Transport of Active Brownian Motors Driven by a Thermal Broadband Noise

Xiao-yang ZHU¹,², Min QIAN¹,² and Dan WU¹,²*

¹School of Optoelectronic Science and Engineering, Soochow University, Suzhou, 215006, China
²Wenzheng College of Soochow University, Suzhou 215104, China

*Corresponding author

Keywords: Thermal broadband noise, Ballistic diffusion, Active brownian motors.

Abstract. The transport properties of active Brownian particles driven by a thermal broadband noise are investigated. The effects of the noise correlation time and the bias on the diffusion and the mean velocity are discussed. It is found that the noise correlation time reduces the ballistic, while the bias enhances the ballistic. The mean velocity decreases as the correlation time increased while for large bias.

Introduction

The Brownian motors have been investigated extensively for many decades due to their possible applications in various fields, such as molecular motors [1-6], nanoscale friction [7,8], surface smoothening [9], and so on. Recently, the transport properties of active Brownian particles (ABP) are investigated [10,11]. For example, the ABP properties for phase and diffusion with different parameters. In these studies, the Langevin model play an important role [10-12] due to sampling degrees of freedom by the use of stochastic differential equations. Thus, it is convenient for numerical simulation and reduce the workload.

Anomalous diffusion is found in different systems, ranging from charge transport processes in amorphous materials to plasma and bio-systems [13-16]. The dynamical origin of the anomalous diffusion is due to the non-locality in time and thus the velocity of the particle has a memory effect, resulting in a non-Markovian Langevin equation [17-19]. There are many different theories reflecting a variety of underlying physical mechanism and one of the fundamental approaches is based on the generalized Langevin equation. It has been reported that the diffusion can be evaluated by the mean square displacement \( \langle \Delta x^2(t) \rangle \propto t^\alpha \), a subdiffusion for \( 0<\alpha<1 \), a normal diffusion for \( \alpha=1 \), a superdiffusion for \( \alpha>1 \), and a ballistic diffusion for \( \alpha=2 \) [20-22]. So far, most of the models deal with Brownian motion cases in a periodic potential, while the anomalous diffusion in active Brownian particles needs to be investigated.

In this paper, the transport properties of non-Markovian active Brownian particles are investigated numerically. In Sec.2, The active Brownian particles is modeled by a generalized Langevin equation with a thermal broadband noise. Actually, the broadband noise is determined by the difference between two Ornstein-Uhlenbeck noises with different time constants. In Sec.3, the effects of the bias \( F \) and the noise correlation time \( \tau_1 \) on the mean velocity \( \langle v \rangle \) are discussed in details. A discussion concludes the paper.

Dynamical Model

The generalized Langevin equation of the active Brownian particle can be written as follows

\[
\ddot{x}(t) + \gamma \int_0^t \eta(t-t') \dot{x}(t') \, dt' = -\frac{dU(v)}{dv} + \xi(t),
\]

where \( \gamma \) is the friction coefficient, \( \eta(t-t') \) is the friction memory kernel, \( \xi(t) \) is a thermal colored noise and its correlation function obeys the fluctuation dissipation theorem \( \langle \xi(t)\xi(t') \rangle = k_B T \eta(|t-t'|) \), where \( k_B \) is the Boltzmann constant, \( T \) is the absolute temperature of the environment. The biased velocity potential \( U(v) = v^4/4 - v^2/2 - Fv \) with a bias \( F \) is plotted in
the inset of Fig.1. It is shown that the potential is symmetric for \( F=0 \), and asymmetric for \( F=0.1 \). Since the influence of the bias \( F \) on the transport of active Brownian particles has already been reported in ref.[10], in the following discussion \( F \) is selected to be 0.1 for a non-zero velocity.

The function \( \eta(t-t') \) is a broadband colored noise with a memory kernel [21], which can be written as

\[
\eta(t-t') = \frac{\tau_1}{\tau_1-\tau_2} \left[ \frac{1}{\tau_1} \exp \left( -\frac{|t-t'|}{\tau_1} \right) - \frac{1}{\tau_2} \exp \left( -\frac{|t-t'|}{\tau_2} \right) \right],
\]

(2)

where \( \tau_1 \) and \( \tau_2 \) are two correlation times. Actually, such broadband noise can be realized from the difference between two Ornstein-Uhlenbeck noises, and the non-Markonian Langevin Eq.1 can be transformed into a set of the Markonian Langevin equations [21] which includes four variables as follows

\[
\dot{x} = v, \quad \dot{v} = -\gamma v - \frac{dU}{dv} + y_1(t) + y_2(t),
\]

\[
\dot{y}_1(t) = -\frac{1}{\tau_1} y_1(t) + \frac{A}{\tau_1} v(t) - \frac{1}{\tau_1} \xi(t),
\]

\[
\dot{y}_2(t) = -\frac{1}{\tau_2} y_2(t) + \frac{A}{\tau_2} v(t) + \frac{1}{\tau_2} \xi(t),
\]

(3)

with \( A = \tau_1^2 / (\tau_1^2 - \tau_2^2) \). \( \xi(t) \) is a white noise that satisfies \( \langle \xi(t) \rangle = 0 \), \( \langle \varepsilon(t)\varepsilon(t') \rangle = 2D\delta(t-t') \). \( D = \gamma k_B T(\tau_1 / (\tau_1 - \tau_2))^2 \) is the noise intensity. In ref.[22], the ballistic diffusion is observed when the noise is in the region of green noise for small \( \tau_2 \) and large \( \tau_1 \). In the following discussion, the noise is chosen in such a region to find whether the anomalous diffusion can also be found in the model of active Brownian particles.

