Performance Investigation of Angle Differential-QAM Scheme in High Mobility Environments

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Abstract. Angle differential quadrature amplitude modulation (ADQAM) is a high spectral efficiency modulation scheme which can be non-coherently demodulated without any channel state information. In this paper, we investigated the performance of ADQAM in high mobility environments where the channel varied rapidly. A comparison has been made among ADQAM, QAM and differential amplitude and phase shift keying (DAPSK) scheme in terms of bit error ratio through numerical simulations. The results showed that ADQAM outperformed DAPSK and QAM significantly in the time rapidly varying channel at high signal noise ratio, while QAM with channel estimation only had 1dB advantage over ADQAM in the time invariant channel. ADQAM was more robust to Doppler shift than DAPSK and QAM. We suggested that ADQAM scheme was more suitable for communication systems in high mobility environments.

Introduction

Support high mobility is an important requirement for wireless communication systems. In high mobility scenarios, such as high speed railway, wireless channels vary rapidly due to the high speed movement between transmitter and receiver in complex terrains. The channel exhibits both time selective fading and frequency selective fading [1-3], i.e., doubly selective fading. The doubly selective fading bring serious interferences and severe performance degradation to communication systems [2-3]. Therefore, it is valuable to introduce a modulation scheme that make the communication system stable, reliable and efficient in the doubly selective channel.

Quadrature amplitude modulation (QAM) is an attractive modulation scheme for many wireless communication systems due to its high spectral efficiency and relatively greater distance between adjacent constellation points [4-7]. The QAM is very sensitive to carrier phase ambiguity. It requires coherent demodulation that channel state information (CSI) is employed to correct the distortion of the phase and amplitude at the receiver. However, in high-speed mobile environments, it is hard to get the accurate knowledge of CSI by the conventional channel tracking and estimating methods [2-3]. The imperfect CSI have a significant impact on the performance of QAM and leads to an irreducible bit error ratio (BER) floor [6-7]. Therefore, the scheme which is free of CSI , such as differential scheme, is desired for high mobility communications because it can be demodulated without CSI and has a simply receiver structure.

The differential modulation scheme means that the transmitted data bits are contained in the difference between two consecutive modulated symbols. Differential amplitude and phase-shift keying (DAPSK) is a combinational scheme of differential amplitude shift keying (DASK) and differential phase shift keying (DPSK) [8-12]. In the DAPSK scheme, the information data to be transmitted is directly mapped to the amplitude ratio and the phase difference of the two consecutive symbols, and a non-coherent demodulation can be adopted to recover the data without any kind of channel equalization and estimation at the receiver side. It is verified that DAPSK has a spectral efficacy as high as QAM and has a better BER performance than DPSK. Compare with QAM, the DAPSK can offer a great advantage in performance over the time varying channel at high signal noise ratio. However, DAPSK is inferior to QAM in performance at least 3dB under the additive white Gaussian noise (AWGN) channel and the time invariant channel [10-12]. Further, an angle differential quadrature amplitude modulation (ADQAM) scheme has been proposed in [13]
to solve phase ambiguity for continuous transmission systems with square QAM constellation. In the ADQAM scheme, the information data are modulated on two or more differential angles, and can be demodulated efficiently without CSI at receiver. It has been found that the ADQAM incurred a little performance degradation in the AWGN channel compared with QAM scheme [13-14]. When the BER is $10^{-4}$, the uncoded 64ADQAM loses about 1 dB of performance compared with 64QAM, the encoded 64ADQAM only loses about 0.5dB. However, little information is available on the performance of ADQAM in time varying fading channels, and few investigations have been made on the performance comparing between DAPSK scheme and ADQAM scheme.

In this paper, we aimed to investigate the performance of ADQAM scheme in high mobility environments. The performance comparison has been made among QAM scheme, DAPSK scheme and ADQAM scheme in the AWGN channel, the time invariant channel and the time varying channel respectively. Moreover, it has also been discussed about the robustness to the variation of the channel for these schemes in the time varying channel.

**ADQAM Scheme**

ADQAM is a non-coherent modulation method with high spectral efficiencies, which can solve the problem of phase ambiguity in the transmission process and reduce the impairments from time-varying channels. Figure 1 shows a constellation diagram for 16ADQAM.

