A CRC-aided Perturbed Decoding of Polar Codes

Nana Kobina Gerrar, Shengmei Zhao and Lingjun Kong

ABSTRACT

We propose a CRC-aided perturbed decoding method for polar codes. Our technique is an iterative process that makes available multiple codeword candidates from a received signal. A CRC decoder is used to assess each decoded sequence. When a decoded sequence fails the CRC a random perturbation noise is added to the received signal, and then the perturbed signal is decoded again by a conventional decoder. This process continues until a valid codeword is found or the threshold for maximum iterations is attained. The simulation results indicate that the proposed scheme has a 0.4dB performance gain over existing schemes at the expense of a minimal increase in complexity. The proposed algorithm has the added advantage of being applicable to most concatenating decoding techniques.

INTRODUCTION

For any given binary-input discrete memoryless channel (B-DMC), polar codes [1-3], which was proposed by Arikan, has been proven to achieve the symmetric capacity with low encoding and decoding complexity. This family of codes applies the phenomenon of channel polarization to create two sets of unique channels. One set of channels is noiseless while the other set offers pure noise. Recently, polar codes have been selected for the next generation of wireless communication standards (5G) [4].

Despite its obvious advantages, polar codes have attracted intense research to improve its performance, and also to reduce its operational latency and complexity. This is because polar codes exhibit poor performance at short-to-medium code lengths. A number of decoding algorithms have therefore been developed. For example, the authors in [5] propose a belief propagation (BP) decoder to improve the finite-length performance of the conventional successive cancellation (SC) decoder and also propose a technique to increase the minimum distance of polar codes. Also in [6] and [7] techniques are suggested which improve the performance

1Nana Kobina Gerrar, College of Overseas Education, Nanjing University of Posts and Telecommunications, Nanjing, China
Shengmei Zhao, Institute of Signal Processing and Transmission, Nanjing University of Posts and Telecommunications, Nanjing, China
Lingjun Kong, Institute of Signal Processing and Transmission, Nanjing University of Posts and Telecommunications, Nanjing, China
of polar codes albeit with an increase in the decoding complexity. In [6] a successive cancellation list (SCL) decoder is presented. An SCL decoder makes available $L$ decoding paths at each decoding stage, settling on the most likely path at the end of the decoding process. SCL produces improved performance compared to SC and also shows comparable performance to the maximum likelihood (ML) decoder even for moderate $L$ values. Successive cancellation stack (SCS) decoding of polar codes is discussed in [7]. Here, a number of candidate partial paths are created in an ordered stack. The objective is to locate the global optimal estimation by locating the best path in the stack. SCS decoding also shows improved performance and is comparable to ML decoding. A concatenation of polar codes with a CRC was presented to improve the performance of polar codes, where CRC was used to check the most likely path in the list [8]. A simplified successive cancellation (SSC) decoder was developed where the focus was to reduce the complexity of the conventional SC decoder [9].

The design of a practical decoder for polar codes that provides a satisfactory balance between decoding complexity and performance is still an open problem. Most existing decoding techniques only makes available one output from the decoder. However, there is a possibility to make available multiple codeword candidates from a received signal by utilizing the principles of perturbation theory. For instance, adding secondary independent noise for the decoding of block codes can be found in [10]. Also in [11], a decoding system is presented where an initial estimate of the transmitted sequence is perturbed by a successive detection method leading to a list of candidate symbol vectors. The most likely candidate is accepted as the transmitted code sequence.

To improve the finite-length performance and reduce the algorithmic complexity of the polar code decoding algorithm, a CRC-aided perturbed decoding algorithm for polar codes is proposed in this paper. SSC is used as the conventional decoder because of its reduced complexity compared to the original SC decoder. A CRC decoder is used to assess each decoded sequence. When a decoded sequence fails the CRC, a random perturbation noise is added to the received signal, and then the perturbed signal is decoded again by the conventional decoder. The proposed algorithm exhibits improved performance over the SSC decoding technique as the numerical simulation results show.

**SYSTEM MODEL**

In this section an overview of polar codes is presented and the proposed perturbed decoding algorithm is discussed.
Polar Codes

Polar codes are able to achieve the symmetric capacity $I(W)$ of any B-DMC by employing the phenomenon of channel polarization. Through channel polarization, it is possible to create additional $N$ binary-input channels $\{I(W_N^{(i)}):1 \leq i \leq N\}$ out of $N$ independent copies of a B-DMC. From the channel polarization process, as $N$ approaches infinity, two sets of channels are obtained; one set for which $I(W_N^{(i)})$ is near 1 and therefore approaches $I(W)$ and another set for which $I(W_N^{(i)})$ is near 0 and therefore approaches $1 - I(W)$. Useful information can then be sent only through channels for which $I(W_N^{(i)})$ is near 1. For the channels for which $I(W_N^{(i)})$ is near 0 it can be used for predetermined bits known to both the sender and the receiver. Therefore in polar codes, there are two categories of bits: information bits that carry useful information and fixed or frozen bits.

