Identification of Linear and Nonlinear Parameters for Systems with Local Non-Linearity

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Abstract. A new method based on dynamic response sensitivity is proposed for the identification of local nonlinear system. The unknown local nonlinear hysteresis parameters are identified together with linear structural parameters. The sensitivities of structural dynamic response with respect to unknown parameters are derived. The identification equation is set up based on Taylor’s first order approximation, and is solved with the damped least-squares method. A four-story shearing system with rubber isolation bearing of nonlinear hysteresis model is studied to validate the proposed method. Numerical simulation with noisy measured accelerations shows that the proposed method can accurately identify both linear and nonlinear parameters of the system. This method provides a new approach for detecting local nonlinear structures with incomplete measured output information.

Introduction

Nonlinear system identification is a fast evolving field of research with contributions from different communities, such as the mechanical engineering, systems and control, and civil engineering communities [1]. Many identification methods have been developed over the last years, for a wide variety of model structures. These methods can be classed into two sets. In the first set, the identification procedure is transformed into a state estimation problem after discretizing the differential equations into discrete state equations and treating the parameters as state variables. In the second set, identification of the nonlinear parameters from the measured data is formulated as an inverse problem and is often fulfilled by solving an optimization problem. Then, various techniques are proposed to deal with the state estimation problem or the optimization problem [2].

Among the various kinds of nonlinear structures, there is a common type that the nonlinearities of the system are caused by several local nonlinearities. For example, the joint connecting different substructures is a major source of the local nonlinearity with features such as friction, gaps, stick-slip behavior. The assumption of local nonlinearities simplifies the decoupling of linear and nonlinear parts of the system and thus makes it possible to utilize some well-developed linear techniques [3]. In this paper, a new method based on dynamic response sensitivity is proposed for the identification of local nonlinear system. The nonlinear hysteresis model used is the Bouc-Wen model. The sensitivities of structural dynamic response with respect to both nonlinear hysteresis parameters and linear structural parameters are derived. The identification equation is set up and is solved with the damped least-squares method in an iterative strategy. A four-story shearing building with rubber isolation bearing of nonlinear hysteresis model is studied to validate the proposed method.

Identification Theories of Local Nonlinear Structures

Equation of Structures with Local Non-Linearity

For a multi-degrees of freedom (DOF) system with local non-linearity under support excitation, its equation of vibration can be expressed as
\[ M\ddot{x} + C\dot{x} + Kx + f(x, v) = -M \cdot G \cdot \ddot{x}_g \]  

where \( M, C \) and \( K \) are respectively the mass, damping and stiffness matrices of the linear parts of the structure. \( f(x, v) \) is the restoring force vector of the local nonlinear elements of the structures, and it can be described based on different nonlinear model. \( \ddot{x}, \dot{x} \) and \( x \) are the relative acceleration, velocity, and displacement vectors of the structure for the ground, respectively. \( \ddot{x}_g \) is the support excitation acceleration contributing inertia forces to the whole structures. \( G \) is the mapping matrix that controls the direction of the support excitation to the structure.

**Bouc-Wen Model Restoring Force**

The expression for restoring force vector \( f(x, v) \) can be described based on different nonlinear model. Here a typical Bouc-Wen Model [4] is used to describe the restoring force. In this model, the restoring force and deformation of the structure are related to a nonlinear differential equation with uncertain parameters and can be expressed as

\[ f(x, v) = \alpha k x + (1 - \alpha) k v \]  

\[ \dot{v} = A \dot{x} - \beta |\dot{x}|^{\alpha-1} v - \ddot{x} |v|^n \]  

where \( k \) is the initial stiffness of the local nonlinear element, \( \alpha \) is the ratio of linearity to nonlinear stiffness, \( x \) the total displacement of the bearing structure, and \( v \) is hysteresis displacement of the support structure. \( A, \beta \) and \( \gamma \) are the parameters of the model and \( n \) is the order of the model.

**Sensitivity Equation of Dynamic Responses to Structural Parameter**

For the linear parts, stiffness and damping parameters are taken as the parameters needed to be identified since mass parameters are easier to be calculated for most structures. The perturbation of the structural stiffness and damping matrix can be respectively described as

\[ \Delta K = \sum_{j=1}^{N_k} \Delta \mu_j k_j^e \quad (0 \leq \Delta \mu_j \leq 1.0) \]  

\[ \Delta C = \sum_{j=1}^{N_c} \Delta \nu_j c_j^e \quad (0 \leq \Delta \nu_j \leq 1.0) \]  

where \( \Delta \mu_j \) is the stiffness change fraction of the \( j \)th structural stiffness parameter \( \mu_j \), and \( \Delta \nu_j \) is the damping change fraction of the \( j \)th structural damping parameter \( \nu_j \). \( N_k \) and \( N_c \) are the total number of the identified structural stiffness and damping parameters.

If accelerations are taken as an example, the sensitivity of the structural dynamic responses with respect to the structural linear parameter \( \mu_j \) and \( \nu_j \) can be computed by the direct differentiation method as

\[ \frac{\partial \ddot{x}}{\partial \mu_j} = \frac{\ddot{x}(\mu_j + \Delta \mu_j) - \ddot{x}(\mu_j)}{\Delta \mu_j} \]  

\[ \frac{\partial \ddot{x}}{\partial \nu_j} = \frac{\ddot{x}(\nu_j + \Delta \nu_j) - \ddot{x}(\nu_j)}{\Delta \nu_j} \]  

where \( \ddot{x}(\cdot) \) is the calculated acceleration from Eq.(1).

