A Model and Algorithm for Passenger-oriented Train Timetabling on Urban Rail Transit Network

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Abstract. This paper focuses on optimizing passenger train timetable on an urban transit network with given predetermined long and short routing operation mode. A double objective integer programming model is established in order to reduce the difficulty of solving train timetabling problem of large transit network. A Lagrangian relaxation algorithm was designed to decrease the complexity of solving the problem by transforming the complicated coupling problem into two independent sub-problems of trains and agents respectively. In response to two sub-problems of Lagrangian relaxation, a time-space network was constructed to solve the shortest path problem of passengers and trains by dynamic programming. Finally, the model and algorithm are verified and analyzed by a simple example.

Introduction

As a key component of public transit operations and management, the passenger train timetabling problem aims to schedule the movements of trains in space and time dimensions and deliver passengers from their origin stations towards their destination stations in a more efficient mode. A lot of researches have been done on the compilation and optimization of urban rail transit diagram under the influence of passenger demand. Niu and Zhou [1] constructs a binary integer programming model considering passenger's time varying demand in order to solve the optimization problem of train operation under single line congestion. Sun and Jun [2] formulate three optimization models to design demand-sensitive timetables by demonstrating train operation using equivalent time. Zhu and Mao [3] proposed a bi-level model to solve the timetable design problem for an urban rail line and designed a two-stage genetic algorithm. In general, existing studies mainly focus on train timetable on a single line or single routing, and a limited number of studies have been devoted to the train timetabling for the urban transit network considering both dynamic passenger demand and routing mode.

In this paper, we first focus on passenger train timetabling for the large-scale urban rail network. In the process of model conformation, the passenger flow OD and passenger departure time and intermediate reentry are fully considered. Then a double objective integer programming model is constructed and the Lagrange relaxation algorithm is designed by decomposing the original problem duality into two sub problems about the shortest path of the passenger and train in order to reduce the computational complexity. A feasible solution can be obtained by constantly adjusting the Lagrange multiplier until the passenger path and the train path match each other. Finally, the model and algorithm are verified by a simple case.

Train Timetabling Model Based on Space-time Network

The Urban rail operation involves two decision-makers, which are obviously different and interdependent, that is, the supply side and the demand side. The operation department aims to
formulate the train timetable according to the passenger demand to minimize the total cost of the network. While the passengers adjust the departure time according to the train timetable to reduce their waiting time. While the travel process of the passenger and the train not only have spatial attributes, but also have time attributes. In order to more clearly describe the travel of passenger and the operation of the train, this paper makes a time discretion for the passenger's starting point and the nodes of each station to construct the space-time network of passenger. The space-time network is defined on the basis of the physical network which is described as $G' = (V', E)$ and $V'$ is node set, $(i, t), (j, s) \in V'$ wherein the node set includes the node of the station layer as shown in Figure 1 and the virtual node. In the process of dealing with the nodes of the station, in order to describe the turn-back operation and the uplink and downlink of the train, each station is divided into four nodes in this paper, which represent Up-In, Up-Out, Down-In, Down-Out respectively. For example, a simple transit line is as Figure 2 shown, there are three stations and the second station is a turn-back station, which are abstracted to obtain the network of the train as shown in Figure 3. The connections of two nodes form an arc, such that the arc $(i, j, t, s) \in E$ wherein the arc set includes virtual arcs, running arcs, stopping arcs, transfer arcs, waiting arcs and turn-back arcs.

