Flight Classification Method Based on Unit Time Delay Cost

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Abstract. Traditional flight classification methods are usually based on the geometry of the aircraft or the size of the wake of the aircraft. In the actual operation of airports, the classification of aircraft based on the delay cost of flights was more conducive to airport operation and management, and improve the economic benefits of airlines and passengers. A new arrival and departure flights classification method based on the transitive closure algorithm was proposed. The fuzzy sets theory and the transitive closure algorithm were introduced. Four different factors were selected to establish the flights classification model. The formulas to calculate the delay cost for each class were given.

Introduction

Congestion problems are becoming increasingly acute in many major American, European and Asian airports. These problems cause inconvenience to passengers, losses to airlines and threats to airspace safety. One way of reducing the amount of congestion and losses is to use air traffic flow management (ATFM), whose objective is to disperse the air traffic flow and minimize the potential delay costs under the current capacity and demand. Odoni gave a systematic description of ATFM problems first in 1987. Since then, many researchers have proposed their own air traffic control models with the aim of reducing the total delay costs. However, among these models, there are still some drawbacks in calculating the cost of flight delays (CFD), i.e., (1) for each flight the delay cost per minute is identical (e.g., $50 for ground delay, $75 for airborne delay)[1,2]; (2) all flights are classified into only three cost classes according to the maximum take-off weight (MTOW)[3,4]. As the foundation of the objective functions in aforementioned models, it is necessary to determine CFDs scientifically.

Motivated by the above problems, we present a new approach to classify the flights. In this approach, all kinds of flights will be categorized by the transitive closure algorithm (TCA) based on 4 different factors, i.e., MTOW, maximum payload, average fuel consumption per hour (or wing span), and grade of importance. It could be used to solve dynamic clustering problems of arrival/departure (A/D) flights in terminal area during congestion periods, and calculate CFDs according to the different classes of flights. To assess the performance of this method, we make a comparison between this method and the traditional classification method, when dealing with sequencing problems of A/D flights in terminal area.

Transitive Closure Algorithm (TCA)

Among the existing fuzzy clustering algorithms, the most frequently used fuzzy clustering technique is TCA, which is based on fuzzy similar relations and fuzzy equivalent relations. It can partition the sample set into several clusters under different threshold values.

Basic Definition

Fuzzy Set and Membership Function. If \( X \) is a collection of objects denoted generically by \( x \) then a fuzzy set \( A \) in \( X \) is a set of ordered pairs: \( A = \{ (x, \mu_A(x)) | x \in X \} \), \( \mu_A \) is the membership function.
function that maps $X$ to the membership space $M$ and $\mu_A(x)$ is the grade of membership of $x$ in $A[5]$. 

**\lambda\text{-Cut and Strong} \lambda\text{-Cut.}** In order to exhibit an element $x \in X$ that typically belongs to a fuzzy set $A$, we may demand that its membership value be greater than some threshold $\lambda \in [0,1]$. The ordinary set of such elements is the $\lambda\text{-Cut} (A)_\lambda$ of $A$, $(A)_\lambda = \{x | \mu_A(x) \geq \lambda, x \in X\}$. One also defines the strong $\lambda\text{-Cut}, (A)_\lambda = \{x | \mu_A(x) > \lambda, x \in X\}$.

**Composite Relation.** Let $R$ and $Q$ be two fuzzy relations on $X \times Y$ and $Y \times Z$ respectively, then $R \circ Q$ is their composite relation on $X \times Z$ and its membership function $(R \circ Q)(x,z)$ is defined by $(R \circ Q)(x,z) = \bigvee_{y \in Y} [R(x,y) \land Q(y,z)]$.

**Similarity Relation.** Let $X_i$ and $X_j$ be elements of $X$, and let $r_{ij}$ denote the grade-membership of the ordered pair $(X_i, X_j)$ in $R$. Then $R = (r_{ij})_{n \times n}$ is a similarity relation in $X$ iff, $r_{ii} = 1$ (reflexive), $r_{ij} = r_{ji}$ (symmetric), $R \circ R \subseteq R$ (transitive)[6].

**Process of TCA**

For describing TCA, we assume hereafter that $X = \{X_1, \cdots, X_k, \cdots, X_n\}$ denotes a set of $n$ objects to be clustered and the $k$th sample in $X$ is represented by $X_k = (x_{k1}, \cdots, x_{kl}, \cdots, x_{km})$, where $k = 1,2,\cdots, n$ and $l = 1,2,\cdots, m$. The characters of $n$ samples constitute a characteristic matrix $X_{n \times m} = (x_{kl})_{n \times m}$, and $x_{kl}$ denotes the $l$th character of the $k$th sample. The clustering process is as follows.