**Figure 1. The constellation diagram for 16ADQAM.**

It can be seen that there are 16 constellation points on the diagram, similarly to 16QAM, the information data can be transmitted as one of 16 symbols, each representing 4 bits of information data. Then a 16ADQAM symbol is given as:

$$S_i = c_i + d_i$$

Where $c_i$ is the center of the quadrant in which the symbol is located, $d_i$ is the rotation vector. Therefore, the 16ADQAM encoding rules can be represented by two recursive update formulas:

$$c_i = c_{i-1}e^{j\alpha_i}$$

$$d_i = d_{i-1}e^{j\beta_i}$$

Where $\alpha_i, \beta_i$ are two differential angles, each can represent 2 bits of data. The mapping relationship between data and differential angles $\alpha_i, \beta_i$ is shown in Table 1:
Given the initial symbol $S_0 = K_1 e^{j\pi/4} + K_2 e^{j\pi/4}$, we can get $c_0 = K_1 e^{j\pi/4}$ and $d_0 = K_2 e^{j\pi/4}$, where $K_1 = 2\sqrt{2}$ denotes the distance between the origin to the center of the quadrant, $K_2 = \sqrt{2}$ denotes the distance between the center of the quadrant to the constellation point, hence $S_0 = 3 + 3j$. The 16ADQAM symbols can be obtained by using the differential encoding scheme with two differential angles, and all the symbols can be expressed as:

$$S_i = a_i + j b_i \quad \text{where} \quad a, b \in \{\pm 1, \pm 3\}$$

(4)

The average energy of the 16ADQAM symbol can be written as:

$$E_{S, av} = \frac{2(M-1)}{3} = 10.$$  

(5)

Where $M$ is the modulation order. If the 16ADQAM symbols are modulated on a carrier and pass through a time varying multipath fading channel, the channel impulse response is given as:

$$h(\tau, t) = \sum \rho_i \mathcal{L}_{i=0} c_i(t) \delta(\tau - \tau_i)$$

(6)

Where $\rho_i$, $\tau_i$ and $c_i$ respectively represent the loss, delay and time-varying attenuation of the $i$-th path, then at the receiver, the received signal can be expressed as:

$$y(t) = \sum \rho_i c_i(t) S_i e^{jw_c(t-\tau_i)}$$

(7)

Where $w_c$ is the carrier frequency. The received signal is demodulated by using a frequency mixer and a low-pass filter, then it yields the following signal:

$$y(t) = \alpha S_i e^{j(w t + \theta)} + n(t).$$

(8)

Where $\alpha$ is the channel attenuation coefficient, related to $\rho_i$ and $c_i$, $w$ and $\theta$ are the carrier frequency offset and phase offset, $w$ is related to Doppler frequency offset, $\theta$ is related to $\tau_i$, and $n(t)$ is additive Gaussian white noise. For the ADQAM system, there are two assumptions on the receiver side: (1) the received ADQAM symbol is correctly positioned and aligned with the I/Q axis via a non-data assisted recursive Costas loop. (2) the received symbol can be adjusted to the correct voltage level by automatic gain control, which converges to the absolute value of a complementary gain channel attenuation. If $y(t)$ is sampled with baseband rate, we can rewrite the receiver signal in discrete form as:

$$y_i = S_i e^{j\phi} + n_i.$$  

(9)

Where $\phi$ is the unknown phase ambiguity. Substituting the equations (2) and (3) into equation (8), the receiving signal can be expressed as:

$$y_i = C_i + D_i + n_i.$$  

(10)

Where $n_i$ is white noise, $C_i$, $D_i$ are the quadrant center and rotation vector after the time varying channel and phase inverse rotation. The quadrant decision can estimate $C_k$, $D_k$ by equations (11) and (12).
\[
\hat{C}_i = K_1 \left[ \text{sgn}(\text{real}(y_i)) + j \text{sgn}(\text{imag}(y_i)) \right] / \sqrt{2}.
\]

\[
\hat{D}_i = K_2 \left[ \text{sgn}(\text{real}(y_i - \hat{C}_i)) + j \text{sgn}(\text{imag}(y_i - \hat{C}_i)) \right] / \sqrt{2}.
\]