For the nonlinear parts, the hysteresis parameters are taken as a parameter vector \( \eta = [A \quad \beta \quad \gamma \quad n]^T \) to be identified. The sensitivity of the structural dynamic responses with respect to the nonlinear parameters are respectively computed as
\[
\frac{\partial \ddot{x}}{\partial \eta_j} = \frac{\ddot{x}(\eta_j + \Delta \eta_j) - \ddot{x}(\eta_j)}{\Delta \eta_j}
\]
where \( \eta_j \) and \( \Delta \eta_j \) is the \( j \)th structural nonlinear parameter and its change fraction.

**Definition and Solution of Identification Equation**

In the inverse identification, the unknown parameters are required to be identified from the measured responses. In other words, the parameters are chosen to best fit the experiment data. Based on Taylor’s first order approximation, the identification problem can be expressed as

\[
\delta \ddot{x} = S \cdot \delta \theta
\]

where \( \delta \ddot{x} \) is the difference between measured and calculated accelerations and \( \delta \theta \) is the vector of perturbation in the unknown parameters. \( S \) is the sensitivity matrix of the structural responses in the measured DOFs with respect to the unknown parameters in the time domain.

In order to provide bounds to the solution of the ill-conditioned problem, the damped least-squares method (DLS) [5] is used. Eq. 9 can be written in the following form in the DLS method:

\[
\delta \theta = (S^T S + \lambda I)^{-1} S^T \delta \ddot{x}
\]

where \( \lambda \) is the regularization parameter governing the participation of least-squares error in the solution. The solution of Eq. 10 is equivalent to minimizing the function

\[
J(\Delta \theta, \lambda) = \|S \Delta \theta - \Delta \ddot{x}\|^2 + \lambda \|\Delta \theta\|^2
\]

with the second term in Eq. 11 proving bounds to the solution. An L-curve method [6] is used in this study to obtain the optimal regularization parameter \( \lambda_{\text{opt}} \).

To overcome the negative effect of the randomly chosen initial values of the unknown parameters, an iterative identification strategy is adopted to obtain \( \delta \theta \).

**Numerical Simulation**

A four-layer shearing system with rubber isolation bearing as shown in Fig.1 is taken as an example to validate the proposed algorithm. The main body of the system with isolation layer is modeled as a 5-DOF system consisting of lumped masses and massless springs and dampers. The concentrated mass of the isolation layer and the other layers all is 500kg. The linear shear stiffness and damping coefficients of the isolation layer respectively is 200kN/m and 50N·m/s, and 240kN/m and 100N·m/s for the other four layers. The hysteresis parameters of the isolated bearing are as following: \( \alpha_{b} = 0.2 \), \( \beta_{b} = 0.5 \), \( \gamma_{b} = 0.5 \) and \( n_{b} = 2 \). The system is subject to the El-Centro seismic acceleration at the support. The sampling rate and time is respectively 0.02Hz and 4s. The accelerations of each layer are taken as the measured responses in the identification of the unknown parameters. 5% and 10% Gaussian white noise is added to the calculated accelerations to simulate the polluted measurements.

Figure 1. Four-layer shear frame with rubber isolation.
Figs. 2 illustrates the identification errors of linear stiffness and damping parameters of the system. As shown in this figure, both linear stiffness and damping parameters can be exactly identified for the noise free cases. For the case with 5% and 10% measurement noise, the identified structural parameters deviate from the true ones. The identification errors increase with the noise level. The maximum relative error of the stiffness and damping parameters is close to 0.5% and 10% respectively. This indicates that the measured noise has some negative effect on the results in comparison with results from the noise free case. However, the identified linear stiffness parameters can more exactly reflect their true values, and the identified damping parameters also are close to their true values.

![Figure 2. Identification errors of linear stiffness and damping parameters.](image)

Table 1 lists the identified hysteresis parameters of local isolated bearings, and Fig. 3 compares the identified hysteresis curves with the true one. As shown in Table 1 and Fig. 3, the identified hysteresis parameters and hysteresis curves both can reflect their true values for the noise free case, and the measured noise only has a little negative effect on the results in comparison with results from the noise free case.

<table>
<thead>
<tr>
<th>Hysteresis parameters</th>
<th>$a_0$</th>
<th>$\beta_0$</th>
<th>$\gamma_0$</th>
<th>$n_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Identified without noise</td>
<td>0.2</td>
<td>0.5</td>
<td>0.5</td>
<td>2</td>
</tr>
<tr>
<td>Identified with 5% noise</td>
<td>0.1960</td>
<td>0.5012</td>
<td>0.4950</td>
<td>2.0400</td>
</tr>
<tr>
<td>Identified with 10% noise</td>
<td>0.1946</td>
<td>0.5030</td>
<td>0.4900</td>
<td>2.1010</td>
</tr>
</tbody>
</table>
Summary

This paper proposed a new method based on dynamic response sensitivity for local nonlinear system identification. A four-story shearing system with rubber isolation bearing of nonlinear hysteresis model is studied to validate the proposed method. Numerical simulation results show the proposed method can accurately identify both structural local nonlinear hysteresis parameters and linear physical parameters from only several responses of the system for the noise free cases. The presence of random noise has some negative effect on the identification results, but the identified accuracy is acceptable. This method can provide a viable and efficient tool for identification of system with local non-linearity from incomplete measured output information.

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