Improved Reachable Set Bounding for Linear Neutral Systems with Disturbances via Triple Integral Functionals

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Abstract. This paper focuses on bound of reachable sets for linear neutral systems with bounded peak disturbances. To best of our knowledge, there were few results related to the ellipsoidal bound of reachable sets for linear neutral systems. By utilizing the Lyapunov-Krasovskii functional theory, delay decomposition technique, reciprocally convex method and free-weighting matrix approach, some new results expressed in terms of linear matrix inequalities are derived. Furthermore, triple integral functionals are introduced first time to derive bound of reachable sets for linear neutral systems. A tighter bound of the reachable set is obtained.

Introduction

Time delay exists in various practical systems such as biological and artificial neural networks, physical and chemical processes, population models, which may lead to an oscillation or instability of systems [1,2]. Therefore, the investigation of control and stability have received growing attention [1–10]. On the other hand, disturbances are unavoidable in a wide range of phenomena in practical and engineer systems [11]. Thus, the problem of reachable set bounding for dynamical systems subject to time delay and disturbance is an important research topic [12–13].

Notations: The notations are used in our paper except where otherwise specified. $\mathbb{R}^n$ is the n-dimensional Euclidean space, $\mathbb{R}^{n \times m}$ denotes the set of $n \times m$-dimension real matrices; Real matrix $P > 0 (\geq 0)$ means that $P$ is a symmetric positive definite (positive semi-definite) matrix. Superscript $'$ denotes transposition of a vector or a matrix; $\star$ represents the elements below the main diagonal of a symmetric block matrix; I denotes an identity matrix; ‘-’ in tables represents no feasible solution for matrix inequality.

Preliminaries

Consider the following time-delayed linear neutral systems with disturbances:

$$i(t) - Ci(t - \delta) = Az(t) + Bz(t - \tau(t)) + D\omega(t), \quad z(\theta) = 0, \theta \in [-d, 0],$$

(1)

Where $z(t) \in \mathbb{R}^n$ is the state vector; $\omega(t) \in \mathbb{R}^m$ is the disturbance. $\tau(t)$ is discrete time delay and $\delta > 0$ is a constant neutral time delay. $A \in \mathbb{R}^{n \times m}, B \in \mathbb{R}^{n \times m}, C \in \mathbb{R}^{n \times m}$ and $D \in \mathbb{R}^{n \times m}$. $A, B, C$ and $D$ are known constant matrices.

$\tau(t)$ is time-varying discrete delay satisfying

$$\tau_m \leq \tau(t) \leq \tau_M, \quad \tau(t) \leq \mu,$$

Where $\tau_m$, $\tau_M$ and $\mu$ are constants. Moreover, $d = \max \{\tau_M, \sigma\}$

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Disturbance $\omega(t) \in \mathbb{R}^{n}$ is the input with bounded peak value, that is,

$$\omega^T(t) \omega(t) \leq \omega^2$$

(2)

Where $\omega^2$ is a constant.

**Main Results**

**Theorem**

If there exist matrices $P > 0, Q_1 > 0, Q_2 > 0, R_1 > 0, K_1 > 0, K_2 > 0, K_3 > 0, K_4 > 0, S, N$ with appropriate dimensions, and a scalar $\alpha > 0$, such that the following inequalities hold:

$$\begin{bmatrix} R_2 + K_3 & S \\ S^T & R_2 + K_4 \end{bmatrix} \geq 0,$$

(3)

$$\Phi = \begin{bmatrix} \Phi_{11} & \Phi_{12} & \Phi_{13} & 0 & \Phi_{15} & 0 & 0 & \Phi_{18} & \Phi_{19} & \Phi_{1,10} \\ * & \Phi_{22} & \Phi_{23} & \Phi_{24} & 0 & \Phi_{26} & \Phi_{27} & \Phi_{28} & 0 & 0 \\ * & * & \Phi_{33} & \Phi_{34} & \Phi_{35} & \Phi_{36} & 0 & 0 & 0 & 0 \\ * & * & * & \Phi_{44} & 0 & 0 & \Phi_{47} & 0 & 0 & 0 \\ * & * & * & * & \Phi_{55} & 0 & 0 & 0 & 0 \\ * & * & * & * & * & \Phi_{66} & 0 & 0 & 0 \\ * & * & * & * & * & * & \Phi_{77} & 0 & 0 & 0 \\ * & * & * & * & * & * & * & \Phi_{88} & NC & NB \\ * & * & * & * & * & * & * & * & - \frac{\alpha}{\omega^2} \\ * & * & * & * & * & * & * & * & * & - \frac{\alpha}{\omega^2} \end{bmatrix} \leq 0$$

(4)