Where \( \text{sgn}(\bullet) \) is the symbol function, \( \text{real}(\bullet) \) and \( \text{imag}(\bullet) \) are the real part and the imaginary part respectively. Multiply the conjugate complex numbers of \( \hat{C}_i \) and \( \hat{C}_{i-1} \), and the result is denoted as \( \lambda_i \). Multiply the conjugate complex number of \( \hat{D}_i \) and \( \hat{D}_{i-1} \), and the result is denoted as \( \varphi_i \). By analyzing the values of \( \lambda_i \) and \( \varphi_i \), \( \alpha_i, \beta_i \) of the transmitted data can be obtained. The specific decision rules are as follows:

\[
\begin{align*}
\text{if } \lambda_i = \begin{cases} 
K_1^2 & \alpha_i = 0 \\
jK_1^2 & \alpha_i = \pi / 2 \\
-K_1^2 & \alpha_i = \pi \\
-jK_1^2 & \alpha_i = 3\pi / 2
\end{cases}
\end{align*}
\]

(13)

\[
\begin{align*}
\text{if } \varphi_i = \begin{cases} 
K_2^2 & \beta_i = 0 \\
jK_2^2 & \beta_i = \pi / 2 \\
-K_2^2 & \beta_i = \pi \\
-jK_2^2 & \beta_i = 3\pi / 2
\end{cases}
\end{align*}
\]

(14)

Thus, the transmitted data can be demodulated after the received signal passes the above rule decision. Figure 2 briefly shows the above two judgment mathematical models.

![Figure 2. Signal detection model.](image)

It is known from the system block diagram that the transmitted data can be demodulated by quadrant decision and delay multiplication. The phase detection rule is the inverse mapping of Table 1, and the transmitted hexadecimal symbol \( \hat{s}_k \) and its corresponding hexadecimal data can be obtained by \( \alpha_k, \beta_k \).
Performance Analysis of ADQAM

First we define the minimum distance of the 16ADQAM constellation point as $d_{\text{min}}$. It is worth noting that at high signal noise ratio, the constellation point quadrant decision error is most likely to be decided to the nearest constellation point, so we need to consider the four constellation points of the 16ADQAM in the same quadrant. Let defining $\alpha = 1$ in the AWGN channel and conform to the $N(0, N_0/2)$ distribution. Then the probability of $C_i$ quadrant decision error can be expressed as:

$$P_e(C_i) = P [C_i \neq \hat{C}_i] \approx \frac{1}{4} * N_p * p = p.$$ (15)

Here we assume that the four constellation points are equally possible in the same quadrant, then:

$$p = \frac{1}{2} \text{erfc} \left( \frac{d_{\text{min}}}{\sqrt{2N_0}} \right)$$ (16)

Where $p$ is the tail probability of the noise distribution in the range $(d_{\text{min}})$. This is because the first phase rotation $\alpha_i$ is determined by the two quadrant centers adjacent to each other, and the approximate error probability of $\alpha_i$ can be expressed as:

$$P_e(\alpha_i) \approx 2P_e(C_i) \approx 2p.$$ (17)

Next we talk about the error probability of the second stage $D_i$, which we can get from the total probability theorem:

$$P_e(D_i) = P [D_i \neq \hat{D}_i] = P_e(D_i | \hat{C}_i) P(\hat{C}_i) + P_e(D_i | C_i) P(C_i) \approx 2p(1-p) + P_e(D_i | \hat{C}_i)p = 2p(1-p) + P_{ed}p.$$ (18)

Where $P_{ed}$ represents the error probability, which produced by the error propagation effect in the first phase, and:

$$P_{ed} = P_e(D_i | \hat{C}_i).$$ (19)

When $p << 1$ and $P_{ed} \approx 1$, the above equation can be simplified as:

$$P_e(D_i) \approx 2p + p = \frac{3}{2} \text{erfc} \left( \frac{d_{\text{min}}}{\sqrt{2N_0}} \right)$$ (20)

So we can get the error probability of $\beta_i$ as:

$$P_e(\beta_i) \approx 2P_e(D_i) \approx 6p.$$ (21)

Finally, the average bit error rate of 16ADQAM can be expressed as:

$$P_{e,16ADQAM} = \frac{1}{\log_216} \left[ 1 - (1 - P_e(\alpha_i))(1 - P_e(\beta_i)) \right] = \frac{1}{4} \left[ P_e(\alpha_i) + P_e(\beta_i) - P_e(\alpha_i)P_e(\beta_i) \right] = 2p - 3p^2.$$ (22)

