Gamma(s) \, dx
$$

where $r, s$ are the positive unknown shape parameters of the $\beta$ distribution, respectively. And function
$\Gamma$ can be expressed as

$$
\Gamma(z) = \int_0^\infty t^{z-1} e^{-t} \, dt
$$

The most reasonable time $m$ can be seen as the mode $M$ of the $\beta$ distribution as

$$
m = M = \frac{b(r-1) + a(s-1)}{r + s - 2}
$$

Three-time estimation method is usually used in the estimation of the duration, by which we
estimate the shortest time, the longest time, the most reasonable time to complete each process
according to the experience in order to reflect the uncertainty of process duration. Hence, we can get
the expected duration and statistical variance after determining the $\beta$ distribution as
\[ \mu = E(x) = \frac{r}{r+s}b + \frac{s}{r+s}a \quad (4) \]
\[ \sigma = D(x) = (b-a)^2 \left( \frac{rs}{(r+s)^2(r+s+1)} \right) \quad (5) \]

Therefore, one of the important hypothesis to make PERT method being valid is that the expert can accurately estimate the value of the three time parameters [11]. According to the definition of shortest time and longest time, parameter a corresponds to the extreme left end of the \( \beta \) function while \( b \) corresponds to the right side of the \( \beta \) function, that is, \( \beta(a)=0 \) and \( \beta(b)=0 \), which means the completion probability of process is equal to 0 and 1 on the period limit condition, respectively. However, in practice, people tend to keep thinking of insurance, and such limit condition cannot be accurately estimated, so the general treatment is to take value \( a \) and \( b \) as minimal value and maximum value as much as possible. But it is hard to say that the boundary value found is representative.

In the case that value \( a \), \( b \), \( r \), and \( s \) is clear, the planned duration can be obtained from Eq. 4. Because the \( \beta \) distribution parameters \( r \) and \( s \) are closely related to value \( m \), the estimated value of this three time parameters therefore decide the planned duration.

Compared with the shortest time and the longest time, the concept of the most reasonable time lacks a unified basis, and there is no fixed probability of \( \beta(m)=0 \) when making an estimate of \( m \). Therefore, it is difficult to guarantee the reasonableness of the obtained result by using the estimated value \( m \) to obtain the expected period value \( \mu \).

In addition, the consideration of resource constraints in the PERT method is generally based on the assumption that the total amount of resource requirements in each process is fixed and the supply of various resources is constant at any time during the project. Therefore, the application of PERT method needs to be improved in the case of resource demand and resource supply change.

### Improved Time Parameter Estimation

#### Basis for Estimation of Time Parameters

As can be seen from the above, the determination of the shortest, longest, most reasonable time in PERT is crucial. But because it is difficult to grasp the forecast scale, it is inevitable to cause errors of the estimated period, thus affecting the progress of planning.

The estimation value of shortest time and longest time can produce a marginal error. But in engineering practice, the longest time \( b \) is controlled with a margin value, because of the hardening requirements of the latest completion time, which is the total duration limitation. The limitation can be expressed as

\[ \sum_{i=1}^{n} h_i \leq T \quad (6) \]

According to the theory of human behavior, people’s psychological pressure under the limitation will keep increasing, so the longest time will be slightly ahead of the theoretical estimates, that is, the cumulative probability corresponding to the longest time \( b \) should be slightly smaller than 1, This paper advises to take it as

\[ P(b) = 0.95 \quad (7) \]

The estimation of shortest time \( a \) does not have a similar marginal limitation like \( b \), so \( P(a) \) Should take a suitable value greater than 0. There are several references given in [6] that \( P(a)=0.01/0.05/0.1 \). But in the actual project duration estimation, it is hard to predict the value in the vicinity of the exact value accurately to the one place of the estimated percentage, as well as taking into account of the construction resources constraints, the value of \( P(a) \) should be adjusted upwards, whose specific
value should be determined by the number of resources occupied. Therefore, assuming that the shortest time taking care of resource constraints is \( a' \), the probability is expressed as

\[
P(a') = \frac{O_i}{R+1}
\]  

(8)

Where \( O_i \) is the number of resources occupied by the \( i \)th process, while \( R \) is the total number of resources for the project, \( P(a') \in (0,1) \).