**Drift and Diffusion**

![Figure 1](image)

Figure 1. (a) The first moment \( \langle x(t) \rangle \) of ABP as a function of time \( t \) for different values of bias \( F \) with the noise correlation time \( \tau_1 = 5 \). (b) The first moment \( \langle x(t) \rangle \) of ABP as a function of time \( t \) for different values of the noise correlation time \( \tau_1 \) with the bias \( F=0.1 \). Inset: The velocity potential \( U(v) \) of ABP for the symmetric case \( (F=0) \) and the asymmetric case \( (F=0.1) \). The other parameters are the friction coefficient \( \gamma = 1 \) and the driving strength \( D=0.5 \).

The first moment \( \langle x(t) \rangle \) of ABP as a function of time \( t \) is plotted in Fig.1 for different bias \( F \) and the noise correlation time \( \tau_1 \). It is found that the first moment \( \langle x(t) \rangle \) exhibits a linear function of time with different slopes when the bias \( F \) and the noise correlation time \( \tau_1 \) increases. From Fig.1(a), it is shown that the first moment \( \langle x(t) \rangle \) is suppressed as the bias \( F \) decreases. However, from Fig.1(b), it is shown that the first moment \( \langle x(t) \rangle \) actively and then repressively as the noise correlation time \( \tau_1 \) increases.

The mean square displacement of the particle can be defined as

\[
\langle \Delta x^2(t) \rangle = \langle x^2(t) \rangle - \langle x(t) \rangle^2,
\]

(4)
Normal diffusion is characterized by a linear time dependence of the mean square displacement, \( \langle \Delta x^2(t) \rangle \approx t \), while anomalous diffusion shows a different time dependence \( \langle \Delta x^2(t) \rangle \approx t^\alpha \), with \( \alpha>1 \) for superdiffusion, \( \alpha<1 \) for subdiffusion, and a ballistic diffusion for \( \alpha=2 \).

Based on the above the theory, the influence of mean square displacement \( \langle \Delta x^2(t) \rangle \) is calculated for different bias \( F \) and noise correlation time \( \tau_1 \). In Fig.2(a), the evolution of \( \langle \Delta x^2(t) \rangle \) vs \( t \) averaged over 5000 different trajectories is shown for different bias \( F \). It is found that there exists non-diffusion as \( F = 0 \), and then the diffusion is active when the bias \( F \) changes from 0.05 to 0.5. From Fig.2(b), it is found that \( \langle \Delta x^2(t) \rangle \) is proportional to \( t^2 \) in the long-time limit, which is the characteristic of the ballistic diffusion. The ballistic diffusion is suppressed as \( \tau_1 \) increases. Our results demonstrate the reported ballistic diffusion in different parameter domains. In the ref.[22], the ballistic diffusion is observed when the noise is in the region of green noise for small \( \tau_2 \) and large \( \tau_1 \).

The drift motion can be characterized by the mean velocity \( \langle v \rangle \). In this paper, we focus mainly our attention on effects induced by the bias \( F \) and the noise correlation time \( \tau_1 \). Fig.3(a) shows the mean velocity \( \langle v \rangle \) as a function of the driving strength \( D \) for different values of the bias \( F \). When the potential is symmetric (\( F = 0 \)), the mean velocity \( \langle v \rangle \) is suppressed (\( \langle v \rangle = 0 \)). When the asymmetry parameter (the bias \( F \)) is increased, the position of the peak moves to a small value of the driving strength \( D \) and its height increases (\( F = 0.05, 0.1, 0.2 \)). However, for a large value of the asymmetry (\( F = 0.5, 0.6 \)), the peak disappears and the mean velocity \( \langle v \rangle \) decreases...
monotonically with increasing the driving strength $D$. The mean velocity $\langle v \rangle$ is plotted as a function of the driving strength $D$ for different values of the noise correlation time $\tau_1$. It is shown that, for small $\tau_1 (\tau_1 < 30)$, the mean velocity $\langle v \rangle$ increases initially as the driving strength $D$ increases, attains maximum, and then decreases as $D$ increases further. Meanwhile, the peak move to left continually with increasing $\tau_1$. For large $\tau_1$ (cf. $\tau_1 = 30, \tau_1 = 40, \tau_1 = 50$), the mean velocity $\langle v \rangle$ decreases monotonically as the driving strength $D$ increases. The inset of Fig.3 shows the contour plot of $\langle v \rangle$ in the $\log_{10}(D) - \tau_1$ plane. It is found that the mean velocity $\langle v \rangle$ decreases monotonically as the driving strength $D$ increases when the noise correlation time $\tau_1 > 26$.