Therefore, this paper proposes a double objective model considering the minimum cost of both sides of supply and demand. The objection is to minimize the total cost of the trains and passengers, which is as follows:

$$\min z = \alpha \sum_{(jts) \in E} \sum_{a} x_{jts}^a \times c_{jts} + \beta \sum_{(jts) \in E} \sum_{k} y_{jts}^k \times \omega_{jts}$$  \hspace{1cm} (1)
Subject to:
Flow balance constraint of passengers
\[
\sum_{i \in A} x_{ijts}^a - \sum_{i \in A} x_{ijts}^d = \begin{cases} 
-1, & j = o(a), s = DT^a, \forall a \\
1, & j = d(a), s = T, \forall a \\
0, & \text{otherwise} 
\end{cases} 
\]
(2)

Arc capacity constraint
\[
\sum_{a} x_{ijts}^a \leq \sum_{k} y_{ijts}^k \times \text{Cap}_{ijts}, \forall (i, j, t, s) \in E
\]
(3)

Flow balance constraint of trains
\[
\sum_{j \in A} y_{ijts}^k - \sum_{j \in A} y_{ijts}^d = \begin{cases} 
1, & \forall i \text{ belongs to depot}, t = T_t^k, \forall k \\
-1, & \forall i \text{ belongs to depot}, t = T_t^k, \forall k \\
0, & \text{otherwise} 
\end{cases}
\]
(4)

Traffic resource constraint
\[
\sum_{(i j t s) \in c_{ijt s}} \sum_{k} y_{ijts}^k \leq L \text{cap}_{ijts}, \forall (i' j' t' s') \in E
\]
(5)

Passenger maximum travel time constraint:
\[
\sum_{(i j t s) \in c_{ijt s}} x_{ijts}^a \times c_{ijts} \leq T_{\text{max}(a)}, \forall a \in A
\]
(6)

Binary variables
\[
x_{ijts}^a = \begin{cases} 
1, & \text{when the passenger } a \text{ chooses the arc} \\
0, & \text{otherwise} 
\end{cases}
\]
(7)

\[
y_{ijts}^k = \begin{cases} 
1, & \text{when the train } k \text{ chooses the arc} \\
0, & \text{otherwise} 
\end{cases}
\]
(8)

Where: $x_{ijts}^a$, $y_{ijts}^k$ are both decision variables, identifying if passenger $a$ choose the service of the arc $(i, j, t, s)$ and if train $k$ runs on the arc $(i, j, t, s)$ respectively, which are also zero-one variables. Among of them, $i, j$ represent station index, $t, s$ represent time index, $E$ represents the set of arcs of space-time network, $c_{ijts}$ represents the general cost that passenger chooses arc $(i, j, t, s)$, only considering travel time here; $\omega_{ijts}$ represents the general cost that train chooses arc $(i, j, t, s)$, only considering travel time here, $\alpha, \beta$ represent the weight of passengers and trains in the objection, which depends on the priority of train planning, $o(a)$ represents the origin station of passenger $a$, $d(a)$ represents the destination of passenger $a$, $DT^a$ represents the start time and $T$ is the end time. $\text{Cap}_{ijts}$ represents the maximum passenger number of the arc $(i, j, t, s)$, that is arc capacity; when the arc satisfies the condition $(i, j, t, s) \in E_r$, $\text{Cap}_{ijts}$ is train carrying capacity while when the arc satisfies the condition $(i, j, t, s) \in E_d$, $\text{Cap}_{ijts}$ is station carrying capacity. $T_t^k$ represents train $k$ departure time from depot, $T_t^h$ represents train $k$ return time to the depot. $C_{ijt's'}$ is the set of other arcs that conflict with this arc $(i, j, t, s)$. $A$ represents the set of all passengers, $T_{\text{max}(a)}$ represents maximum endurance travel time of passenger. The decision variable is a 0-1 variable which represents if passenger choose the service of arc $(i, j, t, s)$. 

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Lagrange Relaxation Algorithm

The train timetable problem has been proved to be a NP-hard problem by a lot of literature, the lower bounds of which can be solved by the Lagrange relaxation algorithm. Therefore, this paper chooses this algorithm to relax couple constraint (2) and capacity constraint(4) to original objection to by two Lagrange multiplier $\mu_{ijts}$ and $\gamma_{ijts}$.

$$Z = \min \alpha \sum_{a} \sum_{(jts) \in E} x_{ijts}^a \times c_{ijts} + \beta \sum_{(jts) \in E} \sum_{k} \omega_{ijts} \times y_{ijts}^k + \sum_{(jts) \in E} [\mu_{ijts} \times (\sum_{a} x_{ijts}^a - y_{ijts}^k \times Cap_{ijts})]$$ (9)