When the amount of resources is continuously changing, we can introduce the theory of constraint Theory of Constrains (TOC) to solve the problem mentioned. Critical Chain Method (CCM) uses the constraint theory TOC for project management, which estimates the duration according to the 50% completion probability and sets it as the schedule, while sets the difference between the owner's duration expectation and the duration estimation to the project buffer. In the event of a deviation or uncertainty factors, the duration will be controlled within the required range by the use of the buffer. Moreover, find out various types of constraints which influence the duration according to the changes in resources at any time. And then centralize the resources or time to eliminate the confines of the constraints to achieve the dynamic optimization of the schedule, reference [12] describes the application of TOC theory in the construction management of power transmission and transformation engineering.

To be specific, for the impact of resource constraints on the shortest time, we can use the idea which relaxes the "resource constraints" or "new resource constraints" in the TOC to tackle with the value \( O_i \) and \( R \) in Eq. 8. Therefore, assuming that the shortest time taking care of resource constraints is \( a'' \), the probability is expressed as

\[
P(a'') = \frac{O_i - R_{\text{release}} + R_{\text{add}}}{R + 1 + R_{\text{newadd}}}
\]  

(9)

Where, \( R_{\text{release}} \), \( R_{\text{add}} \), and \( R_{\text{newadd}} \) represent resources that are released, added resources that original activities didn’t take use of and new resources added by the entire project, respectively.

**Fitting of the \( \beta \) Distribution in Duration**

After determining the gist of the time parameter prediction, the \( \beta \) distribution of the construction duration can be further obtained. In [13], three fixed-time probabilistic estimations are used to solve this problem, that is, estimate the duration on the basis of fixed three-time probabilities. Three fixed-time probability estimation methods can be divided into two categories, the first type is an empirical formula algorithm based on a large number of experimental data, such as several kinds of mean and variance calculation formulas for fixed probability combinations. The second type is the \( \beta \) probability density distribution fitting method using computer numerical solution, to find the parameter \( a, b, r \) and \( s \) which result in the smallest variance of the fit, and then to determine the \( \beta \) distribution. The fundamental difference between the two methods is reflected in the initial data access, where the first category requires fixed probabilities and combinations while the second category of fitting method is relatively flexible. According to the literature [12], the error of the fitting method is smaller and more reasonable. The \( \beta \) distribution variance can be expressed as

\[
\Delta \varepsilon = \frac{1}{n} \sum_{i=1}^{n} [\beta(t_i) - P_i]^2
\]  

(10)

Therefore, fitting the \( \beta \) distribution function with three fixed-time probabilities which derived from the fixed probability \( P=(P_1, P_2, \ldots, P_n) \) and its corresponding estimated duration \( t=(t_1, t_2, \ldots, t_n) \), can fit the \( \beta \) distribution and make \( \Delta \varepsilon \) the smallest., and then get the fitting of the distribution parameters.

Since the estimation is based on the cumulative probability distribution, so in this paper, the cumulative probability distribution function of \( \beta \) is used for fitting as
\[ P = F(x|\alpha, \beta) = \frac{1}{B(\alpha, \beta)} \int_0^{x^{\alpha-1}(1-t)^{\beta-1}} dt \]

(11)

Where \( \alpha, \beta \) are the unknown shape parameters for the cumulative probability distribution model.

In the case of the three given fixed time probabilities are \( P(a') \) or \( P(a'') \), \( P(b) \) and 0.5, respectively, relationship between the estimated duration-probability can be obtained by the cumulative probability distribution, thus completes the determination of the shortest time considering resource allocation, and solves the problem of reasonable allocation of time periods under changing resource constraints. Specially, when \( P(a') \) or \( P(a'') = 0.5 \), that is, \( P(a') \) or \( P(a'') \) coincides with a fixed probability of 0.5, since the three-time estimation requires three different parameters, so we make \( O_i = O_{i+1} \) and revalue \( P(a') \) or \( P(a'') \) upwards to meet the needs. This paper makes the duration \( a' \) or \( a'' \) corresponding with \( P(a') \) or \( P(a'') \) as the lower limit of regulation after resource constraint changing.

In the fitting calculation of the \( \beta \) cumulative probability distribution, we do not use the most reasonable time of the estimated value, and therefore avoided the deviation caused by the estimated uncertainty.