In the M-ary QAM constellation diagram, the average bit energy $E_b$ is related to the minimum distance $d_{\text{min}}$, which can be expressed as:
\[ E_b = \frac{E_{\text{av}}}{\log_2 M} = \frac{2(M-1)}{3 \log_2 M} d_{\text{min}}^2. \]  

(23)

Therefore, when \( M = 16 \), we can get \( E_b = 2.5 d_{\text{min}}^2 \). The bit error rate of 16ADQAM can be approximated as:

\[ P_{e, \text{16ADQAM}} \approx 2p = \text{erfc} \left( \frac{E_b}{10N_0} \right) \]  

(24)

On the other hand, a coherent 16QAM system has an approximate bit error rate, expressed as:

\[ P_{e, \text{16QAM}} \approx \frac{3}{8} \text{erfc} \left( \frac{E_b}{10N_0} \right) \]  

(25)

Then, the ratio of 16ADQAM to 16QAM error rate is:

\[ \frac{P_{e, \text{16ADQAM}}}{P_{e, \text{16QAM}}} = \frac{8}{3} \approx 2.667. \]  

(26)

Next, we consider the bit error rate of 16ADQAM in the time invariant channel. In this condition, the attenuation coefficient \( \alpha \) is a Gaussian variable with zero mean. Then the instantaneous signal noise ratio \( \gamma = \alpha^2 E_b / N_0 \) obeys the following exponential probability density function:

\[ f(\gamma) = \frac{1}{\sqrt{\gamma}} e^{-\gamma / \gamma}, \quad \gamma \geq 0. \]  

(27)

Where \( \gamma = E[\alpha^2] E_b / N_0 \) represents the average signal noise ratio per bit. Therefore, the instantaneous bit error rate can be written as:

\[ P_{e, \text{16ADQAM}}(\gamma) = 2 \left( \frac{1}{2} \text{erfc} \left( \frac{\gamma}{10} \right) \right) + 3 \left( \frac{1}{2} \text{erfc} \left( \frac{\gamma}{10} \right) \right)^2. \]  

(28)

Then, in the time invariant channel, the average bit error rate of 16ADQAM can be expressed as:

\[ P_{f, \text{16ADQAM}} = \int_0^\infty P_{e, \text{16ADQAM}}(\gamma) f(\gamma) d\gamma 
\]

\[ = 2 \left( 1 - \frac{\gamma}{10} \right) - 3 \left( 1 - \frac{\gamma}{10} \right) \tan^{-1} \left( \frac{\gamma}{10} \right). \]  

(29)

Since the time varying channel is more complicated, it is difficult to give an exact error performance for 16ADQAM in the time varying channel. It is not explained here. For the higher-order even-bit ADQAM and the bit error performance analysis, you can refer to the literature [13] and [14], this paper gives the results of the even-bit high-order ADQAM’s error performance in AWGN channel. The results are as follows:

\[ P_{e, \text{MADQAM}} = \frac{1}{\log_2 M} \left[ 1 - \prod_{n=1}^{K/2} \left( 1 - P_c(\theta_{n,i}) \right) \right] \]

\[ \approx \frac{1}{\log_2 M} \left[ 1 - \left( 1 - 2p \right)^{K-1} \left( 1 - 2p \left( \frac{K}{2} + 1 \right) \right) \right] \]

\[ \approx 2p. \]  

(30)
Where $K = \log_2 M$, $K/2$ is the number of differential angles, $\theta_{n,i}$ is the n-th differential angle, and $p$ is the error complement function, which can be expressed as:

$$
p = \frac{1}{2} \text{erfc} \left( \sqrt{\frac{K}{E_s \cdot 4N_0}} \right).
$$

(31)

Simulation Results and Analysis

In this section, we investigate the performance of ADQAM scheme through simulating. Simulation parameters are set as follow: the carrier frequency is 2.4 GHz, the channel is a six-path COST207 typical suburban channel model, the delay of each path is $\{0,0.1,0.2,0.3,0.4,0.5\}$μS, the power of each path is $\{0,-4,-8,-12,-16,-20\}$dB, and the transmit filter and the receive filter both adopt the square root raised cosine filter with the roll-off coefficient $\alpha = 1$. In the simulation, QAM scheme is equipped with a LS channel estimation and equalization, ADQAM and DAPSK scheme do not use any CSI.