After relaxing the couple constraint, we can decompose original problem to two independent sub problem about train and passenger as follows:

$$Z_1 = \sum_{a} \sum_{(jts) \in E} x_{ijts}^a \times (\alpha \times c_{ijts} + \mu_{ijts})$$ (10)

$$Z_2 = \min \sum_{k} \sum_{(jts) \in E} (y_{ijts}^k \times (\beta \omega_{ijts} - \mu_{ijts} \times Cap_{ijts} + \gamma_{ijts}) - \gamma_{ijts})$$ (11)

The sub problem of train or passenger can both be solved by shortest path algorithm in space-time network, then constantly update Lagrange multiplier to get feasible solution. The update method is as follows:

$$\gamma_{ijts}^{n+1} = \max \{0, \gamma_{ijts}^{n} + \alpha^{n} \times \sum_{k} y_{ijts}^k \times Lcap_{ijts}\}, \forall (i, j, t, s) \in E$$ (12)

$$\mu_{ijts}^{n+1} = \max \left\{0, \mu_{ijts}^{n} + \alpha^{n} \times \sum_{a} x_{ijts}^a - \sum_{k} y_{ijts}^k \times Cap_{ijts} \right\}, \forall (i, j, t, s) \in E$$ (13)

where: n represents the number of iterations, $\alpha$ is the step value, and $\alpha^{n} = \frac{1}{n+1}$

<table>
<thead>
<tr>
<th>Station</th>
<th>Station 1</th>
<th>Station 2</th>
<th>Station 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dwell time</td>
<td>0.5min</td>
<td>0.5min</td>
<td>0.5min</td>
</tr>
<tr>
<td>Running time</td>
<td>(1-2) 3min</td>
<td>(2-3) 2min</td>
<td></td>
</tr>
</tbody>
</table>
Example

The model and algorithm in this paper are verified by a simple transit network in Figure 2 with three stations. The time horizon is one hour, from 8:00 am to 9:00 am and the time interval is 5 seconds. The interval of return is 3 min and the minimum departure interval is 2 min. The train carrying capacity is 10 and station carrying capacity is 20. The weight coefficient \( \alpha \) is 1/6 and \( \beta \) is 5/6. The running time and dwell time are as shown in Table 1. The passenger OD and departure time are as shown in Table 2. All Lagrangian multipliers are initially assumed to be 1 and passenger maximum endurance travel time is two times of their free traveling time.

Table 2. Passengers OD.

<table>
<thead>
<tr>
<th>number</th>
<th>Departure time</th>
<th>O</th>
<th>D</th>
<th>number</th>
<th>Departure time</th>
<th>O</th>
<th>D</th>
<th>number</th>
<th>Departure time</th>
<th>O</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>8:01</td>
<td>3</td>
<td>2</td>
<td>2</td>
<td>8:13</td>
<td>2</td>
<td>1</td>
<td>9</td>
<td>8:29</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>6</td>
<td>8:02</td>
<td>3</td>
<td>1</td>
<td>10</td>
<td>8:14</td>
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<td>3</td>
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<td>3</td>
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<td>2</td>
<td>8</td>
<td>8:30</td>
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<td>9</td>
<td>8:51</td>
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<td>3</td>
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</table>

According to the above data, the input of the model can be obtained. Based on the procedure of Lagrangian relaxation proposed in Figure 4, we can obtain the optimal target value is 302.2 minutes,
and the lower bound is 301.8 minutes. Figure 5 demonstrates a comparison between the lower bound and optimal value with a final solution gap of 1.3%.

Conclusions
This paper constructed a passenger-oriented timetabling model, and designed a Lagrange relaxation algorithm to decompose original problem to two sub problem in order to reduce the complexity of solving. Finally, through a simple transit network, we can obtain the lower bound of Lagrange relaxation problem. The difference between lower bound and optimal value is only 1.3% as shown in Figure.5, which can verify the correctness of the model and algorithm.

References