**Step 1** Normalize the given sample set. In order to make the clustering results feasible, every character of each sample must be normalized. The formula (1) is introduced here to guarantee that the sample’s character will be a value between 0 and 1.

$$x_{kl}' = \frac{x_{kl}}{x_{\text{max}l}} \quad (1)$$

where $x_{\text{max}l} = \max(x_{1l}, x_{2l}, \cdots, x_{nl})$.

**Step 2** Construct the fuzzy similar matrix from the normalized sample set. In the fuzzy similar matrix, $R = (r_{ij})_{n \times n}, r_{ij}$ denotes the degree of similarity between $X_i$ and $X_j$, $r_{ij} \in [0,1]$. The formula for $r_{ij}$ is:

$$r_{ij} = \frac{\sum_{l=1}^{m}(x_{il}' \land x_{jl}')}{\sum_{l=2}^{m}(x_{il}' \lor x_{jl}')} \quad (2)$$

**Step 3** Compute the transitive closure with the square method. The transitive closure is usually produced with the square method, where the following matrix synthesis operations are computed in turn $R^2, R^3, \cdots, R^k$. The procedure continues until $R^k \circ R^k$ is equal to $R^k$ for the first time. Thus, the matrix $R^k$ is the transitive closure of $R$, namely $t(R)$.

**Step 4** Generate clustering results for different threshold value $\lambda$. For an arbitrary threshold value $\lambda \in [0,1], (R^k)_{\lambda}$, the $\lambda\text{-cut of } t(R)$, is a crisp equivalent relation, which can partition the sample set into several clusters.

**Flights Classification Model**

In this section, the characteristics of arrival and departure flights are introduced. Then, we set up a
model to divide A/D flights and describe its process. Finally, the formulae to calculate the delay cost per minute for each class are given.

The characteristics of arrival flights include 4 factors: MTOW, maximum payload (MP), average fuel consumption per hour (AFCH), and grade of importance (GI). Similarly, the characteristics of departure flights include MTOW, AFCH, wing span (WS), and GI. So, the parameter \( m \) in this flights classification model is 4. Note that for the factor GI, the values 100, 50, 10, and 5 represent, respectively, special plane flight, very important person flight, international flight and domestic flight.

In each congestion period, if there are \( n \) \((n \geq 5)\) flights waiting for landing or taking-off per hour, we would divide them according to their characteristics by TCA. Fig. 1 shows the process of the classification model for A/D flights.

![Flights' Classification Model](image)

Figure 1. Flow chart of the flights classification model

If \( n \) arrival or departure flights are classified into \( S \) classes by the model, then each class is defined the \( s \) th class according to decreasing mean values of GIs (if equal, according to decreasing mean values of MTOWs), \( s = 1, 2, \cdots, S \), denoted with Roman numerals I,II ,III....

For example, assume that there are 6 arrival flights (F01, F02,…, F06), and they have been classified into 3 classes. The mean values of GIs for each class are 30, 10 and 5 respectively, i.e., the first class (F01, F03) 30, the second class (F02, F05) 10, the third class (F04, F06) 5. So the classification result is I Class\{F01, F03\}, II Class\{F02, F05\} and III Class\{F04, F06\}. For arrival and departure flights the costs of delays per minute (CDPM) to the \( s \) th class are \([4 \times (S - s) + 1]c_a\) and \([4 \times (S - s) + 1]c_g\), respectively. Thereinto, \( c_a \) and \( c_g \) represent the airborne and ground delay cost unit respectively.

The time complexity of this model is \( O(n^3 \log n) \), and its space complexity is \( O(n^2) \)[7]. In most circumstances, the number of arrival or departure flights per hour is no more than 60\((i.e., n \leq 60)\), so we can get the classification result easily in one second by using a PC with 2.0GHz CPU and 1GB RAM.

**Summary**

A new A/D flights classification model based on TCA is proposed in this paper. By using TCA all flights can be classified more exactly than using traditional classification method in consideration of 4 different factors. The optimized flight classification is conducive to improving the level of air traffic control and the efficiency of the airport operation. In view of this method, it is still necessary to use this classification model to do further demonstration research.
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References