Figure 4. (a) The mean velocity $\langle v \rangle$ as a function of the noise correlation time $\tau_1$ for different values of the bias $F$. (b) The contour plot of the mean velocity $\langle v \rangle$ as a function of the noise correlation $\log_{10}(\tau_1)$ and the bias $F$. The black to white grayscale represents the lowest value to the highest value. The other parameters are the friction coefficient $\gamma = 1$, the driving strength $D = 0.5$.

In Fig.4(a), we depict the mean velocity $\langle v \rangle$ as a function of the noise correlation time $\tau_1$ for different values of the bias $F$. As the bias increased, the mean velocity shows a variety of features. In particular, for the bias $F = 0$, $\langle v \rangle = 0$. For very large values of $F$, the peak disappears. In order to further investigate the cooperative effects of the bias $F$ and the noise correlation time $\tau_1$ on the mean velocity $\langle v \rangle$, the contour plot of the mean velocity $\langle v \rangle$ vs the bias $F$ and the noise correlation time $\tau_1$ is plotted in Fig.4(b). It is found that $\langle v \rangle$ is very sensitive to the change of the noise correlation time $\tau_1$ in certain a specific range of $\tau_1$. And we could see clearly that the peak disappears when $F > 0.39$.

Figure 5. (a) The probability distribution $P_{st}(v)$ for different values of the bias $F$ with the noise correlation time $\tau_1 = 5$. (b) The probability distribution $P_{st}(v)$ for different values of the noise correlation time $\tau_1$ with the bias $F = 0.1$. The other parameters are the friction coefficient $\gamma = 1$, the driving strength $D = 0.5$. 

367
The probability distribution \( P_{st}(v) \) for different values of the bias \( F \) and the noise correlation time \( \tau_1 \) is shown in Fig.5. From Fig.5(a), it is found that with increasing of the bias \( F \), the left valley values \((v < 0)\) of the curves for \( P_{st}(v) \) versus \( \langle v \rangle \) descend (and move toward the right), while the right peak values \((v > 0)\) of the curves for \( P_{st}(v) \) versus \( \langle v \rangle \) ascend (and also move to the right). In Fig.5(b), the main feature of this distribution is the peak behavior of \( P_{st}(v) \) as \( \langle v \rangle \approx \pm 1 \) for the varied \( \tau_1 \). The reason is that for small \( \tau_1 \) \((\tau_1 = 0.1)\), the two peaks in \( P_{st}(v) \) are located at \( \langle v \rangle \approx -1 \) and \( \langle v \rangle \approx 1 \). When the noise correlation time \( \tau_1 \) is increased to \( \tau_1 = 1 \), both the left peak and the right peak decrease and move to the middle gradually. When the noise correlation time \( \tau_1 \) is increased further to \( \tau_1 = 50 \), the right peak still decreases and moves to left. While the left peak increases. No matter what values of \( \tau_1 \) are, the right peak is higher than the left one.

![Figure 6](image)

Figure 6. (a) The mean escape time \( T_{esc} \) as a function of the driving strength \( D \) for different bias \( F \) with the noise correlation time \( \tau_1 = 5 \). (b) The mean escape time \( T_{esc} \) as a function of the driving strength \( D \) for different noise correlation time \( \tau_1 \) with the bias \( F = 0.1 \). The other parameter is the friction coefficient \( \gamma = 1 \).

Fig.6(a) shows the mean escape time \( T_{esc} \) as a function of the driving strength \( D \) for different bias \( F \). It is found that the mean escape time \( T_{esc} \) is suppressed as the bias \( F \) decreases. The mean escape time \( T_{esc} \) as a function of the driving strength \( D \) for different values of the noise correlation time \( \tau_1 \) is plotted in Fig.6(b). It is found that there are no effects when the noise correlation time \( \tau_1 \) is varied.

Discussion

The transport properties of active Brownian particles driven by a thermal broadband noise are investigated. The ballistic diffusion is observed and the ballistic diffusion is suppressed as correlation time \( \tau_1 \) increases, which demonstrates the results shown in Ref.[21]. It should be pointed out that the friction is omnipresent in nature. Our results are obtained a reasonable and stable scope of system model. It can be found that the mean velocity \( \langle v \rangle \) presents a variety of characteristics as a function of the noise correlation time \( \tau_1 \) for increasing the bias \( F \). For \( F = 0 \), the mean velocity is \( \langle v \rangle = 0 \). When the asymmetry parameter (the bias \( F \)) is increased, the position of the peak moves to a small value of the noise correlation time \( \tau_1 \) and its height increases \((F = 0.05, 0.1, 0.2)\). However, for a large value of the asymmetry \((F > 0.39)\), the peak disappears and the mean velocity \( \langle v \rangle \) decreases monotonically with increasing the noise correlation time \( \tau_1 \). Our results may help in studies of friction, as well as in designing particle separation techniques.
Acknowledgement

The financial supports from the National Natural Science Foundation of China (Grant Nos.11005077, 11474213 and 11205111) are gratefully acknowledged.

References