For the transmission of an information vector, $u$, over a B-DMC, a polar code $(N, K, A, u^A)$ can be specified where $N$ is the block length, $K$ the information size, $A$ the information set, and $u^A$ the frozen bit set. Polar encoding takes place through a linear transformation of information bits, $u_i^N$, to coded bits, $x_i^N$. As in [2] the polar encoding process can be expressed as:

$$x_i^N = u_i^N G_N$$  

where $G_N$ denotes the generator matrix of order $N$ and is derived from the $n^{th}$ Kronecker power of the kernel matrix $F_2$. $F_2$ can also be expressed in standard form as:

$$F_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

When the fixed and frozen bits are taken into consideration, equation (1) can be rewritten as:

$$x_i^N = u_A^N G_N (A) \oplus u_A^N G_N (A^c)$$  

where $G_N(A)$ is a submatrix of $G_N$ formed by the rows with the indices in $A$.

For SSC, local decoding at rate-one nodes (all descendant leaf nodes for rate-one nodes are information bits) are simplified. The decoding process can be analyzed using a binary code tree at node $w$, with depth $d_w$ and parent $p_w$. Node $w$ serves as the local decoder for its constituent code. During the message passing process node $w$ receives soft information vector, $\alpha_w$, from its parent, $p_w$, and produces a codeword $\beta_w$. However, for rate-one nodes, the evaluation of the output of the binary quantizer
is done directly as $\beta_w = h(\alpha_w)$. Secondly, all the bits that are related to the node, $w$, as its children or descendants are also computed immediately by applying the formula:

$$
(u[\min \gamma_w], \ldots, u[\max \gamma_w]) = \beta_w G_n - d_w
$$

where $\gamma_w$ is the set containing the indices of all leaf nodes that are descendants of node $w$ and $G_n$ is the generator matrix for node $w$.

**Proposed Perturbed Decoding Algorithm**

The block diagram of the proposed decoding algorithm is shown in Figure 1. The system is a concatenated coding system which consists of an outer CRC encoder and an inner SSC decoder. Information bits are sent over the transmission channel to the intended receiver by means of polar codes.

![Block diagram of the proposed perturbed decoding algorithm.](image)

Figure 1. Block diagram of the proposed perturbed decoding algorithm.

Firstly, a CRC sequence is generated out of the information bits. Then, polar encoding is performed on the CRC sequence and the coded sequence is sent over the channel. At the receiver, an SSC decoder generates a decoded sequence. The decoded sequence is then validated by the CRC decoder. If the decoded sequence is a valid CRC sequence then the decoding process is successful. However, if CRC fails, then the received signal is perturbed by the addition of secondary independent random noise and the perturbed received signal is again decoded by the SSC decoder. This perturbation process, which is iterative, continues until a valid CRC sequence is obtained or another set termination condition, the maximum threshold, is achieved. In this decoding scheme, CRC is employed as part of the concatenated coding scheme as it enables the determination of a correct codeword thereby validating the output of the SSC decoder. With CRC there is easy detection of errors, allowing for a convenient termination of the decoding process [12].
For an information bit sequence, \( k \), a CRC bit stream is generated as \( r^N_1 = (r_1,r_2,\ldots,r_N) \). The CRC bit sequence, \( r^N_1 \), under goes polar encoding as \( x^N_1 = (x_1,x_2,\ldots,x_N) \). The list of valid codewords according to the CRC sequence can be specified as \( c = (c_1,c_2,\ldots,c_M) \) for a total of \( M \) codewords. At the receiver a corresponding sequence, \( y^N_1 = (y_1,y_2,\ldots,y_N) \) is obtained. This received signal is fed to the SSC decoder which generates \( d^N_1 = (d_1,d_2,\ldots,d_N) \) as its output sequence. The decoded sequence is then validated by the CRC decoder. If CRC is passed, the decoding process is successful, however, if CRC fails then the perturbed decoding algorithm is initiated. Here, the received signal is perturbed by the addition of secondary independent random noise. If \( y \) is taken as the received signal, then the perturbed signal, \( y_p \), can be expressed as:

\[
y_p = y + n
\]

where \( n \), the perturbation noise added, is assumed to be an i.d.d zero mean Gaussian random variable with variance \( \sigma^2 \). After SSC decoding, the decoded sequence \( d \), of the perturbed received signal \( y_p \) is then validated by the CRC decoder. The process is repeated and only terminates when a valid codeword is found or the threshold for maximum iterations for the algorithm, \( T \), is obtained. The proposed perturbed decoding algorithm is presented in Algorithm 1.