Where

$$\Phi_{11} = \alpha P + PA + A^T P + M_2 + Q_1 - e^{-\alpha t} R_1 - 2e^{-\alpha t} K_1,$$

$$\Phi_{12} = PD, \quad \Phi_{13} = e^{-\alpha t} R_1, \quad \Phi_{15} = 2e^{-\alpha t} R_1,$$

$$\Phi_{18} = A^T N^T, \quad \Phi_{19} = PC, \quad \Phi_{1,10} = PB,$$

$$\Phi_{22} = -(1 - \mu)e^{-\alpha t} M_2 - 2e^{-\alpha t} K_3 - 2e^{-\alpha t} R_2 + 2e^{-\alpha t} K_1, \quad S^T + S,$$

$$\Phi_{23} = e^{-\alpha t} K_4, \quad \Phi_{24} = 2e^{-\alpha t} R_2 - e^{-\alpha t} S,$$

$$\Phi_{26} = 2e^{-\alpha t} K_4, \quad \Phi_{27} = 2e^{-\alpha t} K_4, \quad \Phi_{28} = D^T N^T,$$

$$\Phi_{33} = e^{-\alpha t} Q_2 - e^{-\alpha t} Q_3 - e^{-\alpha t} R_1 - 2e^{-\alpha t} K_2 - 2e^{-\alpha t} K_3 - e^{-\alpha t} R_2,$$

$$\Phi_{34} = e^{-\alpha t} S, \quad \Phi_{35} = 2e^{-\alpha t} K_2, \quad \Phi_{36} = 2e^{-\alpha t} K_2,$$

$$\Phi_{44} = -e^{-\alpha t} Q_2 + 2e^{-\alpha t} K_4 - e^{-\alpha t} R_2, \quad \Phi_{47} = 2e^{-\alpha t} K_4,$$

$$\Phi_{55} = -2e^{-\alpha t} K_2, \quad \Phi_{66} = -2e^{-\alpha t} K_2, \quad \Phi_{67} = -2e^{-\alpha t} K_2,$$

$$\Phi_{77} = -2e^{-\alpha t} K_2, \quad \Phi_{78} = -2e^{-\alpha t} K_2,$$

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\[ \Phi_{ss} = M_1 + r_a^2 R_1 + (\tau_y - \tau_x)^2 R_2 + \frac{1}{2} r_a^2 K_1 + \frac{1}{2} r_a^2 K_2 + \frac{1}{2} (\tau_y - \tau_x)^2 K_3 \]
\[ + \frac{1}{2} (\tau_y - \tau_x)^2 K_4 - N - N^T \]

Then the reachable sets of system (1) is bounded by a ball
\[ B(0, r) = \{ z \in \mathbb{R}^n : \|z\| \leq r \} \]
with
\[ r = \frac{1}{\sqrt{\lambda_{\text{min}}(P)}} \] (5)

Proof

Construct the following Lyapunov-Krasovskii functional
\[ V(z_t) = \sum_{j=1}^{0} V_j(z_t), \]

Where
\[ V_1(z_t) = z^T(t) P z(t), \]
\[ V_2(z_t) = \int_{t=a}^{t} e^{a(s-t)} \hat{z}^T(s) M_1 \hat{z}(s) ds + \int_{t-t(s)}^{t} e^{a(s-t)} \hat{z}^T(s) M_2 \hat{z}(s) ds, \]
\[ V_3(z_t) = \int_{t-t_a}^{t} e^{a(s-t)} \hat{z}^T(s) Q_1 \hat{z}(s) ds + \int_{t-t_a}^{t} e^{a(s-t)} \hat{z}^T(s) Q_2 \hat{z}(s) ds, \]
\[ V_4(z_t) = \tau_a \int_{t-t_a}^{t} e^{a(s-t)} \hat{z}^T(s) R_c \hat{z}(s) ds, \]
\[ V_5(z_t) = (\tau_y - \tau_x) \int_{t-t_a}^{t} e^{a(s-t)} \hat{z}^T(s) \hat{z}(s) ds, \]
\[ V_6(z_t) = \int_{t-t_a}^{t} \int_{t-t_a}^{t} e^{a(s-t)} \hat{z}^T(s) K_1 \hat{z}(s) ds Rd \theta d \eta, \]
\[ V_7(z_t) = \int_{t-t_a}^{t} \int_{t-t_a}^{t} \int_{t-t_a}^{t} e^{a(s-t)} \hat{z}^T(s) K_2 \hat{z}(s) ds Rd \theta d \eta, \]
\[ V_8(z_t) = \int_{t-t_a}^{t} \int_{t-t_a}^{t} \int_{t-t_a}^{t} e^{a(s-t)} \hat{z}^T(s) K_3 \hat{z}(s) ds Rd \theta d \eta, \]
\[ V_9(z_t) = \int_{t-t_a}^{t} \int_{t-t_a}^{t} \int_{t-t_a}^{t} e^{a(s-t)} \hat{z}^T(s) K_4 \hat{z}(s) ds d \theta d \eta, \]

Taking the time derivative of \( V(z_t) \) along the trajectory of system (1), we obtain
\[ \dot{V} + \alpha V - \frac{\alpha}{\omega^2} \omega^T(t) \omega(t) \leq 0 \]

Using the spectral properties of symmetric positive definite matrix \( P \), the following inequality holds
\[ \lambda_{\text{min}}(P) \|z(t)\|^2 \leq V(z_t) \]

This further implies that \( \|z(t)\| \leq r = \frac{1}{\sqrt{\lambda_{\text{min}}(P)}} \). This completes the proof.

Conclusions

In this paper, the problem of reachable set bounding for linear neutral systems with discrete delay...
and disturbances has been investigated. By using Lyapunov-Krasovskii functional theory, delay decomposition technique, reciprocally convex method and free-weighting matrix approach, new reachable set bounds are obtained. Triple integrations are introduced first time in Lyapunov functional to study reachable set bounding for linear neutral systems. Meanwhile, delay decomposition technique is employed, which leads to more tighter reachable set bounds. The results have illustrated advantages compared with the ones in[10-13].

References