**Improved PERT Schedule Formulating Method**

In the classic PRET, we use \( a, b, m \) and \( \mu \) showed in Eq. 5 as schedule value. Some critical processes or some processes which needs lots of resources occupy most of the duration, the range of adjustable duration is small. And on \( \beta \) distribution \( \mu \) corresponding probability is about 0.5. So these kinds of durations which using \( \mu \) as schedule value are obviously unsuitable. In [9], it describe the problem that \( \mu \) compared with the actual statistical period is small. To solve this problem, we combined improved three estimated time values and the time buffer to improve the cost time linear optimization model mentioned in [14], to obtain a method that can distribute the duration and flexible time more reasonable. The specific improvements of the optimization model are as follows

1) Time compression is expressed as the probability constraint, \( P_i(a') \) or \( P_i(a'') \) is upper probability and \( P_i(b) \) is lower probability.

2) The time constraints are expressed as the quality constraints. We proposed duration is proportional to quality, and the conversion coefficient is \( \omega \).

3) Adding flexible time constraints.

The mathematical model is expressed as

\[
\begin{align*}
\min C &= \sum_{i=1}^{n} C_i x_i \\
\sum_{i=1}^{n} x_i &\geq N \\
P(a_i') &\leq P_i \leq P(b_i) \\
P_i &\geq P\left(\frac{1-O_i}{\omega_i}\right)
\end{align*}
\]

(12)

Where \( C_i \) is the \( i \)th’s compression cost, \( x_i \) is the \( i \)th’s compressible time basing on the upper time \( b_i \), \( N \) is the requirement of the total compression time which can be adjusted as buffer according to the actual progress.

After getting the optimization results \( x_i \), the schedule \( H_i \) can be expressed as

\[ H_i = b_i - x_i \]

(13)

Through the new method, the end optimized results \( H_i \) we get include the comprehensive consideration of the three factors: schedule, quality, and cost. Besides the new method can eliminate the deviation of the schedule in the classical method because of its adjustable function.
**Example**

Firstly, we analysis a project according to the longest time $b$. Then we got the critical path expressed as 1→2→3→4→5. The key process flow chart is shown in Figure. 2, which the nodes represent procedures.

![Figure 2. The critical activities’ flow chart.](image)

The optimization results of the classic PERT and the improved method are shown in Table 1 and Figure. 3. In this example, we proposed $N=25$.

![Figure 3. The comparison of completion probability between classic PERT and improved method.](image)

<table>
<thead>
<tr>
<th>Node</th>
<th>The improved method</th>
<th>The classic PERT</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Calculated duration when $P=0$</td>
<td>Calculated duration when $P=1$</td>
</tr>
<tr>
<td>1</td>
<td>40</td>
<td>0.044</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
<td>0.005</td>
</tr>
<tr>
<td>3</td>
<td>14</td>
<td>0.027</td>
</tr>
<tr>
<td>4</td>
<td>25</td>
<td>0.04</td>
</tr>
<tr>
<td>5</td>
<td>3</td>
<td>0.893</td>
</tr>
</tbody>
</table>

*We got the calculated duration through the fitting $\beta$ distribution, actually we set $P=0.00001$ and $P=0.99999$.

There are the conclusions and analyses of the results

1) The sum of durations in the improved method can be expressed as $\sum H=92$, obviously bigger than in the classic PERT expressed as $\sum m=70$. Through analysis, we thought the reason is we set a buffer to respond to emergencies in the improved method. Besides, in most cases the probabilities of $H$, corresponding to $\beta$ distribution is greater than the probabilities of $\mu$ in the classical PERT, it connects better with actual expectations in critical processes;

2) The calculated durations which 0 probabilities correspond in the classic PERT is slightly greater than the calculated values in the improved method, but all close to 0;

3) The calculated durations which 1 probabilities correspond in the classic PERT is obviously greater than the calculated values in the improved method and much larger than $b$, but in the improved method, the values close to $b$;

4) The variance in the improved method is much less than the variance in the classical PERT.
Conclusion

The "three time estimation method" of classical PERT, which ignores resource constraints sometimes results in unreasonable schedule. To solve the problems, we develop an improved method, a well-defined selected scale to the time parameters is determined, and the time distribution restriction is considered by handling the resource constraints as probability constraints in setting the lower bound of duration. Cost—time linear optimization model is built and solved based on the new ideas. Simulation results show that the improved model not only gives balanced attention to quality, time requirement and cost, but also reaches higher completion probability. It is helpful to hope with the change of external resources and the extension of individual processes in power grid construction.

Reference