![Performance curves of different modulation modes in the AWGN channel.](image)

Figure 3. Performance curves of different modulation modes in the AWGN channel.

As can be seen from the figure, QAM performance is the best, followed by ADQAM, and DAPSK performance is the worst. When the BER is $10^{-4}$, ADQAM is better than DAPSK about 3dB, and the advantage is more obvious with the increase of signal noise ratio; QAM is better than ADQAM about 0.5dB; QAM is better than DAPSK about 4dB. It can be seen that ADQAM loses less than DAPSK in the transmission process, and ADQAM modulation has a higher theoretical upper limit than DAPSK modulation. As the signal noise ratio increases, the advantage becomes more obvious.
Figure 4. Performance curves of different modulation modes in the time invariant channel.

Figure 4 shows the performance curves of the QAM, DAPSK, and ADQAM in the time invariant channel. As can be seen from the figure, QAM has the same downward trend as ADQAM, in which QAM performance is the best, ADQAM is second, and DAPSK performance is the worst. Compared to QAM, ADQAM has a system performance loss less than 1 dB. When the BER is $10^{-3}$, the performance of ADQAM is better than DAPSK about 3dB. ADQAM performance is better than DAPSK as the signal noise ratio increases. This is because ADQAM has a higher performance limit than DAPSK, and ADQAM performance will be better than DAPSK in the case of ensuring continuous channel changes.

Figure 5. Performance curves of different modulation modes in the time varying channel.

Figure 5 shows the performance curves of QAM, DAPSK and ADQAM in the time varying channel. Where the moving speed is $V = 300km/h$, as can be seen from the figure, ADQAM has the best performance, followed by DAPSK. QAM performs slightly better than ADQAM at low SNR, but there is a floor effect at around 25dB. This is because in the time varying channel, the channel information changes with time, and the LS channel estimation cannot give the accurate channel state information, which leads to the QAM decision error, and leads to feedback the wrong phase information, affecting the performance of QAM. In the case of inaccurate CSI, ADQAM and DAPSK can also guarantee system performance without floor effect, because ADQAM and DAPSK are non-coherent demodulation and do not require CSI. When the BER is $10^{-3}$, ADQAM improves performance about 2dB compared with DAPSK, and the advantage becomes more obvious as the signal noise ratio increases.
Figure 6 is a plot of bit error rate versus speed for QAM, DAPSK, and ADQAM in the time varying channel. Where, the SNR is 40dB. It can be seen from the figure that when V=0, it is equivalent to the time invariant channel. QAM performance is the best, ADQAM is second, DAPSK is the worst. When V increases, QAM performance is worse, ADQAM and DAPSK performance is better, among which ADQAM performance is the best, because QAM is coherent demodulation and requires accurate CSI, but in high mobility environments, it is difficult to obtain accurate CSI, while ADQAM and DAPSK can resistance to phase ambiguity and have a certain ability to track the channel. However, when V>500Km/h, the performance of ADQAM and DAPSK starts to deteriorate. This is because when the speed is faster, the channel is varying faster in time, and the effect of time selective characteristics is more obvious. It causes the ADQAM and DAPSK constellation points to be shifted to other quadrants, resulting in a decision error, which cannot continue to accurately track the time varying channel and cause error transmission. The ADQAM technology is more suitable for high mobility environments, but the speed cannot be increased all the time.

Conclusions

In the mobility environments, the channel exhibits doubly selective fading, and the time rapidly varying channel influence is difficult to eliminate. In order to study which modulation scheme performs best in different channels, we investigate the performance of ADQAM scheme. The performance comparison has been made among QAM scheme, DAPSK scheme and ADQAM scheme in the AWGN channel, the time invariant channel and the time varying channel respectively. Moreover, it has also been discussed about the robustness to the variation of the channel for these schemes in the time varying channel. It is concluded that QAM performance is better than DAPSK and ADQAM in the AWGN channel and the time invariant channel. In the time varying channel, the performance of ADQAM is better than QAM and DAPSK, especially at high SNR, ADQAM performance is superior, indicating that ADQAM is more suitable for high mobility environments.

In summary, the ADQAM modulation scheme can solve phase ambiguity for continuous transmission systems with square QAM constellation and can offer a great advantage in performance over the time varying channel at high signal noise ratio. We suggested that ADQAM scheme was more suitable for communication systems in high mobility environments.

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