The complexity of the proposed perturbed decoding algorithm can be assessed by the additional computation or the extra computational overhead required compared to the existing SSC decoder. The main additional computation for the perturbed decoding algorithm can be found in the generation of the random perturbation noises. In the worst case scenario, where the maximum threshold is reached before a valid codeword is found, the additional complexity can be expressed as \( O(T) \), where \( T \) is the maximum threshold. However, this additional computation only comes about when a decoded sequence fails the CRC; therefore the frequency of operation is minimal especially at a high SNR regime. The effect on the overall average decoding complexity occasioned by the proposed scheme is not significant.

Algorithm 1: Perturbed Decoding Algorithm

1. \( d \leftarrow \text{SSC}(y) \) \text{ } / \text{ } \text{Conventional SSC Decoding}
2. \( \text{if } \text{CRC}(d) \not\in c \text{ then } \text{CRC} \)
3. \( \text{for } i = 1, \ldots, T \text{ do} \)
4. \( y_p \leftarrow y + n \) \text{ } / \text{ } \text{Addition of Perturbation Noise}
5. \( d_s \leftarrow \text{SSC}(y_p) \) \text{ } / \text{ } \text{SSC Decoding of Perturbed Signal}
6. \( \text{if } \text{CRC}(d_s) \in c \text{ then} \)
7: \text{return} \ d_s \text{ as codeword}

8: \text{terminate} \text{ Perturbed Decoding Algorithm}

9: \text{end if}

10: \text{end for}

11: \text{end if}

NUMERICAL SIMULATIONS

The proposed perturbed decoding algorithm is verified by numerical simulations in this section. The parameters and associated values used in the simulations are presented in Table I. A (1024, 512) polar code is used, an AWGN channel is assumed for the simulations and BPSK modulation is applied. The perturbation noise level that was found optimal after experimentation is 0.25.

Figure 2 shows the BER performance of the proposed perturbed decoding algorithm and the SSC decoding algorithm. As can be seen from the figure, the proposed algorithm exhibits improved performance over the existing SSC decoding algorithm. At a BER of $10^{-4}$ there is a 0.40dB performance gain with the proposed algorithm. In Figure 3, the FER performance of the proposed decoding algorithm and the SSC decoding algorithm are shown against each other. At an FER of $10^{-3}$ there is a 0.41dB performance gain for the proposed perturbed decoding algorithm as far as the FER performance is concerned.

An SSC decoder is used as the conventional decoder in this study as it offers reduced algorithmic complexity compared to the original SC decoder of polar codes. The proposed decoder maintains the lower algorithmic complexity of SSC decoder while improving the performance by adding secondary independent noise to produce other highly likely sequences.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rate of Polar Code</td>
<td>0.5</td>
</tr>
<tr>
<td>Code Length</td>
<td>1024</td>
</tr>
<tr>
<td>Decoding Algorithm of Polar Code</td>
<td>SSC</td>
</tr>
<tr>
<td>Modulation</td>
<td>BPSK</td>
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<tr>
<td>Mode of Transmission</td>
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<tr>
<td>Perturbation Noise Level</td>
<td>0.25</td>
</tr>
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</table>
CONCLUSION

This paper considered the improvement of the decoding performance of the SSC decoder. A CRC-aided perturbed decoding algorithm was proposed. It makes available multiple likely codeword candidates in the event of a failed CRC from the output of an SSC decoder. A perturbed decoding algorithm is applied in generating multiple codeword candidates from which a valid CRC sequence may be found. The simulation results show improved BER and FER performance over existing techniques such as the SSC decoding algorithm. There is at least a 0.4dB performance gain for the proposed scheme compared to existing schemes. The additional computation from the proposed algorithm has little effect on the overall average decoding complexity. It is a practical scheme that is applicable to most concatenating decoding algorithms.
Figure 3. FER performance.

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REFERENCES

4. Chairman’s Notes 2016. “3GPP TSG RAN WG1 Meeting no. 87, Chairman’s Notes of Agenda Item 7.1.5 Channel Coding and Modulation.”